RATE OPTIMIZATION FOR MASSIVE MIMO RELAY NETWORKS: A MINORIZATION-MAXIMIZATION APPROACH

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ABSTRACT

We consider the problem of sum-rate maximization in massive MIMO two-way relay networks with multiple (communication) operators employing the amplify-and-forward (AF) protocol. The aim is to design the relay amplification matrix (i.e., the *relay beamformer*) to maximize the achievable communication sum-rate through the relay. The design problem for the case of single-antenna users can be cast as a non-convex optimization problem, which in general, belongs to a class of NP-hard problems. We devise a method based on the minorization-maximization technique to obtain quality solutions to the problem. Each iteration of the proposed method consists of solving a strictly convex unconstrained quadratic program; this task can be done quite efficiently such that the suggested algorithm can handle the beamformer design for relays with up to ~ 70 antennas within a few minutes on an ordinary PC. Such a performance lays the ground for the proposed method to be employed in massive MIMO scenarios.

Index Terms— Beamforming, minorization-maximization, massive MIMO, relay networks, sum-rate.

1. INTRODUCTION

Sum-rate maximization is a fundamental task arising in signal design for communication, and particularly relay networks, in which relays are often used to enhance the quality of communication between pairs of users within the network. In such networks, two-way relaying is shown to achieve better spectral efficiency as compared to one-way relaying [1]. Various protocols including decode-and-forward (DF), and amplify-and-forward (AF) have been proposed in the literature for two-way relay networks [2, 3]. Contrary to the DF case, the AF relaying does not perform any signal decoding at the relay, and hence enjoys a lower hardware and software complexity, as well as smaller transmission delay. As a result of such simple processing requirement, AF relaying is a more suitable scheme for large-scale or massive MIMO systems.

Note that the sum-rate of a MIMO relay system depends on the amplification matrix, i.e. the *beamformer* of the relay. However, an optimal design of the beamformer leads to a non-convex (in general NP-hard [1]) optimization problem. The authors of [1] developed a polynomial-time iterative method based on a semidefinite relaxation (referred to as POTDC) to tackle the problem. POTDC guarantees a rank-one solution only for the special case of single (communication) operator and hence, its solution is generally associated with a synthesis loss. Furthermore, each iteration of POTDC consists of solving a convex MAXDET optimization that has a large computational burden. On the other hand, POTDC results outperform those obtained by the approximate (projection-based) algorithm suggested in [4]. Additionally, [1] includes heuristic algorithms based on one/two dimensional search for the case with single operator.

In the case of an arbitrary number of operators, the literature does not offer efficient methods that can lead to (some strong type of) optimality of the obtained solutions. Furthermore, most of the proposed methods in the literature are merely suitable for small scale problems (see e.g. [1, 5]). In this paper, the problem is considered in a rather general form enabling the user to freely choose the number of operators L and the structure of the associated matrices (i.e., the channel parameters). We devise an iterative method based on the minorization-maximization technique to tackle the design problem. Applying the proposed method increases the value of the objective function at each iteration. Consequently, it can be shown that the obtained solution is a stationary point of the problem for arbitrary L. The proposed method is computationally efficient and hence can be applied to large-scale MIMO systems[1] (with M_R antennas). Indeed, each iteration of the devised method consists of solving a convex unconstrained quadratic program (QP); which can be performed efficiently for instance with an $\mathcal{O}(n^{2.3})$ complexity (where n is the problem dimension, $n = M_R^2$) [6]. As a result, the

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[1]This paper can address the beamforer design problem in large-scale scenarios where the near optimality of zero-forcing does not hold, e.g., low-middle regime massive MIMO systems.

method can handle problems with $n \sim 10^3$ variables (i.e., $M_R \sim 70$) on an ordinary PC within a few minutes.

2. PROBLEM FORMULATION

We consider a MIMO AF two-way relay network consisting of M_R antennas, L operators and pairs of user terminals. We assume single-antenna user terminals and flat fading channels between the k^{th} user of the l^{th} operator and the relay, which are denoted by $\{\mathbf{h}_{k,l}\}$ [1]. The received signal at the relay can be expressed as [1, 4],

$$\mathbf{r} = \sum_{l=1}^{L}\sum_{k=1}^{2} \mathbf{h}_{k,l} x_{k,l} + \mathbf{n}_R \quad (1)$$

where $x_{k,l}$ is the transmitted symbol by the k^{th} user of the l^{th} operator with power $p_{k,l}$ (given by $\mathrm{E}\{|x_{k,l}|^2\}$), and $\mathbf{n}_R$ denotes the circularly symmetric white Gaussian noise with covariance matrix $\sigma_R^2\mathbf{I}$ at the relay. By employing the AF protocol, the transmit signal of the relay is given by $\widetilde{\mathbf{r}} = \mathbf{G}\mathbf{r}$ with $\mathbf{G} \in \mathbb{C}^{M_R \times M_R}$ being the relay amplification matrix, which is to be designed. We assume reciprocal channels between the relay and users [4]; thus, the received signal $y_{k,l}$ of the k^{th} user at the l^{th} operator becomes

$$y_{k,l} = \mathbf{h}_{k,l}^T \widetilde{\mathbf{r}} + n_{k,l} \quad (2)$$

where $n_{k,l}$ is the associated (white) noise component (with variance $\sigma_{k,l}^2$) and $(.)^T$ stands for transpose. The sum-rate of the system can be formulated as [4]

$$R_{sum} = \frac{1}{2}\sum_{l=1}^{L}\sum_{k=1}^{2}\log_2(1+\eta_{k,l}). \quad (3)$$

Herein $\eta_{k,l}$ denotes the signal-to-interference-plus-noise ratio (SINR) for the k^{th} user of the l^{th} operator and it has the following expression [4]

$$\eta_{k,l} = \frac{\mathbf{g}^{\mathrm{H}}\boldsymbol{\Phi}_{k,l}\mathbf{g}}{\mathbf{g}^{\mathrm{H}}(\boldsymbol{\Upsilon}_{k,l}+\boldsymbol{\Delta}_{k,l})\mathbf{g}+\sigma_{k,l}^2} \quad (4)$$

where $\mathbf{g} = \mathrm{vec}(\mathbf{G})$ that vec(.) operator stacks the columns of a matrix into a vector, $(.)^H$ stands for Hermitian transpose, and the matrices $\boldsymbol{\Phi}_{k,l}, \boldsymbol{\Upsilon}_{k,l}, \boldsymbol{\Delta}_{k,l}$ are defined as

$$\boldsymbol{\Phi}_{k,l} = p_{k,l}\left(\mathbf{h}_{3-k,l}^T \otimes \mathbf{h}_{k,l}^T\right)^H \left(\mathbf{h}_{3-k,l}^T \otimes \mathbf{h}_{k,l}^T\right) \quad (5)$$

$$\boldsymbol{\Upsilon}_{k,l} = \sum_{\widetilde{k}}\sum_{\widetilde{l}\neq l} p_{\widetilde{k},\widetilde{l}}\left(\mathbf{h}_{\widetilde{k},\widetilde{l}}^T \otimes \mathbf{h}_{k,l}^T\right)^H \left(\mathbf{h}_{\widetilde{k},\widetilde{l}}^T \otimes \mathbf{h}_{k,l}^T\right)$$

$$\boldsymbol{\Delta}_{k,l} = \sigma_R^2\left(\mathbf{I}_{M_R} \otimes (\mathbf{h}_{k,l}\mathbf{h}_{k,l}^T)\right).$$

The sum-rate maximization is constrained via the total available power P_R at the relay, viz.

$$\begin{aligned}\mathrm{E}\{\|\widetilde{\mathbf{r}}\|_2^2\} &= \mathrm{tr}\{\mathrm{E}\{\mathbf{G}\mathbf{r}\mathbf{r}^H\mathbf{G}^H\}\} \quad (6)\\ &= \sum_{l=1}^{L}\sum_{k=1}^{2} p_{k,l}\|\mathbf{G}\mathbf{h}_{k,l}\|_2^2 + \sigma_R^2\|\mathbf{G}\|_F^2 \leq P_R\end{aligned}$$

wherein $\|.\|_2$ and $\|.\|_F$ denote the Euclidean norm of the vector and the Frobenius norm of the matrix arguments, respectively. The latter equation can be expressed with respect to (w.r.t.) $\mathbf{g}$ as $\mathbf{g}^H\mathbf{C}\mathbf{g} \leq P_R$ where

$$\mathbf{C} = \sigma_{\mathrm{R}}^2\mathbf{I}_{M_{\mathrm{R}}^2} + \sum_{l=1}^{L}\sum_{k=1}^{2} p_{k,l}((\mathbf{h}_{k,l}\mathbf{h}_{k,l}^{\mathrm{H}})^{\mathrm{T}} \otimes \mathbf{I}_{M_{\mathrm{R}}}). \quad (7)$$

Therefore, the design problem (i.e., sum-rate maximization) in MIMO AF relay networks with L operators can be cast as

$$\begin{aligned}\max_{\mathbf{g}} \quad & \frac{1}{2}\sum_{l=1}^{L}\sum_{k=1}^{2}\log_2\left(1+\frac{\mathbf{g}^{\mathrm{H}}\boldsymbol{\Phi}_{k,l}\mathbf{g}}{\mathbf{g}^{\mathrm{H}}(\boldsymbol{\Upsilon}_{k,l}+\boldsymbol{\Delta}_{k,l})\mathbf{g}+\sigma_{k,l}^2}\right)\\ \text{s. t.} \quad & \mathbf{g}^H\mathbf{C}\mathbf{g} \leq P_R. \quad (8)\end{aligned}$$

Note that the inequality constraint in the above problem is active (i.e. satisfied with equality) at the optimal point. More precisely, assume that $\mathbf{g}$ is an optimal solution to (8) with $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_0 < P_R$. Then a scaled version of $\mathbf{g}$ which satisfies the constraint with equality, i.e. $\mathbf{g}_1 = \sqrt{P_R/P_0}\,\mathbf{g}$, will lead to a larger objective value which is a contradiction.

3. SUM-RATE MAXIMIZATION

The aim is to design the AF amplification matrix $\mathbf{G}$ in order to maximize the sum-rate R_{sum}. Considering the fact that the inequality constraint in (8) is satisfied with equality at the optimal solution, the optimization in (8) can be recast as

$$\max_{\mathbf{g}} \quad \sum_{l=1}^{L}\sum_{k=1}^{2}\Big[\log\left(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}\right) - \log\left(\mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}\right)\Big] \quad (9)$$

where we have used the following definitions:

$$\mathbf{B}_{k,l} = \boldsymbol{\Upsilon}_{k,l} + \boldsymbol{\Delta}_{k,l} + \frac{\sigma_{k,l}^2}{P_R}\mathbf{C}, \ \mathbf{A}_{k,l} = \mathbf{B}_{k,l} + \boldsymbol{\Phi}_{k,l} \quad (10)$$

The above optimization problem is non-convex and belongs to a class of NP-hard problems in general [1]. Note that the objective function of (9) is invariant with respect to scaling; therefore, we can deal with the unconstrained problem and then scale the solution $\mathbf{g}$ such that it satisfies the constraint $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$. In this paper, we use the minorization-maximization technique to tackle the non-convex design problem formulated in (9). Minorization-maximization (MaMi) is an iterative technique that can be used for obtaining a solution to the general maximization problem [7, 8]:

$$\max_{\mathbf{z}} \ \widetilde{f}(\mathbf{z}) \ \ \textit{subject to} \ \ c(\mathbf{z}) \leq 0. \quad (11)$$

Each iteration of MaMi consists of two steps:

- Minorization Step: Finding $\widetilde{p}^{(\kappa)}(\mathbf{z})$ such that its maximization is simpler than that of $\widetilde{f}(\mathbf{z})$ and $\widetilde{p}^{(\kappa)}(\mathbf{z})$ minorizes $\widetilde{f}(\mathbf{z})$, i.e.,

$$\widetilde{p}^{(\kappa)}(\mathbf{z}) \leq \widetilde{f}(\mathbf{z}), \ \forall\mathbf{z}, \ \widetilde{p}^{(\kappa)}\left(\mathbf{z}^{(\kappa-1)}\right) = \widetilde{f}\left(\mathbf{z}^{(\kappa-1)}\right)$$

with $\mathbf{z}^{(\kappa-1)}$ being the value of $\mathbf{z}$ at the $(\kappa-1)^{th}$ iteration.

- Maximization Step: Solving the following optimization problem to obtain $\mathbf{z}^{(\kappa)}$:

$$\max_{\mathbf{z}} \; \widetilde{p}^{(\kappa)}(\mathbf{z}) \;\; subject \; to \;\; c(\mathbf{z}) \leq 0$$

Note that the following inequality holds due to the concavity of $\log(x)$ for all $x, x_0 \in \mathbb{R}^+$:

$$\log(x) \leq \log(x_0) + \frac{1}{x_0}(x - x_0). \tag{12}$$

Setting $x_0 = \mathbf{g}_0^H \mathbf{B}_{k,l}\mathbf{g}_0$ and $x = \mathbf{g}^H \mathbf{B}_{k,l}\mathbf{g}$ leads to a minorizer for $-\log(\mathbf{g}^H \mathbf{B}_{k,l}\mathbf{g})$. By substituting the minorizer into (9), we have the following maximization problem at the $(\kappa+1)^{th}$ iteration:

$$\max_{\mathbf{g}} \; \sum_{l=1}^{L}\sum_{k=1}^{2}\left[\log(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g}) - \frac{1}{(\mathbf{g}^{(\kappa)})^H \mathbf{B}_{k,l}\mathbf{g}^{(\kappa)}}\; \mathbf{g}^H\mathbf{B}_{k,l}\mathbf{g}\right]. \tag{13}$$

Inspired by the rich literature on semidefinite relaxation, we note that by considering $\mathbf{X} = \mathbf{g}\mathbf{g}^H$ as the optimization variable in (13) and dropping the rank-1 constraint, a convex alternative of (13) can be obtained at each iteration. However, there is no guarantee for a rank-1 solution $\mathbf{X}$, and hence, this approach is associated with a synthesis loss. In addition, applying the relaxation leads to iteratively solving a MAXDET problem possessing a high computational burden. Instead, in the sequel, we devise a computationally efficient method that increases the objective value at each iteration and guarantees the first-order optimality condition for the solution $\mathbf{g}$. To this end, we proceed by finding a minorizer for the term $\log(\mathbf{g}^H\mathbf{A}_{k,l}\mathbf{g})$ as a function of $\mathbf{g}$ using the following lemma (whose proof is omitted for the sake of brevity).

Lemma 1. *Let $s(\mathbf{x}) = -\log(\mathbf{x}^H\mathbf{T}\mathbf{x})$ and $\mathbf{x}^H\mathbf{C}\mathbf{x} = P$ for positive-definite matrices $\mathbf{T}, \mathbf{C}$ in $\mathbb{C}^{N\times N}$, and $P \in \mathbb{R}^+$. Then, the following inequality holds $\forall\, \mathbf{x}, \mathbf{x}_0$:*

$$s(\mathbf{x}) \leq s(\mathbf{x}_0) + \Re\left(\mathbf{b}^H(\mathbf{x}-\mathbf{x}_0)\right) + (\mathbf{x}-\mathbf{x}_0)^H\mathbf{U}(\mathbf{x}-\mathbf{x}_0)$$

where $\mathbf{b} = \left(\frac{-2}{\mathbf{x}_0^H\mathbf{T}\mathbf{x}_0}\right)\mathbf{T}\mathbf{x}_0$, $\mathbf{U} = \left(\frac{4P}{\mathbf{w}_1^H\mathbf{C}\mathbf{w}_1} + \epsilon\right)\mathbf{I}$, $\mathbf{w}_1$ is the principal eigenvector of $\mathbf{T}$, and $\epsilon > 0$ being an arbitrary scalar.

Assume that $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$ at each iteration (see Remark 1 below). Now by using Lemma 1 for minorizing the objective of (13), the following unconstrained QP will be obtained:

$$\min_{\mathbf{g}} \quad \mathbf{g}^H\mathbf{Q}^{(\kappa)}\mathbf{g} + \Re\left(\left(\mathbf{q}^{(\kappa)}\right)^H \mathbf{g}\right) \tag{14}$$

where

$$\mathbf{Q}^{(\kappa)} = \sum_{l=1}^{L}\sum_{k=1}^{2}\left[\frac{\mathbf{B}_{k,l}}{\left(\mathbf{g}^{(\kappa)}\right)^H\mathbf{B}_{k,l}\mathbf{g}^{(\kappa)}} + \mathbf{U}_{k,l}\right], \tag{15}$$

$$\mathbf{q}^{(\kappa)} = \sum_{l=1}^{L}\sum_{k=1}^{2}\left[\mathbf{b}_{k,l} - 2\mathbf{U}_{k,l}\mathbf{g}^{(\kappa)}\right],$$

$$\mathbf{b}_{k,l} = \left(\frac{-2}{\left(\mathbf{g}^{(\kappa)}\right)^H\mathbf{A}_{k,l}\mathbf{g}^{(\kappa)}}\right)\mathbf{A}_{k,l}\mathbf{g}^{(\kappa)}, \tag{16}$$

$$\mathbf{U}_{k,l} = \left(\frac{4P_R}{\widetilde{\mathbf{w}}_{k,l}^H\mathbf{C}\widetilde{\mathbf{w}}_{k,l}} + \epsilon\right)\mathbf{I}$$

and $\widetilde{\mathbf{w}}_{k,l}$ denotes the principal eigenvector of $\mathbf{A}_{k,l}$. Note that $\mathbf{B}_{k,l} \succeq \mathbf{0}$, and also, $\mathbf{U}_{k,l} \succ \mathbf{0}$ as it is a scaled version of identity matrix $\mathbf{I}$ with a positive scalar. Therefore, the matrix $\mathbf{Q}^{(\kappa)}$ is positive-definite at each iteration. Consequently, the problem in (14) is strictly convex w.r.t. $\mathbf{g}$. The unique solution to this optimization is obtained by solving the system of linear equations $2\mathbf{Q}^{(\kappa)}\mathbf{g} + \mathbf{q}^{(\kappa)} = \mathbf{0}$, viz.

$$\mathbf{g} = -\frac{1}{2}\left(\mathbf{Q}^{(\kappa+1)}\right)^{-1}\mathbf{q}^{(\kappa)}. \tag{17}$$

Remark 1: Note that the above solution $\mathbf{g}$ does not necessarily satisfy the constraint $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$ of the original problem (9) at each iteration. As mentioned before, we can scale the obtained solution at the convergence to deal with this issue as the objective function in (9) is scale invariant. However, the derivation of the matrix $\mathbf{U}_{k,l}$ in Lemma 1 requires the satisfaction of the constraint at each iteration. Therefore, we need to scale the obtained $\mathbf{g}$ at each iteration such that $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$. Note also that the scaling does not affect the convergence of the sequence of the objective function values.

Table 1 summarizes the steps of the proposed method for relay beamformer design to maximize the communication sum-rate. The suggested method improves the value of the sum-rate at each iteration. As a result, employing the proposed method will lead to the convergence of the network sum-rate value due to the upper boundedness of the sum-rate metric (see [7–9] and references therein for details of the convergence of MaMi technique).

4. SIMULATIONS

In this section, the performance of the proposed method is evaluated via Monte-Carlo simulations. An AF based bidirectional MIMO relay network with L operators and M_R antennas at the relay is considered. The variances of the Gaussian noises for the relay and users are assumed to be equal, i.e., $\sigma_R^2 = \sigma_{k,l}^2 = \sigma_n^2$. We assume that the transmit powers of the relay and users are identical, i.e., $P_R = p_{k,l} = p$. The SNR is defined as p/σ_n^2. Moreover, the normalized distance

Table 1: Relay Beamformer Design Algorithm

Step 0: Initialize $\mathbf{g}$ with a random vector in $\mathbb{C}^{M_R^2}$ (and scale it such that $\mathbf{g}^H\mathbf{C}\mathbf{g} = P_R$); set $\kappa = 0$.
Step 1: Compute $\mathbf{Q}^{(\kappa)}$ and $\mathbf{q}^{(\kappa)}$ using (15).
Step 2: Solve the convex problem in (14) using either the closed-from expression (17) or the direct methods to obtain $\mathbf{g}^{(\kappa+1)}$.
Step 3: Scale the obtained solution $\mathbf{g}^{(\kappa+1)}$ such that $(\mathbf{g}^{(\kappa+1)})^H\mathbf{C}\mathbf{g}^{(\kappa+1)} = P_R$; set $\kappa \leftarrow \kappa + 1$.
Step 5: Repeat steps 1-3 until a pre-defined stop criterion is satisfied, e.g. $|f^{(\kappa+1)} - f^{(\kappa)}| \leq \xi$ (where f denotes the objective function of the problem (9)) for some $\xi > 0$.

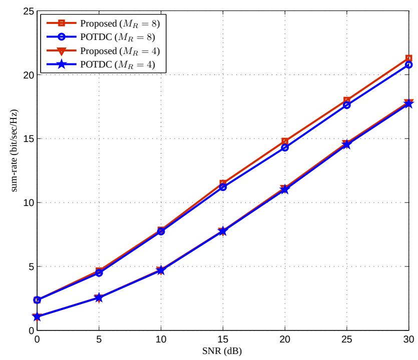

Fig. 1: The sum-rate values associated with the proposed method and POTDC [1] versus SNR for $L = 2$.

between k^{th} user of the l^{th} operator and the relay is represented by $d_{k,l}$. For simplicity and without loss of generality, we assume that $d_{1,l} = d_1$ and $d_{2,l} = d_2$ (with $d_1 + d_2 = 1$). Therefore, the near-far (N/F) ratio is defined as d_1/d_2. The Rayleigh flat fading channel vectors $\{\mathbf{h}_{k,l}\}$ are reciprocal and spatially uncorrelated and the path loss exponent is assumed to be 3 in all simulations. All the results are presented considering 100 realizations of the associated fading channels. We begin by investigating the effect of the SNR on the sum-rate in a symmetric scenario (i.e., $d_1 = d_2$). The sum-rate values associated with the proposed method as well as the POTDC method of [1] (which is dealt with via CVX toolbox [10]) versus SNR are shown in Fig. 1 for $M_R = 4$ and $M_R = 8$ with $L = 2$. As expected, the sum-rate is increasing with respect to SNR. Furthermore, the results of the proposed method are slightly better than those of the method in [1] because the proposed method circumvents the synthesis loss associated with POTDC. Next, we study the effect of the N/F ratio. Fig. 2 plots the sum-rate values versus different N/F ratios ($L = 2$). The proposed method achieves better results in the whole interval of the N/F ratio. Moreover, Fig. 1 and Fig. 2 show that a larger number of antennas M_R leads to a larger sum-rate value of the network—as expected. The computational times of both methods are investigated in Fig. 3, which plots the average computational times by considering 10 runs of the

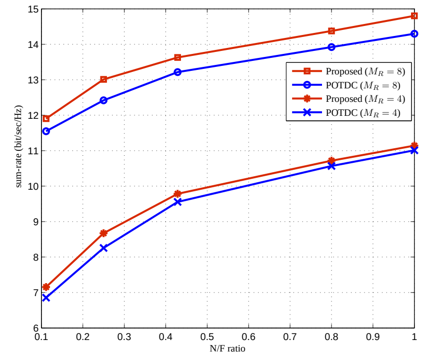

Fig. 2: The sum-rate values versus N/F ratio for $L = 2$ and SNR=20dB.

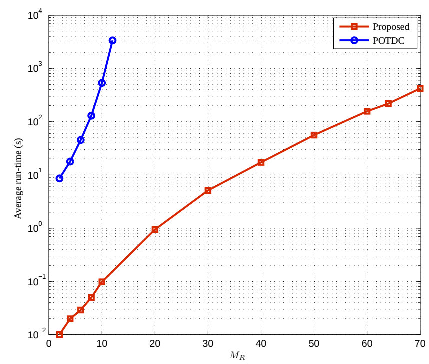

Fig. 3: The average run-time (s) versus the number of antennas M_R for the case of $L = 2$.

methods with random initializations on an ordinary PC (with 8GB RAM and CPU CoRe i5). It can be observed that the proposed method exhibits a low computational cost compared to its rival (note that the values for POTDC correspond to 10 iterations). The presented computational results illustrate the applicability of the proposed method to currently available prototypes of massive MIMO (e.g. Argos [11]).

5. CONCLUSION

The problem of relay beamformer design for sum-rate maximization in massive MIMO AF relay networks was considered. An iterative method based on the minorization-maximization (MaMi) technique was devised to deal with the design problem. The proposed method provides quality solutions to the design problem for an arbitrary number of operators L. Numerical examples confirmed the effectiveness of the proposed method when compared to other methods in terms of the solution quality and the computational efficiency.

6. REFERENCES

[1] A. Khabbazibasmenj, F. Roemer, S. Vorobyov, and M. Haardt, "Sum-rate maximization in two-way AF MIMO relaying: Polynomial time solutions to a class of DC programming problems," *IEEE Transactions on Signal Processing*, vol. 60, no. 10, pp. 5478–5493, October 2012.

[2] B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," in *IEEE International Symposium on Information Theory (ISIT)*, Seattle, USA, 2006, pp. 1668–1672.

[3] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Transactions on Wireless Communications*, vol. 6, no. 4, pp. 1398–1407, 2007.

[4] J. Zhang, F. Roemer, and M. Haardt, "Relay assisted physical resource sharing: Projection based separation of multiple operators (ProBaSeMO) for two-way relaying with MIMO amplify and forward relays," *IEEE Transactions on Signal Processing*, vol. 60, no. 9, pp. 4834–4848, September 2012.

[5] F. Roemer and M. Haardt, "Algebraic norm-maximizing (ANOMAX) transmit strategy for two-way relaying with MIMO amplify and forward relays," *IEEE Signal Processing Letters*, vol. 16, no. 10, pp. 909–912, October 2009.

[6] A. M. Davie and A. J. Stothers, "Improved bound for complexity of matrix multiplication," *Proceedings of the Royal Society of Edinburgh: Section A Mathematics*, vol. 143, no. 02, pp. 351–369, 2013.

[7] P. Stoica and Y. Selen, "Cyclic minimizers, majorization techniques, and the expectation-maximization algorithm: a refresher," *IEEE Signal Processing Magazine*, vol. 21, no. 1, pp. 112 – 114, January 2004.

[8] M. M. Naghsh, M. Modarres-Hashemi, S. Shahbaz-Panahi, M. Soltanalian, and P. Stoica, "Unified optimization framework for multi-static radar code design using information-theoretic criteria," *IEEE Transactions on Signal Processing*, vol. 61, pp. 5401–5416, November 2013.

[9] M. M. Naghsh, M. Modarres-Hashemi, A. Sheikhi, M. Soltanalian, and P. Stoica, "Unimodular code design for MIMO radar using Bhattacharyya distance," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2014, pp. 5282–5286.

[10] M. Grant and S. Boyd. (2012, February) CVX package. [Online]. Available: http://www.cvxr.com/cvx

[11] C. Shepard, H. Yu, N. Anand, E. Li, T. Marzetta, R. Yang, and L. Zhong, "Argos: Practical many-antenna base stations," in *Proceedings of the 18th annual international conference on Mobile computing and networking*. ACM, 2012, pp. 53–64.