NETWORK TOPOLOGY ADAPTATION AND INTERFERENCE COORDINATION FOR ENERGY SAVING IN HETEROGENEOUS NETWORKS

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ABSTRACT

Interference coupling in heterogeneous networks introduces the inherent non-convexity to the network resource optimization problem, hindering the development of effective solutions. A new framework based on multi-pattern formulation has been proposed in this paper to study the energy efficient strategy for joint cell activation, user association and multicell multiuser channel allocation. One key feature of this interference pattern formulation is that the patterns remain fixed and independent of the optimization process. This creates a favorable opportunity for a linear programming formulation while still taking interference coupling into account. A tailored algorithm is developed to solve the formulated network energy saving problem in the dual domain by exploiting the problem structure, which gives a significant complexity saving compared to using standard solvers. Numerical results show a huge improvement in energy saving achieved by the proposed scheme.

Index Terms— cell activation, user association, power minimization, interference coordination, cutting plane methods

1. INTRODUCTION

The densification and expansion of wireless networks pose new challenges on interference management and reducing energy consumption. In a dense heterogeneous network (HetNet), base stations (BSs) are typically deployed to satisfy the peak traffic volume and they are expected to have low activity outside rush hours such as nighttime. There is a high potential for energy saving if BSs can be switched off according to the traffic load.

Obviously, cell activation is coupled with user association: the users in the muted cells must be re-associated with other BSs. In addition, cell muting and user re-association impose further challenges on interference management, since the user may not be connected to the BS with the strongest signal strength. This interference issue can be resolved by interference coordination, i.e., properly sharing the channels among multiple cells and then distributing them to the associated users in each cell. Hence, to obtain energy-efficient resource management strategies, multicell multiuser channel assignment should be integrated into the optimization of the cell activation and user association.

However, the resource management that considers the above elements jointly is very challenging mathematically because the intercell interference coupling leads to the inherent non-convexity in the optimization problems. To make the problems tractable, the previous

studies relied on worst-case interference assumption [1, 2], average interference assumption [3, 4], or neglecting inter-cell interference [5]. In these works, the interference was assumed *static* (or absent), i.e., independent of the resource allocation decisions in each cell, when estimating the user achievable rate. Clearly, this is a suboptimal design because the BS deactivation will cause interference fluctuation in the network, hence affecting the user rate.

This paper is developing a new framework for energy-efficient resource management to consider the interference coupling caused by cell deactivation. The idea is to pre-calculate the user rate under each possible *interference pattern* (i.e. an interference scenario in the network, described as one combination of ON/OFF activities of the BSs), and then perform resource allocation among these patterns. This allocation yields the actual interference and the corresponding user achievable rates that well match the interference at the same time.

2. SYSTEM MODEL

Consider a downlink HetNet, where a number of small cells are embedded in the conventional macro cellular network. The set of all (macro and small) cells is denoted by $\mathcal{B} = \{1, 2, \dots, B\}$. The cells can be switched on or off every time period T (say, many minutes). In this relatively long decision period, we adopt test points as an abstract concept to represent demands of users [2]. The test points can be chosen from typical user locations, or we can simply partition the geographic region into pixels and then each pixel becomes one test point. The set of test points is denoted by $\mathcal{K} = \{1, 2, \dots K\}$ (In our model, each test point can represent multiple co-located users). The traffic demand of each test point $k \in \mathcal{K}$ is represented by a minimum required average rate d_k during one period of T, which is assumed known via traffic estimation algorithms. We are interested in developing adaptive strategies for every period of T to accommodate the traffic requirement with minimum network energy consumption, taking into account the inter-cell interference coupling.

The enabling mechanism is to characterize the interference by specifying the interference patterns, each of which defines a particular ON/OFF combination of BSs. We use the pattern activity vector $\mathbf{a}_i = (a_{i1}, a_{i2}, \cdots, a_{iB})^T$ to indicate the ON/OFF activity of the BSs under pattern i, where

$$a_{ib} = \begin{cases} 1 & \text{if BS } b \text{ is ON under pattern } i \\ 0 & \text{otherwise} \end{cases}$$
 (1)

We denote the set of pre-defined patterns by $\mathcal{I} = \{1, 2, \cdots, I\}$ and further define the matrix $\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_I)$ to combine the activity vectors for all candidate patterns. In order to fully characterize

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the interference scenarios in a network of B cells, generally speaking, 2^B patterns are needed. However, since BSs with large distance have weak mutual interference, omitting some patterns will not affect the accurate estimation of user achievable rates. We will discuss more on this next (see Proposition 1 in Section 3).

Based on the above pattern definition, we establish a framework to optimize the cell activation, test point (user) association and multicell multiuser channel assignment jointly.

Firstly, the multi-cell channel allocation is translated into partitioning the spectrum across all patterns. In a slow timescale considered in this paper, all frequency resources can be assumed to have equal channel conditions. Denote the spectrum allocation profile by $\boldsymbol{\pi} = (\pi_1, \dots, \pi_i, \dots, \pi_I)^T \in \Pi$, where π_i represents the fraction of the total bandwidth allocated to pattern i and $\Pi = \{\boldsymbol{\pi} : \sum_{i \in \mathcal{I}} \pi_i = 1, \pi_i \geq 0, \forall i\}$. Then the total bandwidth fraction allocated to BS b is $\mathbf{A}(b,:) \times \boldsymbol{\pi}$, where $\mathbf{A}(b,:)$ denotes the b-th row of the matrix \mathbf{A} .

Secondly, denote by $\alpha_{kbi} \geq 0$ the fraction of resources that BS b allocates to test point k under pattern i. Naturally, each BS is allowed to use up to π_i resources under pattern i for its associated test points, expressed as $\sum_{k \in \mathcal{K}} \alpha_{kbi} \leq \pi_i, \forall b, \forall i$. Note that the association is implicitly indicated by α_{kbi} , i.e., $\alpha_{kbi} > 0$ means test point k is associated with BS b under pattern i, while zero value of α_{kbi} means that they are not connected. In this formulation, test point k is allowed to be connected to multiple BSs. This can be equivalently viewed as multiple users co-located at the same test point, and each BS serves one user individually. In this paper, we assume a single-user detector at each receiver.

Finally, we define the usage of BS b as $\rho_b = \sum_k \sum_i \alpha_{kbi}$. The definition of α_{kbi} leads to $0 \le \rho_b \le 1, \forall b \in \mathcal{B}$.

2.1. Rate model

Assuming flat power spectral density (PSD) of BS transmit power and the noise, the received SINR of the link connecting BS b to test point k under pattern i is

$$SINR_{kbi} = \frac{a_{ib}P_bG_{bk}\|h_{bk,n}\|^2}{\sigma^2 + \sum_{l \neq b} a_{il}P_lG_{lk}\|h_{lk,n}\|^2}$$
(2)

where a_{ib} is the cell activation indicator as given in (1), P_b is the PSD of BS b, σ^2 is the received noise PSD. We denote the channel gain between BS b and test point k over the n-th frequency resource by $\sqrt{G_{bk}}h_{bk,n}$ where G_{bk} is the large-scale coefficient including antenna gain, path loss and shadowing, and $h_{bk,n}$ accounts for the small-scale fading. We assume $\{h_{bk,n}, \forall b, \forall k, \forall n\}$ are independent and identically distributed (i.i.d.). Hence, the ergodic rate of test point k served by the b-th BS under pattern i can be written as

$$\bar{r}_{kbi} = \alpha_{kbi} \underbrace{W\mathbb{E}_{\mathbf{h}} \left[\log_2 \left(1 + \text{SINR}_{kbi} \right) \right]}_{\triangleq r_{kbi}} = \alpha_{kbi} r_{kbi}$$
 (3)

where W is the system bandwidth, $\mathbf{h} \triangleq (h_{1k,n}, h_{2k,n}, \dots, h_{Bk,n})$.

Finally, the total rate of test point k is obtained by summing up the contributions from all associated BSs and patterns, as

$$R_k = \sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} \bar{r}_{kbi} = \sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} \alpha_{kbi} r_{kbi}. \tag{4}$$

Note that r_{kbi} can be pre-calculted using (3) and hence treated as constants during the optimization.

2.2. Energy consumption model

As mentioned previously, the BS usage vector is defined as $\rho = (\rho_1, \cdots, \rho_B)^T$, where $\rho_b = \sum_k \sum_i \alpha_{kbi}$. A typical power consumption model for BSs consists of two types of power consumption: fixed power consumption and dynamic power consumption that is proportional to BS's utilization [3]. Denote by $P_b^{\rm OP}$ the maximum operational power of BS b if it is fully utilized (i.e., $\rho_b = 1$), which includes power consumption for transmit antennas as well as power amplifier, cooling equipment and so on. We can then express the total power consumption by all BSs as

$$P^{\text{tot}} = \sum_{b \in \mathcal{B}} \left[(1 - q_b) \rho_b P_b^{\text{OP}} + q_b |\rho_b|_0 P_b^{\text{OP}} \right]$$
 (5)

where $q_b \in [0,1]$ is the portion of the fixed power consumption for BS b as long as it is switched on, and $|x|_0$ is the function that takes the value of 0 if x=0 or the value 1 otherwise. Note that by setting $q_b=1$ we arrive at a constant energy consumption model considered in [1,6], which is a reasonable assumption for macro BSs. However, the small BSs such as pico or femto BSs may have smaller value of q_b because they do not usually have a big power amplifier or cooling equipment.

3. RATE-CONSTRAINED ENERGY SAVING

3.1. Problem formulation

The joint optimization of cell activation, user association and interference coordination for energy saving can be formulated as

minimize
$$P^{\text{tot}} = \sum_{b \in \mathcal{B}} \left[(1 - q_b) \rho_b P_b^{\text{OP}} + q_b |\rho_b|_0 P_b^{\text{OP}} \right]$$
 (6a)

subject to
$$\rho_b = \sum_{b \in \mathcal{K}} \sum_{i \in \mathcal{I}} \alpha_{kbi}, \forall b$$
 (6b)

$$\sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} \alpha_{kbi} r_{kbi} \ge d_k, \ \forall k$$
 (6c)

$$\sum_{k \in \mathcal{K}} \alpha_{kbi} \le \pi_i, \forall b, \ \forall i$$
 (6d)

$$\sum_{i \in \mathcal{I}} \pi_i = 1 \tag{6e}$$

$$\pi_i \ge 0, \ \forall i, \quad \alpha_{kbi} \ge 0, \ \forall k, b, i$$
 (6f)

where (6c) specifies the traffic demand of all test points, and all variables and parameters have been explained in Section 2.

The difficulty of solving (6) lies in two facts. The first is the combinatorial objective function involving the ℓ_0 -norm. The other is that the number of all possible patterns in the network grows exponentially with the number of cells as 2^B , resulting in huge problem dimension for a moderate-sized network. Fortunately, the following Proposition 1 identifies that only a small number of patterns out of 2^B are needed for resource allocation to achieve the optimality.

Proposition 1. There exists an optimal solution to problem (6) that activates at most K+B+1 patterns, i.e., $|\{i \in \mathcal{I} : \pi_i > 0\}| \le K+B+1$.

The proof is provided in the extended manuscript [7]. The sparsity structure identified by this proposition is exploited for the proposed dual cutting plane method to reduce the computational complexity (see Section 3.3 for more discussion).

3.2. Solving network energy saving problem

We now turn the attention to solving (6) assuming it is feasible. The idea is to apply reweighted ℓ_1 -norm minimization methods [8], originally proposed to enhance the data acquisition in compressed sensing. It is known that for nonnegative scalar $x \geq 0$, $|x|_0 = \lim_{\epsilon \to 0} \frac{\log(1+x\epsilon^{-1})}{\log(1+\epsilon^{-1})}$ [1]. With a small design parameter $\epsilon > 0$, we neglect the limit and then approximate the ℓ_0 -norm as

$$|x|_0 \approx \frac{\log(1 + x\epsilon^{-1})}{\log(1 + \epsilon^{-1})}. (7)$$

Relying on (7) and ignoring unnecessary constants, the problem (6) can be approximately solved by the following problem:

minimize
$$f(\boldsymbol{\rho}) \triangleq \sum_{b \in \mathcal{B}} \left[(1 - q_b) P_b^{\text{OP}} \rho_b + \frac{q_b P_b^{\text{OP}} \log(\epsilon + \rho_b)}{\log(1 + \epsilon^{-1})} \right]$$
subject to
$$\rho_b = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \alpha_{kbi}, \ \forall b$$

$$\sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} \alpha_{kbi} r_{kbi} \ge d_k, \ \forall k.$$
 (8)

where \mathcal{X} is defined by (6d), (6e) and (6f).

Note that (8) is a continuous problem unlike the one in (6) involving combinatorial terms. However, problem (8) is not convex since it minimizes a concave function. Fortunately, it falls into the framework of difference-of-convex (DC) functions and therefore can be efficiently solved by the convex-concave procedure [9].

Specifically, by applying the first-order Taylor expansion to the objective function in (8) at the point $\rho^{(t-1)}$ obtained in (t-1)-th iteration, we arrive at the following problem for the t-th iteration:

$$\underset{(\boldsymbol{\alpha}, \boldsymbol{\pi}) \in \mathcal{X}}{\text{minimize}} \quad \sum_{b \in \mathcal{B}} w_b^{(t)} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \alpha_{kbi}$$
 (9a)

subject to
$$\sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} \alpha_{kbi} r_{kbi} \ge d_k, \ \forall k$$
 (9b)

where

$$w_b^{(t)} = (1 - q_b)P_b^{\text{OP}} + \frac{q_b P_b^{\text{OP}}}{\log(1 + \epsilon^{-1})(\epsilon + \rho_b^{(t-1)})}$$
(10)

with

$$\rho_b^{(t-1)} = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \alpha_{kbi}^{(t-1)}.$$
 (11)

It can be shown, by applying the results in [10], that any limiting point of $(\boldsymbol{\alpha}^{(t)}, \boldsymbol{\pi}^{(t)})$ generated by the above convex-concave procedure as $t \to \infty$ is a stationary point of the problem (8). In practice, the reweighted ℓ_1 method converges typically within 6-10 iterations and the largest improvement in sparsity is obtained in the first few iterations.

Problem (9) is a linear program. It can be efficiently solved by, e.g., interior-point methods, if the problem dimension $\mathcal{O}(IKB)$ is small. However, it is also desirable to solve (9) by involving a large number of patterns. This could happen when we consider all possible 2^B patterns in order to calculate an optimal performance benchmark in a reasonable-sized network, or when the pre-selection still results in lots of candidate patterns for a large-scale network. In such case, the state-of-the-art interior-point solvers cannot be applied, since they typically have cubic computational complexity in the problem dimension [11]. Fortunately, the problem has an interesting structure that facilitates a tailored cutting plane method to solve the dual problem.

By dualizing the constraint of (9b), we can express the dual function as

$$h(\boldsymbol{\mu}) = \inf_{(\boldsymbol{\alpha}, \boldsymbol{\pi}) \in \mathcal{X}} \left\{ \sum_{k,b,i} \alpha_{kbi} w_b^{(t)} - \sum_{k,b,i} \alpha_{kbi} r_{kbi} \mu_k + \sum_k d_k \mu_k \right\}$$
(12)

where $\mu = (\mu_1, \dots, \mu_K)^T$ is the Lagrangian multiplier. The corresponding dual problem can be stated as

$$\underset{\boldsymbol{\mu}>0}{\text{maximize}} \quad h(\boldsymbol{\mu}). \tag{13}$$

Since the strong duality holds for this linear program, the original primal problem (9) can be alternatively solved by the dual problem (13). Following the idea of cutting plane methods, we formulate a master problem as

subject to
$$\sum_{k,b,i} \alpha_{kbi}^{(t,j)} w_b^{(t)} - \sum_{k,b,i} \alpha_{kbi}^{(t,j)} r_{kbi} \mu_k + \sum_k d_k \mu_k \geq z,$$

$$\forall j \in \{0,\cdots,l-1\} \quad (14b)$$

and an inner problem as

$$\underset{(\boldsymbol{\alpha}, \boldsymbol{\pi}) \in \mathcal{X}}{\text{minimize}} \quad \sum_{k.b.i} \alpha_{kbi} \left(w_b^{(t)} - r_{kbi} \mu_k^{(l)} \right) + \sum_k d_k \mu_k^{(l)} \quad (15)$$

respectively, where we denote the solution to (14) by $(\boldsymbol{\mu}^{(l)}, z^{(l)})$ and the solution to (15) by $(\boldsymbol{\alpha}^{(t,l)}, \boldsymbol{\pi}^{(t,l)})$. The master problem (14) is refined for the next iteration by adding $(\boldsymbol{\alpha}^{(t,l)}, \boldsymbol{\pi}^{(t,l)})$ to the constraint (14b). In this way, we iteratively solve (14) and (15) until $h(\boldsymbol{\mu}^{(l)}) \geq z^{(l)}$, implying that we have solved the problem (9) in the dual domain.

The difficulty with huge dimension has now been encapsulated in problem (15) and nicely resolved thanks to the following Proposition 2. The master problem (14) is a linear problem with small dimension (not involving 2^B term) that can be trivially solved using any standard solver.

Proposition 2. The problem (15) has a closed-form solution that can be expressed as

$$\alpha_{kbi}^{(t,l)} = \begin{cases} 1 & \text{if } i = \bar{i}, k = \bar{k}(b, \bar{i}), \text{ and } \tilde{r}_{kbi} < 0 \\ 0 & \text{otherwise} \end{cases}$$
 (16)

ana

$$\pi_i^{(t,l)} = \begin{cases} 1 & \text{if } i = \bar{i} \\ 0 & \text{otherwise} \end{cases}$$
 (17)

where $\bar{k}(b,i) = \arg\min_{k} \tilde{r}_{kbi}$ with $\tilde{r}_{kbi} = w_b^{(t)} - r_{kbi}\mu_k^{(l)}$, and $\bar{i} = \arg\min_{i} \sum_{b} \left[\tilde{r}_{\bar{k}(b,i)bi}\right]_{-}$, where $[x]_{-} = \min(0,x)$.

After solving (13) by the cutting plane, the primal solution can be found as [12, Ch.6]: $\alpha^{(t)} = \sum_{j=0}^{l-1} \kappa_j \alpha^{(t,j)}$ and $\pi^{(t)} = \sum_{j=0}^{l-1} \kappa_j \pi^{(t,j)}$, where $\{\kappa_j\}$ are the dual variables corresponding to the inequality constraints of (14b), which are available if we solve the problem (14) by off-the-shelf interior-point solvers.

Finally, the outermost iteration is to adjust the weights according to (10) and (11) and then the problem (9) is solved again with the new weights until convergence.

Table 1. Algorithm running time.

| | | 0 | | |
|------------------------------|-----|---------|---------|----------|
| Number of candidate patterns | 19 | 2^{6} | 2^{9} | 2^{15} |
| Proposed algorithm (sec) | 4.2 | 10.2 | 13.8 | 313 |
| Interior-point solver (sec) | 0.3 | 1.2 | 19.6 | 6346 |

3.3. Complexity of the algorithm

If problem (9) considering all possible pattern is directly solved by interior-point methods, the complexity is roughly $\mathcal{O}(I^3K^3B^3)$. By contrast, every iteration of the proposed dual algorithm requires finding a solution to (15) by Proposition 2, and a solution to (14) by interior-point solvers. Specifically, solving (15) requires $\mathcal{O}(IKB)$, while the complexity of solving (14) depends on the number of constraints in (14b), which is increased by one inequality per iteration. Our numerical experiment suggests that the number of iterations is roughly proportional to K. (This can be explained by the inherent sparsity structure of the solution identified by Proposition 1. Since the proposed algorithm activates one pattern per iteration (see (17)), the number of iteration is unsurprisingly much lower than I if I is large). Consequently, it is safe to bound the complexity of solving (14) as $\mathcal{O}(K^3)$ per iteration. Hence, the overall complexity of the proposed algorithm for solving (9) is $\mathcal{O}(IK^2B+K^4)$, much smaller than directly applying interior-point solvers to (9). In Table 1, we report the algorithm running time for a network consisting of 50 users and 15 cells, where the proposed algorithm outperforms a commercial solver (Gurobi [13] with the barrier method), as I increases.

3.4. Initialization

The cutting plane method should be initialized with a strictly primal feasible solution in terms of (9b), otherwise the master problem will become unbounded in the first iteration. We can solve the following rate balancing problem to test the feasibility of (6) and obtain a strictly primal feasible solution if the original problem is feasible:

$$\underset{\leftarrow}{\text{minimize}} \quad -R_{\text{sum}} \tag{18a}$$

subject to
$$\beta_k R_{\text{sum}} - \sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} \alpha_{kbi} r_{kbi} \le 0, \ \forall k$$
 (18b)

where $\beta_k = d_k / \sum_{k \in \mathcal{K}} d_k$. Note that problem (18) is always feasible. We can again apply cutting plane method to solve it, but without worrying about the initialization (since we can always decrease R_{sum} to make sure (18b) is strictly satisfied).

4. NUMERICAL RESULTS AND DISCUSSIONS

4.1. Simulation setup

We consider a network consisting of 3 macro cells, each of which contains 4 randomly dropped pico cells as shown in Fig. 1.

The parameters for propagation modeling and simulations follow the suggestions in 3GPP evaluation methodology, and summarized in Table I of [14]. Based on the linear relationship between transmit power and operational power consumption¹, we calculate the maximum operational power P^{OP} as 439W and 38W for macro and pico BSs, respectively. We further assume each macro BS has a constant power consumption, i.e., $q_b = 1$, $\forall b \in \mathcal{B}_{\mathrm{macro}}$, and the

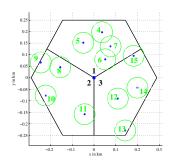


Fig. 1. A heterogeneous network consisting of 15 cells.

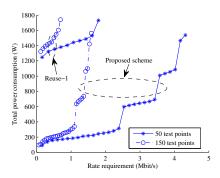


Fig. 2. A comparison of network power consumption between the proposed scheme and Reuse-1 scheme in [2].

fixed power consumption of a pico takes 50% of the maximum operational power, i.e. $q_b = 0.5, \forall b \in \mathcal{B}_{pico}$. Note that these assumptions are made for providing concrete numerical results, and they are not from the restriction of our formulation.

4.2. Performance comparison

The baseline strategy in comparison is the energy saving optimization scheme proposed by [2], where worst-case estimates of the user rates resulted from no intercell interference coordination are used to calculate the QoS requirements. This scheme can be cast into the proposed framework by restricting the candidate pattern to a single *Reuse-1* pattern.

Fig.2 plots the network power consumption versus the rate requirement of the test points, where 50 and 150 test points are uniformly distributed within the network, and all test points are assumed to have the same requirement for simplicity.

As shown, the network power consumption increases with the user rate requirement for both schemes, but the proposed scheme has a significantly power saving compared to the existing Reuse-1 scheme. For example, to satisfy 1Mbit/s for 50 test points, the proposed scheme only consumes 200W, whereas the Reuse-1 needs more than 1400W. Moreover, the maximum rate requirement that the network can support has been greatly improved by the proposed scheme. We observe, for example, the maximum feasible rate in 50-test-point case increases from 1.8Mbit/s to 4.3Mbit/s by using the proposed scheme. The performance gains of the proposed strategy come from its ability to manage the interference by resource allocation and explicitly take into account the interference coupling caused by cell (de)activation when estimating the user rate.

We adopted the linear model in [15]: $P_b^{\rm OP}=\alpha_bP_b+\beta_b$, where P_b is the transmit power for BS $b,\,\alpha_b=\frac{22.6}{3}$ and $\beta_b=\frac{412.4}{3}{\rm W}$ if b is a macro; otherwise $\alpha_b=5.5$ and $\beta_b=32{\rm W}$ if b is a pico.

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