OPTIMAL RESOURCE BLOCK ALLOCATION AND MUTING IN HETEROGENEOUS NETWORKS

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ABSTRACT

In this paper, we investigate user association and resource block (RB) allocation in downlink heterogeneous cellular networks. Our goal is to jointly optimize user association and RB allocation to maximize network throughput while taking into account fairness among users. To effectively control interference, RB muting is employed. The problem is addressed in an integer linear programming (ILP) framework. Due to the combinatorial nature of the problem, the computational complexity of solving it becomes prohibitively high even for medium size networks. Therefore, we propose to decouple user association from the original problem, and formulate a low-complexity ILP problem for the optimal RB allocation and muting. Simulation results show that the proposed RB allocation and muting scheme achieves significantly higher throughput and better fairness compared with conventional schemes.

***Index Terms*—** Heterogeneous networks, resource block allocation and muting, user association, interference management, integer linear programming.

1. INTRODUCTION

Spatial reuse and cell splitting in cellular networks are effective mechanisms for increasing system capacity. They are foreseen to be essential for accommodating the ever-increasing volume of mobile data in 4G and 5G cellular networks [1, 2]. In light of this, heterogeneous cellular networks (HetNets) are introduced and currently deployed globally to boost the system capacity of cellular networks [3, 4]. However, the co-channel deployment of high-power macro base stations (BSs) and low-power BSs (micro, pico, and femto BSs) in HetNets introduces technical challenges, such as load balancing and interference management [5–9].

The maximum signal-to-interference-plus-noise ratio (max-SINR) based user association scheme, in which each user selects the BS providing the strongest SINR, performs well in the traditional homogeneous networks. However it is experienced that, in HetNets, due to high transmit power, most users associate with the macro BSs, which are thus overloaded. Meanwhile, the resources of low-power BSs remain underutilized [5]. Many techniques have been proposed in the literature to deal with this problem, which achieve near optimal load balancing in HetNets [5–7, 10]. Similarly, the interference scenario in HetNets is disparate from that of the traditional homogeneous networks, and alternative approaches are needed to minimize it. One of the popular techniques discussed in the literature to reduce the interference in HetNets involves the introduction of so-called almost blank subframes, in which some of the frequency (or time) resource blocks (RBs) of the macro BSs are muted [3, 7, 8, 11, 12].

The interference in HetNets is the combined effect of user association, RB allocation, and RB muting. Therefore, only the joint optimization of all these procedures can yield the optimal throughput in HetNets. In this paper, we formulate an integer linear programming (ILP) problem to jointly optimize user association, RB allocation, as well as RB muting, in frequency selective channels to maximize the proportional fair throughput. One of the main contributions of our work lies in employing RB muting in the joint optimization problem. The RB muting offers more degrees of freedom to improve throughput and fairness in HetNets. By muting RBs of BSs those cause severe interference in the network, the optimization problem not only increases the average SINR level in the network, but also reduces the transmitted powers of the BSs. However, due to the combinatorial nature of the problem, the computational complexity of solving the formulated ILP becomes prohibitively high for practical cellular networks. To address this issue, we decouple the problem of user association from RB allocation and muting. The user association is carried out independently with any existing techniques, and a low-complexity ILP problem is formulated for the optimal RB allocation and muting. Simulation results show that the proposed RB allocation and muting scheme yields significantly higher network throughput and better fairness compared with the conventional RB allocation schemes.

2. SYSTEM MODEL

Consider a downlink HetNet with B single antenna BSs and K single antenna users. Let $\mathcal{B} \triangleq \{1, 2, \dots, B\}$ denote the

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set of all BSs, and $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$ denote the set of all users. With co-channel deployments, i.e., with a frequency reuse factor of one, each BS uses the whole available bandwidth. The bandwidth is divided into F equally sized RBs, and $\mathcal{F} \triangleq \{1, 2, \ldots, F\}$ represents the set of indexes of all the RBs. Let $h_{k,b,f} \in \mathbb{C}$ indicate the channel response between the kth user and the bth BS on the fth RB. All the active RBs of a BS are assumed to have equal powers. Let ϕ_b be a constant, which denotes the transmit power of each active RB of the bth BS. The transmit power of any muted RB is zero.

With link adaptation in the form of adaptive modulation and coding, each user is assigned a discrete rate from the set of candidate rates $\mathcal{R} \triangleq \{R_1, R_2, \ldots, R_L\}$ associated with a specific modulation and coding scheme (MCS) [13, 14]. In order to meet the prescribed block error rate (BLER) requirements, an MCS with a data rate R_ℓ can be assigned to a user, only if the received SINR at the user is above the predetermined threshold γ_ℓ, where $\ell \in \mathcal{L} \triangleq \{1, 2, \ldots, L\}$.

We introduce binary indicators $\{\alpha_{k,b} \in \{0,1\}, \forall k \in \mathcal{K}, \forall b \in \mathcal{B}\}$ to model user association, with $\alpha_{k,b} = 1$ when the kth user is associated with the bth BS, and $\alpha_{k,b} = 0$ otherwise. It is assumed that each user associates with at most one BS. This can be expressed as

$$\sum_{b=1}^{B} \alpha_{k,b} \leq 1, \quad \forall k \in \mathcal{K}. \tag{1}$$

We use binary indicators $\{\beta_{k,b,f,\ell} \in \{0,1\}, \forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \forall f \in \mathcal{F}, \forall \ell \in \mathcal{L}\}$ to represent RB allocation and discrete data rate assignment. The variable $\beta_{k,b,f,\ell} = 1$ indicates the fth RB of the bth BS is assigned to the kth user with the ℓth MCS, and $\beta_{k,b,f,\ell} = 0$ otherwise. We assume, a BS assigns each of its RBs to at most one user, which can be modelled as

$$\sum_{k=1}^{K} \sum_{\ell=1}^{L} \beta_{k,b,f,\ell} \leq 1, \quad \forall b \in \mathcal{B}, \forall f \in \mathcal{F}. \tag{2}$$

A BS can assign RBs to a user, only if the user is associated with that BS. In addition, the maximum number of RBs that can be granted to the user from the associated BS is F. This constraint can be expressed as

$$\sum_{f=1}^{F} \sum_{\ell=1}^{L} \beta_{k,b,f,\ell} \leq \alpha_{k,b} F, \quad \forall k \in \mathcal{K}, \forall b \in \mathcal{B}. \tag{3}$$

3. PROBLEM FORMULATION

In this section, we consider the joint optimization of user association and RB allocation with RB muting to maximize the proportional fair throughput.

Let $\mathrm{SINR}_{k,b,f}$ be the SINR received by the kth user on the fth RB when the user is associated with the bth BS. With single user detection, the $\mathrm{SINR}_{k,b,f}$ is given by

$$\mathrm{SINR}_{k,b,f} = \frac{\sum_{\ell=1}^{L} \beta_{k,b,f,\ell}\, \phi_b\, |h_{k,b,f}|^2}{\sum_{i=1}^{K} \sum_{j=1, j\neq b}^{B} \sum_{\ell=1}^{L} \beta_{i,j,f,\ell}\, \phi_j |h_{k,j,f}|^2 + \sigma_k^2}, \tag{4}$$

where σ_k^2 denotes additive white Gaussian noise at the kth user. The binary variables $\beta_{i,j,f,\ell}$ in (4) facilitate considering only the active RBs, and excluding the muted RBs while calculating the interference. A rate R_ℓ can be assigned to the kth user by the bth BS on the fth RB only when $\mathrm{SINR}_{k,b,f} \geq \gamma_\ell$, which can be expressed as

$$\frac{\phi_b\, |h_{k,b,f}|^2}{\sum_{i=1}^{K} \sum_{j=1, j\neq b}^{B} \sum_{\ell=1}^{L} \beta_{i,j,f,\ell}\, \phi_j |h_{k,j,f}|^2 + \sigma_k^2} \geq \beta_{k,b,f,\ell} \gamma_\ell. \tag{5}$$

The inequality (5) represents a bilinear integer inequality constraint in the variables $\beta_{k,b,f,\ell}$ and $\beta_{i,j,f,\ell}$. We can reformulate (5) and express it in an equivalent integer linear form using the big-M technique as [15]

$$\frac{1}{\gamma_\ell} \phi_b |h_{k,b,f}|^2 + (1 - \beta_{k,b,f,\ell}) M_{k,f} \geq \sum_{i=1}^{K} \sum_{j=1, j\neq b}^{B} \sum_{\ell=1}^{L} \beta_{i,j,f,\ell}\, \phi_j |h_{k,j,f}|^2 + \sigma_k^2. \tag{6}$$

In (6), $M_{k,f}$ is a sufficiently large constant computed from the right-hand-side of (6) by setting $\{\beta_{i,j,f,\ell} = 1, \forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \forall f \in \mathcal{F}, \forall \ell \in \mathcal{L}\}$. The constant $M_{k,f}$ upper bounds the right-hand-side of the inequality constraint (6). Note that, when $\beta_{k,b,f,\ell} = 0$, both constraints (5) and (6) become redundant, and are satisfied $\forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \forall f \in \mathcal{F}, \forall \ell \in \mathcal{L}$; when $\beta_{k,b,f,\ell} = 1$, constraints (5) and (6) are identical.

We adopt the proportional fair [16] throughput as our objective function, which is denoted by Ω. It can be formulated in a linear form as

$$\Omega \triangleq \sum_{k=1}^{K} \frac{r_k}{\bar{r}_k^{\mu}}. \tag{7}$$

In (7), r_k is the instantaneous rate achievable for the kth user, which can be computed as

$$r_k = \sum_{b=1}^{B} \sum_{f=1}^{F} \sum_{\ell=1}^{L} \beta_{k,b,f,\ell} R_\ell, \quad \forall k \in \mathcal{K}, \tag{8}$$

and the constant $\bar{r}_k$ is the weighted average rate [17] of the kth user over the previous time slots. The weighting factor μ in (7) is a constant, and it can be used to adjust the fairness among the users.

The joint optimization problem of user association and RB allocation can be mathematically cast as

$$\begin{aligned} &\underset{\{\alpha_{k,b}, \beta_{k,b,f,\ell}, \forall k, \forall b, \forall f, \forall \ell\}}{\text{maximize}} \quad \Omega \\ &\text{s. t.} \quad (1), (2), (3), (6), (8), \alpha_{k,b} \in \{0,1\}, \beta_{k,b,f,\ell} \in \{0,1\}. \end{aligned} \tag{9}$$

The optimal solution of problem (9) provides the best combination of user association, RB allocation and muting, and discrete data rate assignment to achieve the largest proportional fair throughput. However, due to the combinatorial nature of the problem, the computational complexity of solving it becomes prohibitively high, and it is impossible to compute optimal solutions in reasonable time even for medium size networks with the existing ILP solvers.

4. OPTIMAL RB ALLOCATION AND MUTING

In this section, we decouple the problem into two parts, namely, 1) user association, and 2) RB allocation and muting. In the first phase, the user association is carried out based on any existing techniques, such as the conventional max-SINR based user association scheme, the range expansion scheme, etc. [4, 5, 18]. We introduce binary variables $\{\tilde{\alpha}_{k,b} \in \{0,1\}, \forall k \in \mathcal{K}, \forall b \in \mathcal{B}\}$ as the user association indicators, with $\tilde{\alpha}_{k,b} = 1$ if the kth user is associated with the bth BS, and $\tilde{\alpha}_{k,b} = 0$ otherwise. After the user association phase, each $\tilde{\alpha}_{k,b}$ is assigned to a respective value, and thus it becomes a constant in the next phase.

In the second phase, we perform the optimal RB allocation and muting. Since the RBs are orthogonal to each other, and the objective function (7) is linear with the optimization variable $\beta_{k,b,f,\ell}$, the allocation and muting of any RB is independent of that of the other RBs. This motivates us to divide the optimization problem into F independent sub-problems, one for each RB, which can be solved in parallel. In each sub-problem we optimize the allocation or muting of single RB for all the BSs to maximize the objective function (7).

Let us consider the fth sub-problem, in which the problem of allocation or muting of the fth RB is solved. We introduce binary variables $\{\lambda_{k,b,f} \in \{0,1\}, \forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \forall f \in \mathcal{F}\}$ as RB allocation indicators. The variable $\lambda_{k,b,f} = 1$ indicates the bth BS has assigned the fth RB to the kth user, and $\lambda_{k,b,f} = 0$ otherwise. The bth BS can assign the RB to the kth user only if the kth user is associated with the bth BS, which can be expressed as

$$\lambda_{k,b,f} \leq \tilde{\alpha}_{k,b}, \quad \forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \forall f \in \mathcal{F}. \qquad (10)$$

A BS will assign a single RB to at most one user, i.e.,

$$\sum_{k=1}^{K} \lambda_{k,b,f} \leq 1, \quad \forall b \in \mathcal{B}, \forall f \in \mathcal{F}. \qquad (11)$$

We use binary variables $\{\delta_{k,f,\ell} \in \{0,1\}, \forall k \in \mathcal{K}, \forall f \in \mathcal{F}, \forall \ell \in \mathcal{L}\}$ to indicate the data rate assignment to each user. The variable $\delta_{k,f,\ell} = 1$, if for the kth user, connected to a particular BS, the ℓth data rate R_ℓ is assigned to the fth RB. If the fth RB is not assigned to the kth user from the associated BS, $\delta_{k,f,\ell} = 0, \forall \ell \in \mathcal{L}$. This can be mathematically expressed as

$$\sum_{\ell=1}^{L} \delta_{k,f,\ell} = \sum_{b=1}^{B} \lambda_{k,b,f}, \quad \forall k \in \mathcal{K}, \forall f \in \mathcal{F}. \qquad (12)$$

As in Section 3, the SINR constraints to ensure the prescribed BLER can be formulated as

$$\frac{1}{\gamma_\ell} \phi_{b_k} |h_{k,b_k,f}|^2 + (1 - \delta_{k,f,\ell}) M_{k,f} \geq \sum_{j=1, j \neq b_k}^{B} \left(\sum_{i=1}^{K} \lambda_{i,j,f} \right) \phi_j |h_{k,j,f}|^2 + \sigma_k^2, \quad \forall k \in \mathcal{K}, \forall f \in \mathcal{F}, \forall \ell \in \mathcal{L}, \qquad (13)$$

where b_k is the BS associated with the kth user. The instantaneous data rate of the kth user on the fth RB can be expressed as

$$r_k = \sum_{\ell=1}^{L} \delta_{k,f,\ell} R_\ell, \quad \forall k \in \mathcal{K}. \qquad (14)$$

The term $\bar{r}_k, \forall k \in \mathcal{K}$ in (7) remains constant over all the sub-problems. The sub-problem of a single RB allocation and muting for all the BSs can be formulated as

$$\underset{\{\lambda_{k,b,f}, \delta_{k,f,\ell}, \forall k, \forall b, \forall \ell\}}{\text{maximize}} \quad \Omega$$
$$\text{s. t. } (10), (11), (12), (13), (14), \lambda_{k,b,f} \in \{0,1\}, \delta_{k,f,\ell} \in \{0,1\}. \qquad (15)$$

The computational complexity of problem (15) is significantly lower than that of the original problem (9). Problem (15) can be efficiently solved using commercial software like CPLEX employing the branch-and-cut algorithm [19–21].

5. NUMERICAL RESULTS

In this section, we present simulation results to compare the network throughput and the fairness of our proposed RB allocation and muting scheme with two standard RB allocation schemes, namely, a) the round robin (RR) scheme, and b) the proportional fair (PF) scheme [16, 17, 22, 23]. The range expansion user association technique is used prior to all the three schemes.

Based on [24], our simulation setting consists of a three-sector hexagonal cell of 500 meters macro-layer inter-site distance. Each sector has one macro BS having a transmit power of 46 dBm, and an antenna gain of 14 dB. Four pico BSs with a transmit power of 35 dBm, and an antenna gain of 5 dB each, are randomly distributed inside the cell. Each BS is equipped with 12 RBs. Hence, transmit power on an active RB ϕ_b of a macro BS is 35.2 dBm, and that of a pico BS is 24.2 dBm. We consider a total number of users in the network of K=30. Hotspot distribution of the users are assumed around the pico BSs, with $2/3$ of the total users inside the hotspots and the remaining users distributed randomly and uniformly within the cell. We adopt the 3GPP path loss models: $L(d) = 128.1 + 37.6 \log_{10}(d)$ for macro BSs and $L'(d) = 140.7 + 36.7 \log_{10}(d)$ for pico BSs, with d being

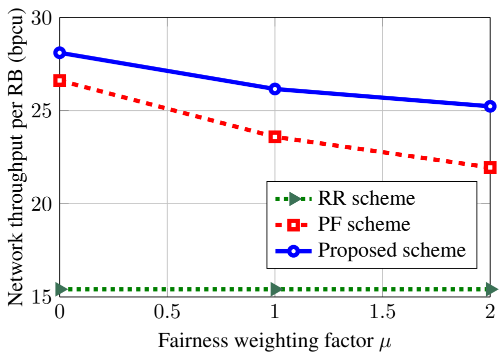

Fig. 1: Network throughput vs. fairness weighting factor μ.

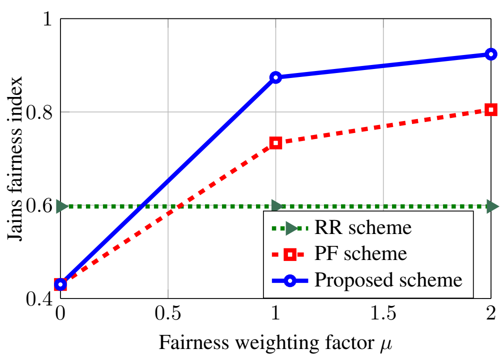

Fig. 2: Jain's fairness index vs. fairness weighting factor μ.

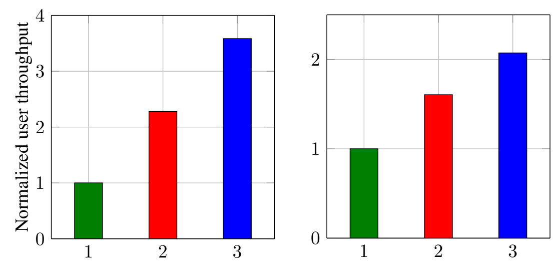

Fig. 3: Normalized user throughput (w.r.t. RR scheme). (left) 5th percentile. (right) 50th percentile.
1 - RR scheme, 2 - PF scheme, 3 - Proposed scheme.

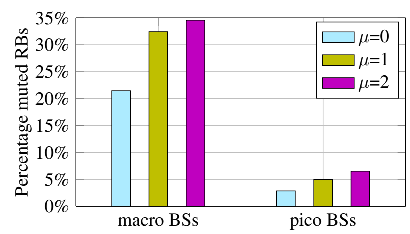

Fig. 4: Percentage of muted RBs with the proposed scheme.

Table 1: Power saved (in Watt) with the proposed scheme.

	Macro BSs	Pico BSs	Total
$\mu = 0$	8.50 W	0.08 W	8.58 W
$\mu = 1$	12.90 W	0.15 W	13.05 W
$\mu = 2$	13.70 W	0.20 W	13.90 W

the distance between a user and its associated BS in km. The simulations are carried out over 2000 Monte Carlo runs.

Fig. 1 shows the network throughput per RB, and Fig. 2 shows the Jain's fairness index (JFI) [25] obtained by the three schemes, for different values of fairness weighting factor μ. We see from Fig. 1 and Fig. 2 that the proposed scheme yields a higher network throughput and a larger JFI compared to the RR scheme and the PF scheme for various values of μ. We also observe the trade-off between the network throughput and the fairness. The substantial improvements in 5%-ile and 50%-ile user throughput (normalized w.r.t. RR scheme) in Fig. 3 with the proposed scheme reveal the fact that the cell edge users, which typically experience low data rate, are the major beneficiaries of our proposed scheme. Since we apply the same user association technique prior to all the three RB allocation schemes, we can conclude that the improvement in the throughput and the fairness in the proposed scheme are due to the muting of appropriate RBs and sophisticated RB allocation.

We observe from Fig. 4 that the percentage of muted RBs in macro BSs are significantly higher compared to that of the pico BSs. Since the macro BSs have higher transmit power, they cause severe interference to the pico BS users. With some of the RBs of the macro BSs muted, the data rates of the users of pico BSs are significantly improved. As the fairness weighting factor μ increases, more RBs of the macro BSs are muted, which improves the fairness index further. Moreover, muting of the RBs also reduces the transmitted power of the BSs linearly, and hence, enhances the power efficiency of the network. Table 1 shows the power saved by the macro and the pico BSs, which are computed by multiplying the number of muted RBs of each BS with the transmit power per RB of the corresponding BSs.

6. CONCLUSION

Joint optimization of user association, RB allocation, and RB muting has been considered in downlink HetNets to maximize the proportional fair throughput. The problem has been formulated as an ILP, based on which, a low-complexity optimal scheme has been developed for the practical RB allocation and muting. The simulation results show that the proposed scheme yields a higher network throughput and an improved fairness over the conventional schemes. By muting a substantial percentage of RBs of macro BSs those cause severe interference in the network, the proposed scheme enhances the power efficiency of the network as well.

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