LONG-TERM GENERAL RANK MULTIUSER DOWNLINK BEAMFORMING WITH SHAPING CONSTRAINTS USING QOSTBC

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ABSTRACT

This paper addresses multiuser downlink beamforming with shaping constraints under the assumption that the transmitter has long-term covariance based channel state information (CSI). Beamformers are designed to maximize the minimum average signal-to-interference-plus-noise ratio (SINR) of users subject to a total transmit power constraint and additional shaping constraints. We combine beamforming with full-rate quasi-orthogonal space time block coding (QOSTBC) to increase the number of beamforming weight vectors and associated degrees of freedom much beyond the limits achieved by the Alamouti code in the beamformer design. The use of QOSTBC destroys the full-orthogonality structure of the corresponding equivalent channel matrix such that generally maximum-likelihood (ML) pairwise decoding has to be applied for optimal decoding. As an alternative to the pairwise decoding, we propose a simple phase rotation scheme on beamformers at the transmitter side that enables simplified symbol-wise decoding. The original beamforming problem is transformed to a semidefinite programming (SDP) problem which can be solved optimally for a massive number of shaping constraints. Simulation results demonstrate a significant performance improvement over the existing approaches.

Index Terms— Downlink beamforming, general rank beamforming, shaping constraints, semidefinite relaxation (SDR), quasi-orthogonal space time block coding (QOSTBC).

1. INTRODUCTION

Multiuser downlink beamforming has been extensively studied due to its potential for improving the spectral efficiency [1–3]. There are typically two quality-of-service based downlink beamforming problem formulations [4, 5]: the problem of minimizing the total transmit power subject to SINR constraints, and the problem of maximizing the minimum SINR of all users subject to a total transmit power constraint. Besides the SINR and power constraints, additional shaping constraints are considered in certain practical applications, e.g., to limit the interference power or guarantee the charging power [6–10].

The rank-one beamforming problem of minimizing the total transmit power subject to SINR constraints and additional shaping constraints has been investigated in [11–14]. As a massive number of constraints is incorporated, the degrees of freedom in the rank-one beamformer design can be rather deficient which may cause the optimization problem either to be infeasible or be difficult to solve optimally. To increase the degrees of freedom in the beamformer design, a general rank beamforming approach is proposed in [15] which combines beamforming with full-rate high dimensional real-valued orthogonal space time block coding (OSTBC) which outperforms the conventional rank-one and rank-two approaches [16–23]. The general rank beamforming approach in [15] is designed based on the assumption that instantaneous CSI is available at the transmitter. However, instantaneous CSI can be difficult to acquire in practical cases. In frequency division duplexing systems, instantaneous CSI needs to be fed back from the users to the base station resulting in a prohibitive signaling overhead especially in fast fading scenarios [1, 4]. Since the long-term covariance based CSI changes at a significantly lower rate as compared to the instantaneous CSI, only infrequent feedback from users is required. Therefore, the use of covariance based CSI is more practical generally.

In this paper, we propose a non-trivial extension of the general rank beamforming approach proposed in [15] to the case when long-term covariance based CSI is available at the transmitter. In this work, we consider the problem of maximizing the minimum SINR among all users while satisfying the total transmit power constraint and additional shaping constraints. The key problem associated with the general rank beamforming approach in [15], when it is applied in the case of covariance based CSI, is that due to the absence of instantaneous CSI at the transmitter, the orthogonality of the code matrix of the equivalent channel can no longer be guaranteed and thus inter-symbol interference is present which leads to performance degradation. To address this issue, a new general rank beamforming approach is developed in this work to solve the downlink beamforming problem by combining downlink beamforming with full-rate QOSTBC. Instead of the real-valued OSTBC employed in [15], QOSTBC is used in this work because the inter-symbol interference in QOSTBC induced by the orthogonality loss of the coding matrix can be much smaller than that in the real-valued OSTBC. A new phase rotation procedure on beamformers associated with QOSTBC is designed to ensure that the average inter-symbol interference is eliminated and correspondingly a simple symbol-wise decoder is developed for QOSTBC. In our proposed QOSTBC based general rank beamforming approach, the original beamforming problem is transformed to a convex optimization problem using semidefinite relaxation (SDR) which can be solved efficiently. The SDR solution after the rank reduction procedure is optimal for the original problem if all SDR solution matrices do not exhibit a rank larger than eight which can be guaranteed if the number of additional shaping constraint does not exceed 79, c.f. [15].

2. RANK-ONE BEAMFORMING

Let us consider a cellular communication system where a base station equipped with an antenna array of N elements simultaneously communicates independent information symbols to M single-antenna receivers. We assume that the channels are random, covariance based CSI is available at the transmitter and individual instantaneous CSI is available at each receiver. The information symbol intended for the i-th receiver is denoted as s_i with zero mean and unit variance. Then, the signals $\{s_i\}_{i=1}^{M}$ are steered to different receivers in a spatially separated way using the respective $N \times 1$ beamforming vectors $\{\mathbf{w}_i\}_{i=1}^{M}$. The received signal at the i-th

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receiver is then given by [1]

$$y_i = s_i \mathbf{w}_i^H \mathbf{h}_i(t) + \sum_{m=1, m\neq i}^{M} s_m \mathbf{w}_m^H \mathbf{h}_i(t) + n_i \quad (1)$$

where $\mathbf{h}_i(t)$ and n_i are the $N \times 1$ time-varying channel vector and complex circularly white Gaussian noise with the variance σ_i^2 of the i-th receiver, respectively, and $(\cdot)^H$ denotes the Hermitian transpose. Based on (1), the long-term average SINR at the i-th receiver in the conventional rank-one beamforming approach is derived as

$$\mathrm{SINR}_{\mathrm{c},i}(\{\mathbf{w}_m\}_{m=1}^M) \triangleq \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum\limits_{m=1, m\neq i}^{M} \mathbf{w}_m^H \mathbf{R}_i \mathbf{w}_m + \sigma_i^2} \quad (2)$$

where $\mathbf{R}_i = \mathrm{E}\{\mathbf{h}_i(t)\mathbf{h}_i^H(t)\}$ [3]. The total transmit power at the base station equals $\sum\limits_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i$. Then, the problem of finding the weight vectors that maximize the minimum average SINR of all users subject to the total transmit power constraint $\mathrm{P}_{\max}$ and additional shaping constraints can be formulated as

$$\max_{\{\mathbf{w}_i\}_{i=1}^M, t} \; t \quad \text{s.t. } \mathrm{SINR}_{\mathrm{c},i}(\{\mathbf{w}_m\}_{m=1}^M) \geq t, \forall i = 1, \ldots, M \quad (3a)$$

$$\sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i \leq \mathrm{P}_{\max} \quad (3b)$$

$$\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \trianglerighteq_l b_l, \forall l = 1, \ldots, L \quad (3c)$$

where L additional shaping constraints are formulated in (3c) for appropriately chosen $N \times N$ Hermitian and possibly indefinite matrices $\mathbf{A}_{lm}$ with corresponding thresholds b_l, and $\trianglerighteq_l$ denotes a sign in the set $\{\geq, \leq, =\}$. The additional shaping constraints in (3c) can be constructed for different applications, e.g., to describe interference suppression towards concurrent co-channel users in coexisting hierarchical networks [6, 7], or to formulate the charging power guarantees at the harvesting nodes in energy harvesting networks [8–10]. Problem (3) is a non-convex quadratically constrained quadratic programming problem and can be approximated by a SDP problem using the SDR technique [24, 25].

3. GENERAL RANK BEAMFORMING

The central idea of combining downlink beamforming with QOSTBC in this work follows the general framework of [15–23] in which beamformers are designed by combining beamforming with OSTBC. By combining beamforming with e.g., Alamouti coding, each user is simultaneously served with two Alamouti coded symbols from two beamformers over two time slot, as described below. The so-called, rank-two beamforming approaches introduced in [16–23] can be applied in various beamforming applications to double the degrees of freedom in the beamformer design. However, the drawback of Alamouti-based rank-two beamforming is that an optimal solution can only be obtained if all ranks of the SDR solution are no greater than two. When applying general (> 2) rank beamforming using real-valued OSTBC to further increase the degrees of freedom in the beamformer design as proposed in [15], the effective channel vectors have to be adjusted to real-valued vectors by specific phase rotations on beamformers to ensure that the corresponding coding matrix becomes orthogonal such that symbol-by-symbol decoding can be performed. The phase rotation procedure in real-valued OSTBC is based on instantaneous CSI available at the transmitter, thus it cannot be applied in the problem considered in this paper since only covariance based CSI is assumed to be available at the transmitter. Meanwhile, the SINR expression for the real-valued OSTBC case can be difficult to obtain. In this paper, we apply QOSTBC and a new phase rotation procedure is designed to eliminate the average inter-symbol interference such that symbol-by-symbol decoding can be used at the receivers.

3.1. Full-rate QOSTBC

Full-rate orthogonal codes with complex symbol constellations in its code matrix are impossible to be obtained for systems with more than two transmit antennas. To design full-rate codes, QOSTBC is proposed in which the strict requirement of full orthogonality of the code matrix is slightly relaxed [26]. Correspondingly, the simple symbol-by-symbol decoding property is lost. However, pairs of symbols can optimally be decoded independently for 4×4 and 8×8 in the QOSTBC [27]. One example of the 4×4 QOSTBC matrix is as follows

$$\mathcal{X}([s_1, s_2, s_3, s_4]^T) \triangleq \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}. \quad (4)$$

3.2. Equivalent System Model

Denote $\mathbf{s}_i = [s_{i1}, \ldots, s_{iK}]^T$ as the $K \times 1$ complex symbol vector for the i-th user with $K \leq N$ and $K \in \{4, 8\}$ in accordance with the dimension of QOSTBC matrices. Instead of weighting each symbol by a beamforming vector as in (1), a QOSTBC matrix $\mathcal{X}(\mathbf{s}_i)$ is transmitted for each user by using K beamformers of length N, denoted as $\mathbf{w}_{i1}, \ldots, \mathbf{w}_{iK}$. In this case, each of the K beams can be regarded as a virtual antenna from which QOSTBC is transmitted. In our scenario, we consider a block fading channel model where the channels remain constant over K time slots. The received signal y_{ik} at the i-th user in the k-th time slot is given by

$$y_{ik} = \sum_{m=1}^{M} \sum_{k'=1}^{K} [\mathcal{X}(\mathbf{s}_m)]_{kk'} \mathbf{w}_{mk'}^H \mathbf{h}_i(t) + n_{ik} \quad (5)$$

where n_{ik} is the noise of the i-th user in the k-th time slot. The received signal vector $\mathbf{y}_i \triangleq [y_{i1}, \ldots, y_{iK}]^T$ at the i-th user within the transmission period of K time slots can be written in matrix form as

$$\mathbf{y}_i = \mathcal{X}(\mathbf{s}_i)\mathbf{W}_i^H \mathbf{h}_i(t) + \sum_{m=1, m\neq i}^{M} \mathcal{X}(\mathbf{s}_m)\mathbf{W}_m^H \mathbf{h}_i(t) + \mathbf{n}_i \quad (6)$$

where $\mathbf{W}_i \triangleq [\mathbf{w}_{i1}, \ldots, \mathbf{w}_{iK}]$ is the beamforming matrix, and the noise vector $\mathbf{n}_i \triangleq [n_{i1}, \ldots, n_{iK}]^T$. The above system model can be reformulated in the following equivalent form [26]

$$\tilde{\mathbf{y}}_i = \mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i(t))\mathbf{s}_i + \tilde{\mathbf{i}}_i + \tilde{\mathbf{n}}_i \quad (7)$$

where $\mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i(t))$ denotes the quasi-orthogonal equivalent channel matrix and

$$\tilde{\mathbf{y}}_i \triangleq \left[y_{i1}, -y_{i2}, \ldots, -y_{iK}\right]^T, \quad (8)$$

$$\tilde{\mathbf{i}}_i \triangleq \sum_{m=1, m\neq i}^{M} \mathcal{X}(\mathbf{W}_m^H \mathbf{h}_i(t))\, \mathbf{s}_m, \quad (9)$$

$$\tilde{\mathbf{n}}_i \triangleq \left[n_{i1}, -n_{i2}, \ldots, -n_{iK}\right]^T. \quad (10)$$

Employing the 4×4 QOSTBC matrix in (4) and multiplying $\frac{\mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i(t))}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2}$ on both sides of (7), we have

$$\begin{aligned} \hat{\mathbf{s}}_i &\triangleq \frac{1}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2} \mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i(t))\tilde{\mathbf{y}}_i \\ &= \mathbf{G}_i \mathbf{s}_i + \frac{1}{\|\mathbf{W}_i^H \mathbf{h}_i\|_2^2} \mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i)(\tilde{\mathbf{i}}_i + \tilde{\mathbf{n}}_i) \end{aligned} \quad (11)$$

where

$$\mathbf{G}_i \triangleq \frac{\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i)\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i)}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^2} = \begin{bmatrix} 1 & 0 & -g_i & 0 \\ 0 & 1 & 0 & g_i \\ g_i & 0 & 1 & 0 \\ 0 & -g_i & 0 & 1 \end{bmatrix}, \quad (12)$$

$$g_i \triangleq \frac{2\mathrm{Im}\{\mathbf{w}_{i1}^H\mathbf{h}_i(t)\mathbf{h}_i(t)^H\mathbf{w}_{i3} - \mathbf{w}_{i2}^H\mathbf{h}_i(t)\mathbf{h}_i^H(t)\mathbf{w}_{i4}\}}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^2} j, \quad (13)$$

$\mathrm{Im}\{\cdot\}$ denotes the imaginary part of a complex scalar and $j = \sqrt{-1}$. We observe in (12) that g_i and $-g_i$ represent inter-symbol interference terms for $\hat{\mathbf{s}}_i$. Due to the quasi-orthogonal property of the equivalent channel matrix as in (12), pairwise ML detection is the optimum detection for information symbols transmitted with QOSTBC, however, it is associated with a decoding complexity increase as compared to symbol-wise decoding [26]. To enhance the characteristics of the equivalent MIMO channel in (7) and reduce the decoding complexity by enabling simple symbol-by-symbol detection, we design the beamforming matrices $\mathbf{W}_i$ such that the quasi-orthogonal equivalent channel matrix is further orthogonalized. The orthogonalization of (12) requires knowledge of instantaneous CSI, i.e., $\mathbf{h}_i(t)$, which is not known at the transmitter. Therefore, here we consider the average inter-symbol interference power defined as

$$\bar{g}_i \triangleq \mathrm{E}\{g_i\} = \frac{2\mathrm{Im}\{\mathbf{w}_{i1}^H\mathbf{R}_i\mathbf{w}_{i3} - \mathbf{w}_{i2}^H\mathbf{R}_i\mathbf{w}_{i4}\}}{\mathrm{Tr}(\mathbf{W}_i^H\mathbf{R}_i\mathbf{W}_i)} j. \quad (14)$$

In order to achieve the best decoding performance, the average inter-symbol interference in $\hat{\mathbf{s}}_i$ should be adjusted to null, i.e.,

$$|\bar{g}_i|^2 = 0. \quad (15)$$

For a given beamformer $\mathbf{W}_i^\star \triangleq \left[\mathbf{w}_{i1}^\star, \ldots, \mathbf{w}_{iK}^\star\right]$, a sufficient but not necessary condition for satisfying (15) is

$$\begin{cases} \mathrm{Im}\{\mathbf{w}_{i1}^{\star H}\mathbf{R}_i\mathbf{w}_{i3}^\star\} = 0 \\ \mathrm{Im}\{\mathbf{w}_{i2}^{\star H}\mathbf{R}_i\mathbf{w}_{i4}^\star\} = 0. \end{cases} \quad (16)$$

To satisfy (16), phase rotation can be performed on beamformer $\mathbf{W}_i^\star$ in various ways, e.g.,

$$\begin{cases} \mathbf{w}_{i1}'^\star \triangleq \mathbf{w}_{i1}^\star \exp(j\angle(\mathbf{w}_{i1}^{\star H}\mathbf{R}_i\mathbf{w}_{i3}^\star)) \\ \mathbf{w}_{i2}'^\star \triangleq \mathbf{w}_{i2}^\star \exp(j\angle(\mathbf{w}_{i2}^{\star H}\mathbf{R}_i\mathbf{w}_{i4}^\star)) \end{cases} \quad (17)$$

where $\angle(\cdot)$ denotes the argument of a complex scalar.

Based on (11), the covariance matrix of the received multiuser interference contained in $\hat{\mathbf{s}}_i$ is given by

$$\begin{aligned} \mathbf{C}_i^{(\mathrm{I})} &\triangleq \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^4}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i(t))\mathrm{E}\{\tilde{\mathbf{i}}_i\tilde{\mathbf{i}}_i^H\}\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i(t)) \\ &= \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^4}\Big[\sum_{m=1,m\neq i}^{M}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i(t))\mathcal{X}(\mathbf{W}_m^H\mathbf{h}_i(t)) \times \\ &\quad \mathcal{X}^H(\mathbf{W}_m^H\mathbf{h}_i(t))\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i(t))\Big]. \end{aligned} \quad (18)$$

Note that $\mathbf{C}_i^{(\mathrm{I})}$ exhibits the same sparsity structure as $\mathbf{G}_i$ in (12). Furthermore, it can be shown that as the number of users increases $\mathbf{C}_i^{(\mathrm{I})}$ converges to a scaled identity matrix, i.e., the interference covariance matrix becomes spatially white. Applying Lemma 1 in [15], the average multiuser interference power of the i-th user in the k-th time slot can be expressed as

$$[\mathbf{C}_i^{(\mathrm{I})}]_{kk} \triangleq \frac{1}{\mathrm{Tr}(\mathbf{W}_i^H\mathbf{R}_i\mathbf{W}_i)}\sum_{m=1,m\neq i}^{M}\mathrm{Tr}(\mathbf{W}_m^H\mathbf{R}_i\mathbf{W}_m). \quad (19)$$

Based on (11), the covariance matrix of the noise in $\hat{\mathbf{s}}_i$ is given by

$$\begin{aligned} \mathbf{C}_i^{(\mathrm{N})} &\triangleq \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^4}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i(t))\mathrm{E}\{\tilde{\mathbf{n}}_i\tilde{\mathbf{n}}_i^H\}\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i(t)) \\ &= \frac{\sigma_i^2}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^2}\mathbf{I}_K. \end{aligned} \quad (20)$$

The average noise power of the i-th user in the k-th time slot can be expressed as

$$[\mathbf{C}_i^{(\mathrm{N})}]_{kk} \triangleq \frac{\sigma_i^2}{\mathrm{Tr}(\mathbf{W}_i^H\mathbf{R}_i\mathbf{W}_i)}. \quad (21)$$

Then, the average post detection SINR corresponding to symbol s_{ik} in the proposed general rank beamforming approach is given by

$$\begin{aligned} \mathrm{SINR}_{\mathrm{g},i}(s_{ik}) &\triangleq \frac{\mathrm{E}\{s_{ik}s_{ik}^*\}}{|\bar{g}_i|^2\,\mathrm{E}\{s_{ik'}s_{ik'}^*\} + [\mathbf{C}_i^{(\mathrm{I})}]_{kk} + [\mathbf{C}_i^{(\mathrm{N})}]_{kk}} \\ &= \frac{\mathrm{Tr}(\mathbf{W}_i^H\mathbf{R}_i\mathbf{W}_i)}{\sum_{m=1,m\neq i}^{M}\mathrm{Tr}(\mathbf{W}_m^H\mathbf{R}_i\mathbf{W}_m) + \sigma_i^2}, \end{aligned} \quad (22)$$

where k' is the index number of the entry g_i or $-g_i$ in the k-th row of G_i in (12). Note that the designed average orthogonality property resulting from (15) is used in deriving $\mathrm{SINR}_{\mathrm{g},i}(s_{ik})$. Since the expression of $\mathrm{SINR}_{\mathrm{g},i}(s_{ik})$ in (22) is independent of the time index k, the average post detection SINR for the i-th user is identical for all symbols in the QOSTBC block. The total transmit power in each time slot equals $\sum_{i=1}^{M}\mathrm{Tr}(\mathbf{W}_i\mathbf{W}_i^H)$ which can be computed in a similar way as in [15]. With multiple beamformers designed for each user, the additional shaping constraints in (3c) can be expressed as

$$\sum_{m=1}^{M}\mathrm{Tr}(\mathbf{A}_{lm}\mathbf{W}_m\mathbf{W}_m^H) \trianglerighteq_l b_l,\ \forall l = 1, \ldots, L. \quad (23)$$

4. BEAMFORMER OPTIMIZATION

The optimization problem of maximizing the minimum average SINR in (22) of all users subject to the power constraint and additional shaping constraints can be formulated as

$$\max_{\{\mathbf{W}_i\}_{i=1}^M, t}\ t \text{ s.t. } \frac{\mathrm{Tr}(\mathbf{W}_i^H\mathbf{R}_i\mathbf{W}_i)}{\sum_{m=1,m\neq i}^{M}\mathrm{Tr}(\mathbf{W}_m^H\mathbf{R}_i\mathbf{W}_m) + \sigma_i^2} \geq t, \forall i = 1, ..., M \quad (24a)$$

$$\sum_{i=1}^{M}\mathrm{Tr}(\mathbf{W}_i\mathbf{W}_i^H) \leq \mathrm{P}_{\max} \quad (24b)$$

$$\sum_{m=1}^{M}\mathrm{Tr}(\mathbf{A}_{lm}\mathbf{W}_m\mathbf{W}_m^H) \trianglerighteq_l b_l, \forall l = 1, ..., L. \quad (24c)$$

To solve problem (24), let us employ the SDR approach and define $\mathbf{X}_i \triangleq \mathbf{W}_i\mathbf{W}_i^H$. By substituting $\mathbf{X}_i$ and omitting the rank constraints $\mathrm{rank}(\mathbf{X}_i) \leq K$, a relaxed optimization problem is obtained as

$$\max_{\{\mathbf{X}_i\}_{i=1}^M, t}\ t \quad \text{s.t. } \frac{\mathrm{Tr}(\mathbf{X}_i\mathbf{R}_i)}{\sum_{m=1,m\neq i}^{M}\mathrm{Tr}(\mathbf{X}_m\mathbf{R}_i) + \sigma_i^2} \geq t, \forall i = 1, \ldots, M \quad (25a)$$

$$\sum_{i=1}^{M}\mathrm{Tr}(\mathbf{X}_i) \leq \mathrm{P}_{\max} \quad (25b)$$

$$\sum_{m=1}^{M}\mathrm{Tr}(\mathbf{A}_{lm}\mathbf{X}_m) \trianglerighteq_l b_l, \forall l = 1, \ldots, L \quad (25c)$$

$$\mathbf{X}_i \succeq \mathbf{0}, \forall i = 1, \ldots, M \quad (25d)$$

which can be solved efficiently by performing a one-dimensional bisection search over t as in [5] using convex optimization solvers such as CVX [28]. Denote $\{\mathbf{X}_i^\star\}_{i=1}^M$ as an optimal solution to problem (25). Then we can apply the rank reduction algorithm in [15] with the input $\{\mathbf{X}_i^\star\}_{i=1}^M$ to reduce the rank of the optimal solution. If the updated $\{\mathbf{X}_i^\star\}_{i=1}^M$ after the rank reduction procedure satisfies $4 < \max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) \le 8$, we choose K=8; if $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) \le 4$, we choose K=4. Note that if $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) \le 2$, the proposed approach is equivalent to the rank-one or rank-two approaches. The corresponding beamforming matrices are calculated by eigenvalue decomposition on $\{\mathbf{X}_i^\star\}_{i=1}^M$ followed by the proposed phase rotation procedure as defined in (17). In the case that $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) > 8$, we choose K=8 and a randomization procedure can be used to obtain a suboptimal solution to problem (24). Similar as the general rank beamforming approach in [15], each user is served with up to eight beamformers in the proposed general rank beamforming, and a maximum number of 79 additional shaping constraints can be accommodated for which an optimal solution can be obtained.

5. SIMULATIONS

In the simulation, we consider the downlink beamformer design that limits the interference to co-channel users which is similar to Example 2 in Sec. VI in [15]. The base station is equipped with a uniform linear array of N=15 antennas spaced half a wavelength apart. There are three downlink users located at θ_1=$-7°$, θ_2=$10°$ and θ_3=$27°$ relative to the array broadside. The downlink users are assumed to be surrounded by a large number of local scatterers corresponding to an angular spread of σ_θ, as seen from the base station. The channel covariance matrices $\mathbf{R}_i$ are calculated in the same way as in [2]. Moreover, there are 19 co-channel users connected to a neighboring base station which are located at $\mu_{1,\ldots,19} = [-89.375°, -80°, -70.625°, -61.25°, -51.875°, -42.5°, -33.125°, -30°, -23.75°, -15°, 2°, 18°, 36°, 43.75°, 49°, 53.125°, 62.5°, 71.875°, 81.25°]$. The interference power at the direction μ_l in each time slot $f(\mu_l) = \sum\limits_{m=1}^{3} \operatorname{Tr}(\mathbf{h}_{\mu_l}\mathbf{h}_{\mu_l}^H\mathbf{X}_m)$ is upper bounded by $b_l = 0.5$, and $\mathbf{h}_{\mu_l}$ is the channel vector corresponding to μ_l. In addition to these constraints, the interference power at μ_l is ensured to obtain a local minimum value by adding interference derivative constraints, i.e., $-\epsilon_a \le \frac{df(\mu_l)}{d\mu_l} \le \epsilon_a$ and $\frac{d^2 f(\mu_l)}{d\mu_l^2} > 0\ \forall l$ where the threshold is set to $\epsilon_a = 10^{-5}$, and $\frac{df(\mu_l)}{d\mu_l}$ and $\frac{d^2 f(\mu_l)}{d\mu_l^2}$ are computed in the same way as in Example 2 in Sec. VI in [15]. We assume σ_i^2=0.1 $\forall i$ and $\mathrm{P}_{\max}$=1. The results are averaged over 100 independent Monte-Carlo runs in which all angles of departures are subject to variations defined in the same way as in Example 1 in Sec. VI in [15]. In each run, 200 instantaneous channel realizations are generated for each downlink user obeying the distribution corresponding to $\mathbf{R}_i$, and 100 symbols are transmitted within each instantaneous channel realization. The number of randomization samples in each run is set to 100 for all approaches if necessary and QPSK modulation is used.

In this example, we compare the proposed approach with the existing ones. The code dimension K in the proposed approach is chosen as K = 4 since $2 < \max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) \le 4$. In Fig. 1, the worst SINR for different spread angles is displayed. As shown in Fig. 1, the proposed approach achieves much higher SINR than that of the rank-one and rank-two approaches which is zero for all spread angles which can be understood as infeasible in practice. In Fig. 2, the worst-user symbol-error-rate (SER) for different spread angles is displayed. In the legend of Fig. 2, 'GR' refers to the general rank approach; 'qs' and 'rl' refer to the use of QOSTBC and real-valued OSTBC, respectively; 'PCR' refers to the phase rotation when $\mathbf{R}_i$ is approximated by its principal component $\mathbf{h}_i^{(p)}$ and the phases of beamformers are rotated to fulfill $\operatorname{Im}\{\mathbf{W}_i^H\mathbf{h}_i^{(p)}\}$=0 $\forall i$ as in [15]; 'PR', 'RR' and 'AR' refer to the proposed phase rotation in (17), random phase rotation, and the phase rotation of using instantaneous CSI which is an ideal case, respectively; 'SW' and 'ML' refer to symbol-wise and ML decoder, respectively. As shown in Fig. 2, QOSTBC based beamforming approaches achieve much better performance than real-valued OSTBC based beamforming approaches. 'GR (qs PR ML)' achieves only slightly worse performance than 'GR (qs AR SW)' which serves as the unachievable lower bound, and is better than all other approaches. 'GR (qs PR SW)' achieves better performance than all other symbol-wise decoders.

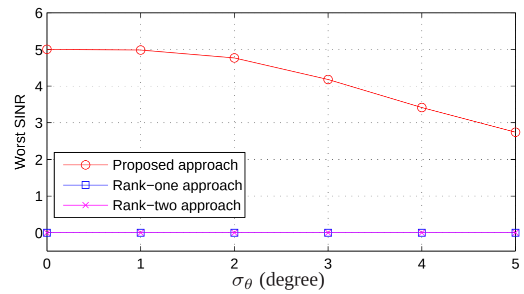

Fig. 1: Worst SINR versus varying spread angles

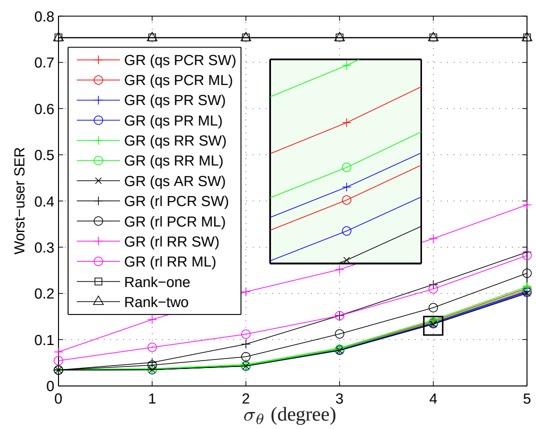

Fig. 2: Worst-user SER versus varying spread angles

6. CONCLUSION

In this paper, we propose a general rank beamforming approach for the multiuser downlink beamforming problem with additional shaping constraints exploiting covariance based CSI at the transmitter. The proposed general rank beamforming approach increases the degrees of freedom in the beamformer design by using QOSTBC. Besides the pairwise decoding for QOSTBC, a phase rotation procedure on beamformers is proposed to enable simplified symbol-wise decoding. The proposed general rank beamforming approach significantly outperforms the conventional rank-one and rank-two approaches and real-valued OSTBC based general rank approach.

References

[1] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*. CRC Press, 2001.

[2] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using semidefinite optimization," in *Annual Allerton Conference on Communication, Control and Computing*, 1999, vol. 37, pp. 987–996.

[3] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming: From receive to transmit and network designs," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 62–75, May 2010.

[4] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[5] A Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 161–176, Jan. 2006.

[6] J. Y. A. Zhang and A. M.-C. So, "Optimal spectrum sharing in MIMO cognitive radio networks via semidefinite programming," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 2, pp. 362–373, Feb. 2011.

[7] S. M. Cai and Y. Gong, "Cognitive beamforming for multiple secondary data streams with individual MIMO constraints," *IEEE Transactions on Signal Processing*, vol. 61, no. 17, pp. 4189–4198, Sep. 2013.

[8] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Transactions on Wireless Communications*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[9] C. Xing, N. Wang, J. Ni, Z. Fei, and J. Kuang, "MIMO beamforming designs with partial CSI under energy harvesting constraints," *IEEE Signal Processing Letters*, vol. 20, no. 4, pp. 4010–4024, Apr. 2013.

[10] S. Zhang, A. Seyedi, and B. Sikdar, "An analytical approach to the design of energy harvesting wireless sensor nodes," *IEEE Transactions on Wireless Communications*, vol. 12, no. 8, pp. 4010–4024, Aug. 2013.

[11] Y. Huang and D.P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 664–678, Feb. 2010.

[12] Y. Huang and D.P. Palomar, "A dual perspective on separable semidefinite programming with applications to optimal downlink beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4254–4271, Aug. 2010.

[13] D. Hammarwall, M. Bengtsson, and B. Ottersten, "An efficient algorithm for solving the downlink beamforming problem with indefinite constraints," in *IEEE International Conference on Acoustics, Speech, Signal Processing (ICASSP 2005)*, 2005, vol. 4, pp. iv/905–iv/908.

[14] D. Hammarwall, M. Bengtsson, and B. Ottersten, "On downlink beamforming with indefinite shaping constraints," *IEEE Transactions on Signal Processing*, vol. 54, no. 9, pp. 3566 –3580, Sep. 2006.

[15] K. L. Law, X. Wen, M. T. Vu, and M. Pesavento, "General rank multiuser downlink beamforming with shaping constraints using real-valued OSTBC," *IEEE Transactions on Signal Processing*, vol. 63, no. 21, pp. 5758–5771, Nov. 2015.

[16] S. X. Wu, A. M.-C. So, and W.-K. Ma, "Rank-two transmit beamformed Alamouti space-time coding for physical-layer multicasting," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2012)*, Mar. 2012, pp. 2793–2796.

[17] X. Wen, K.L. Law, S.J. Alabed, and M. Pesavento, "Rank-two beamforming for single-group multicasting networks using OSTBC," in *IEEE 7th Sensor Array and Multichannel Signal Processing Workshop (SAM 2012)*, 2012, pp. 69–72.

[18] S. X. Wu, W.-K. Ma, and A. M.-C. So, "Physical-layer multicasting by stochastic transmit beamforming and Alamouti space-time coding," *IEEE Transactions on Signal Processing*, vol. 61, no. 17, pp. 4230–4245, Sep. 2013.

[19] A. Schad, K.L. Law, and M. Pesavento, "A convex inner approximation technique for rank-two beamforming in multicasting relay networks," in *Proceedings of the 20th European Signal Processing Conference (EUSIPCO 2012)*, Aug. 2012, pp. 1369–1373.

[20] S. Ji, S. X. Wu, A. M.-C. So, and W.-K. Ma, "Multi-group multicast beamforming in cognitive radio networks via rank-two transmit beamformed alamouti space-time coding," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2013)*, May 2013, pp. 4409–4413.

[21] K.L. Law, X. Wen, and M. Pesavento, "General-rank transmit beamforming for multi-group multicasting networks using OSTBC," in *IEEE 13th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2013)*, Jun. 2013, pp. 475–479.

[22] B. Zhu, J. Ge, X. Huang, Y. Yang, and M. Lu, "Rank-two beamformed secure multicasting for wireless information and power transfer," *IEEE Signal Processing Letters*, vol. 21, no. 2, pp. 3435–3447, Feb. 2014.

[23] A. Schad, K.L. Law, and M. Pesavento, "Rank-two beamforming and power allocation in multicasting relay networks," *IEEE Transactions on Signal Processing*, vol. 63, no. 13, pp. 3435–3447, Jul. 2015.

[24] Z.-Q. Luo, W.-K. Ma, A.M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, May 2010.

[25] S. Boyd and L. Vandenberghe, *Convex optimization*, Cambridge University Press, 2004.

[26] H. Jafarkhani, *Space-Time Coding: Theory and Practice*, Cambridge University Press, New York, NY, USA, 1st edition, 2005.

[27] R. Grover, W. Su, and D. A. Pados, "An 8×8 Quasi-Orthogonal STBC form for transmissions over eight or four antennas," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 4777–4785, Dec. 2008.

[28] M. Grant, S. Boyd, and Y. Ye, "CVX: Matlab software for disciplined convex programming," *Online accessiable: http://stanford. edu/˜ boyd/cvx*, 2008.