# ACHIEVING GLOBAL OPTIMALITY FOR WIRELESSLY-POWERED MULTI-ANTENNA TWRC WITH LATTICE CODES

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## ABSTRACT

In this paper, we consider the joint optimization of relay transmit-receive beamformers, users' transmit powers, and users' power splitting ratios in wirelessly-powered two-way relay channel under data-rate quality-of-service constraints. In order to solve the problem, we first establish that the uplink data-rate constraints would be active at the global optimum. Then we transform it into an equivalent problem by introducing slack variables and applying the linear matrix inequalities. Based on the transformed problem, the global optimal solution is derived. Numerical results on network power consumption versus circuit power and data-rate QoS show that the proposed algorithm outperforms existing algorithms.

***Index Terms***— Wireless power transfer, two-way relay channel, lattice codes, global optimal

## 1. INTRODUCTION

While embracing wireless power transfer (WPT) in energy-constrained communication networks brings opportunities [1], a critical challenge that needs to be overcome is the high propagation path-loss during energy transmission [2]. Fortunately, the beamforming gain in multiple-input-multiple-output (MIMO) systems offers a viable option for mitigating such problem [3]. Therefore, WPT combined with MIMO systems has been a focus of research lately [4].

In the context of WPT, the harvest-then-transmit protocol is recently proposed in [5], which enables energy-harvesting terminals to transmit data, and ignites the researches on wirelessly-powered communication networks (WPCNs) [6]. In WPCNs, the harvested energy at users from the access point supports subsequent uplink transmission, and thus its uplink and downlink are coupled. Furthermore, when the access points are equipped with multiple antennas, the optimization becomes challenging [7]. Specifically, in wirelessly-powered multi-antenna two-way relay channel (TWRC) [8], the joint optimization of relay transmit-receive beamformers, users' transmit powers, and users' power splitting ratios is very difficult to solve, and an alternating optimization algorithm converging to a suboptimal solution is proposed in [9].

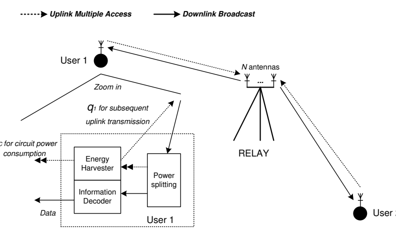



**Fig. 1**. System model of wirelessly-powered TWRC

However, obtaining the global optimal solution is still an open problem.

In this paper, we first prove that the uplink date-rate constraints would be active at the global optimum. Then, we transform the problem by introducing slack variables, and applying the linear matrix inequalities (LMIs). Finally, global optimal solution is obtained via rank-one guaranteed semi-definite relaxation (SDR). Numerical results show that the proposed scheme achieves the lowest power consumption compared to existing algorithms.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a TWRC consisting of a multi-antenna relay station with $N$ antennas, and 2 single-antenna users. As shown in Fig. 1, the 2 users intend to exchange messages with each other through the relay using lattice based compute-and-forward network coding, as it achieves Shannon capacity within 1/2 bit [10] and has a lower complexity than decode-and-forward scheme. The transmission consists of uplink multiple access phase and downlink broadcast phase, each with time duration of $M$ symbols.

### 2.1. Uplink Multiple Access Phase with Lattice Codes

In uplink multiple access phase, given $M \times 1$ doubly nested lattice [10] $\Lambda_1 \subseteq \Lambda_2 \subseteq \Lambda$, we generate lattice codebook $L_i = \{\Lambda \bmod \Lambda_i\}$ for $i = 1, 2$. For user $i$, the source data is mapped into $\mathbf{c}_i \in L_i$ and the symbols to be transmitted are

$$\mathbf{x}_i = (\mathbf{w}^H \mathbf{h}_i)^{-1}[(\mathbf{c}_i + \mathbf{d}_i) \text{mod } \Lambda_i] \in \mathbb{C}^{M\times 1}, \quad (1)$$

where $\mathbf{d}_i \in \mathbb{C}^{M\times 1}$ is a pre-generated random dither vector known to the users and relay. The term $(\mathbf{w}^H\mathbf{h}_i)^{-1}$ is applied to offset the spatial gains, where $\mathbf{w}^H \in \mathbb{C}^{1\times N}$ with $||\mathbf{w}|| = 1$ is the receive beamforming vector at relay. With the second moments of $\Lambda_i$ set to $\sigma^2(\Lambda_i) = q_i|\mathbf{w}\mathbf{h}_i|^2$, the transmit power of $\mathbf{x}_i$ is $\frac{1}{M}\mathbb{E}[||\mathbf{x}_i||^2] = q_i$. Then the received signal $\mathbf{Y} \in \mathbb{C}^{N\times M}$ at the relay is given by

$$\mathbf{Y} = \mathbf{h}_1\mathbf{x}_1^T + \mathbf{h}_2\mathbf{x}_2^T + \mathbf{N}, \quad (2)$$

where $\mathbf{h}_i \in \mathbb{C}^{N\times 1}$ denotes the uplink channel vector, and $\mathbf{N} \in \mathbb{C}^{N\times M}$ is the Gaussian noise with $\mathbb{E}[\text{vec}(\mathbf{N})\text{vec}(\mathbf{N})^H] = \sigma_r^2\mathbf{I}_{MN}$. To achieve spatial gains, a receive beamformer $\mathbf{w}^H$ is applied to $\mathbf{Y}$. Putting (1) into (2), we have $\mathbf{w}^H\mathbf{Y} = [\sum_{i=1}^{2}(\mathbf{c}_i + \mathbf{d}_i)\text{mod } \Lambda_i]^T + \mathbf{w}^H\mathbf{N}$, where $\frac{1}{M}\mathbb{E}[||\mathbf{w}^H\mathbf{N}||^2] = \sigma_r^2$. Therefore we can express the uplink SINR of the $i^{\text{th}}$ user as $\Gamma_i^{MA} = q_i|\mathbf{w}^H\mathbf{h}_i|^2/\sigma_r^2$. Furthermore, applying the results from [10, Theorem 3], the uplink achievable rate $R_i^{MA}$ from the $i^{\text{th}}$ user to the relay can be computed to be

$$R_i^{MA} = \frac{1}{2}\Big[\log\Big(\frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sum_{j=1}^{2} q_j|\mathbf{w}^H\mathbf{h}_j|^2} + \Gamma_i^{MA}\Big)\Big]^+, \quad (3)$$

where $[x]^+ = \max(x, 0)$.

### 2.2. Downlink Broadcast Phase with WPT

In the downlink broadcast phase, the relay first computes

$$\Big\{(1 + \frac{\sigma_r^2}{\sum_{j=1}^{2} q_j|\mathbf{w}^H\mathbf{h}_j|^2})^{-1}\Big(\mathbf{w}^H\mathbf{Y}\Big)^T - \Sigma_{i=1}^{2}\mathbf{d}_i\Big\}\text{mod } \Lambda_1, \quad (4)$$

and then maps the result of (4) to a symbol $\mathbf{s} \in L_r$, where $L_r$ is the lattice codebook at relay. The detail procedure can be found in [9, 10].

After the above procedure, the relay broadcasts $\mathbf{s}$ through the corresponding transmit beamforming vector $\mathbf{v} \in \mathbb{C}^{N\times 1}$ with $||\mathbf{v}|| = 1$. The received signal $\mathbf{r}_i^T \in \mathbb{C}^{1\times M}$ at the user $i$ is

$$\mathbf{r}_i^T = \mathbf{g}_i^H\mathbf{v}\mathbf{s}^T + \mathbf{n}_i^T, \quad (5)$$

where $\mathbf{g}_i^H \in \mathbb{C}^{1\times N}$ is the downlink channel vector from the relay to the $i^{\text{th}}$ user, and $\mathbf{n}^T \in \mathbb{C}^{1\times M}$ is the Gaussian noise at the $i^{\text{th}}$ user with $\mathbb{E}[\mathbf{n}_i\mathbf{n}_i^H] = \sigma_u^2\mathbf{I}_M$. In the spirit of WPT, the received signal at the $i^{\text{th}}$ user in the downlink is further splitted into two branches, one for the information decoder and the other for the energy harvester.

At the information decoder side, the signal is given by

$$\widetilde{\mathbf{r}}_i^T = \sqrt{\beta_i}\mathbf{g}_i^H\mathbf{v}_k\mathbf{s}^T + \sqrt{\beta_i}\mathbf{n}_i^T + \mathbf{z}_i^T, \quad (6)$$

where $\beta_i$ is the splitting factor, and $\mathbf{z}_i^T \in \mathbb{C}^{1\times M}$ is Gaussian noise introduced by the power splitter, with $\mathbb{E}[\mathbf{z}_i\mathbf{z}_i^H] = \sigma_z^2\mathbf{I}_M$. Based on (6), the downlink SINR for the $i^{\text{th}}$ user is $\Gamma_i^{BC} = \beta_i p|\mathbf{g}_i^H\mathbf{v}|^2/(\beta_i\sigma_u^2 + \sigma_z^2)$. Then applying [10, Theorem 3], the downlink achievable rate $R_i^{BC}$ at the $i^{\text{th}}$ user is expressed as

$$R_i^{BC} = \frac{1}{2}\log\Big(1 + \Gamma_i^{BC}\Big). \quad (7)$$

On the other hand, the average harvested power from the wireless signals at user $i$ can be expressed as $\eta(1 - \beta_i)\mathbb{E}[||\mathbf{r}_i||^2]/M$, where $0 < \eta < 1$ is power conversion efficiency. Based on (5), it can be further expressed as $\eta(1-\beta_i)\Big(p|\mathbf{g}_i^H\mathbf{v}|^2 + \sigma_u^2\Big)$.

### 2.3. Problem Formulation

With the system model above, and assuming that the rate requirement from the $i^{\text{th}}$ user is $\overline{R}_i > 0$, then we have $R_i^{MA} \geq \overline{R}_i$ and $R_{3-i}^{BC} \geq \overline{R}_i$. On the other hand, since the user transmit power $q_i$ is harvested from the downlink wireless signal, we must have

$$\eta(1-\beta_i)\Big(|\mathbf{g}_i^H\mathbf{v}|^2) + \sigma_u^2\Big) - 2p_c \geq q_i, \quad (8)$$

where $p_c$ is the circuit power consumption per symbol time, and the coefficient 2 is due to the two phases of transmission.

Having the QoS and power harvesting requirements satisfied, it is crucial to reduce the total transmit power at relay and users because energy efficiency translates to cost reduction and environmental benefits:

$$\begin{aligned}
\mathcal{P}1: \min_{\mathbf{v},\mathbf{w},p,\{q_i,\beta_i\}} \quad & p + q_1 + q_2 \\
\text{s.t.} \quad & \frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sum_{j=1}^{2} q_j|\mathbf{w}^H\mathbf{h}_j|^2} + \frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sigma_r^2} \geq 2^{2\overline{R}_i}, \ \forall i \in \{1,2\} \\
& 1 + \frac{\beta_i p|\mathbf{g}_i^H\mathbf{v}|^2}{\beta_i\sigma_u^2 + \sigma_z^2} \geq 2^{2\overline{R}_{3-i}}, \ \forall i \in \{1,2\} \\
& \eta(1-\beta_i)\Big(p|\mathbf{g}_i^H\mathbf{v}|^2) + \sigma_u^2\Big) - 2p_c \geq q_i, \ \forall i \in \{1,2\} \\
& q_i \geq 0, \ \beta_i \in (0,1), \ \forall i \in \{1,2\} \\
& p \geq 0, \ ||\mathbf{v}|| = 1, \ ||\mathbf{w}|| = 1,
\end{aligned}$$

where the first and second constraints are the uplink and downlink data-rate constraints, respectively.

Problem $\mathcal{P}1$ is difficult to solve, since the first three constraints contain nonconvex quadratic terms of $\mathbf{v}$ and $\mathbf{w}$, which are further nonlinearly coupled with $\beta_i$, $p$ and $q_i$. Even its special case of dualcast problem is shown to be hard [11].

## 3. GLOBAL OPTIMAL SOLUTION

To proceed to solve $\mathcal{P}1$, we first have the following property.

**Property 1.** *The optimal solution of $\mathcal{P}1$ must activate the first constraint for both users.*

*Proof.* For convenience of presentation, we define the left hand side of the first constraint of $\mathcal{P}1$ as

$$\Delta_i := \frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sum_{j=1}^2 q_j|\mathbf{w}^H\mathbf{h}_j|^2} + \frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sigma_r^2} \tag{9}$$

$$= 1\Big/\Big(1 + \frac{q_{3-i}|\mathbf{w}^H\mathbf{h}_{3-i}|^2}{q_i|\mathbf{w}^H\mathbf{h}_i|^2}\Big) + \frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sigma_r^2}, \tag{10}$$

which is obtained by dividing the numerator and denominator of the first term in (9) by $q_i|\mathbf{w}^H\mathbf{h}_i|^2$. Now it is clear that $\Delta_i$ is a increasing function of $q_i$, and a decreasing function of $q_{3-i}$. That is, decreasing $q_i$ would decrease the $i^{\text{th}}$ user's uplink data-rate, while benefitting the $(3-i)^{\text{th}}$ user.

Using the above result, if the first constraint is not active for user $i_0$, we can always decrease $q_{i_0}$ until $\Delta_{i_0} = 2^{2\overline{R}_{i_0}}$ holds while the uplink data-rate constraints of user $(3-i_0)$ will remain satisfied. Therefore at the optimal solution of $\mathcal{P}1$, the first constraints of both users must be activated. □

Using the result from **Property 1**, we can transform the first inequality constraint into equality, i.e.,

$$\frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sum_{j=1}^2 q_j|\mathbf{w}^H\mathbf{h}_j|^2} + \frac{q_i|\mathbf{w}^H\mathbf{h}_i|^2}{\sigma_r^2} = 2^{2\overline{R}_i}, \ \forall i, \tag{11}$$

without changing the optimal solution of $\mathcal{P}1$ [12]. Dividing (11) with $i=1$ by that with $i=2$, we have $\frac{q_1|\mathbf{w}^H\mathbf{h}_1|^2}{q_2|\mathbf{w}^H\mathbf{h}_2|^2} = \frac{2^{2\overline{R}_1}}{2^{2\overline{R}_2}}$, which implies that

$$\frac{q_i|\mathbf{w}^H\mathbf{h}_1|^2}{\sum_{j=1}^2 q_j|\mathbf{w}^H\mathbf{h}_j|^2} = \frac{2^{2\overline{R}_i}}{2^{2\overline{R}_1} + 2^{2\overline{R}_2}} = \alpha_i, \ \forall i. \tag{12}$$

Putting (12) back into (11), the first constraint of $\mathcal{P}1$ becomes

$$q_i|\mathbf{w}^H\mathbf{h}_i|^2 = \alpha_i\sigma_r^2, \ \forall i. \tag{13}$$

To reduce the problem dimension of $\mathcal{P}1$, the following property can be established for the beamforming vectors.

**Property 2.** *The optimal $\mathbf{v}^* \in \text{span}\{\mathbf{g}_1, \mathbf{g}_2\}$, and the optimal $\mathbf{w}^* \in \text{span}\{\mathbf{h}_1, \mathbf{h}_2\}$.*

The proof of this property can be completed by contradiction. That is, by assuming the beamformers contain orthogonal elements, and if we find a solution with smaller objective value, then the contradiction is obtained. Due to the space limitation, the detail is omitted here.

Based on **Property 2**, we put $\mathbf{v} = a_1\mathbf{g}_1 + a_2\mathbf{g}_2$ and $\mathbf{w} = b_1\mathbf{h}_1 + b_2\mathbf{h}_2$ into $\mathcal{P}1$, where $a_1, a_2, b_1, b_2 \in \mathbb{C}$ are parameterization coefficients. Furthermore, since $\alpha_i > 0$ in (13), we must have $q_i \neq 0$. Now, letting $q_i = \frac{1}{\xi_i}$ to decouple user power and beamformer in (13), we arrive at the following equivalent problem to $\mathcal{P}1$

$$\mathcal{P}2: \min_{a_1,a_2,b_1,b_2,p,\{\xi_i,\beta_i\}} \ p + \frac{1}{\xi_1} + \frac{1}{\xi_2}$$

$$\text{s.t.} \quad [b_1\ b_2]\mathbf{\Xi}_i[b_1\ b_2]^H = \alpha_i\sigma_r^2\xi_i, \ \forall i \tag{14a}$$

$$\beta_i\Big(\frac{p[a_1\ a_2]\mathbf{\Phi}_i[a_1\ a_2]^H}{\theta_i} - \sigma_u^2\Big) \geq \sigma_z^2, \ \forall i \tag{14b}$$

$$\eta(1-\beta_i)\Big(p[a_1\ a_2]\mathbf{\Phi}_i[a_1\ a_2]^H + \sigma_u^2\Big) \geq \frac{1}{\xi_i} + 2p_c, \ \forall i \tag{14c}$$

$$p \geq 0, \ \xi_i > 0, \ \beta_i \in (0,1), \ \forall i \tag{14d}$$

$$||\mathbf{G}[a_1\ a_2]^T|| = ||\mathbf{H}[b_1\ b_2]^T|| = 1, \tag{14e}$$

where the channel vectors are stacked into $\mathbf{G} = [\mathbf{g}_1\ \mathbf{g}_2]$ and $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2]$, and the constant matrices are given by $\mathbf{\Phi}_i = \mathbf{G}^H\mathbf{g}_i\mathbf{g}_i^H\mathbf{G}$ and $\mathbf{\Xi}_i = \mathbf{H}^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{H}$.

However, since the terms $a_1, a_2, b_1, b_2$ in $\mathcal{P}2$ are quadratic and still nonconvex, we need further change of variables. More specifically, introducing $\mathbf{A} = p[a_1\ a_2]^H[a_1\ a_2] \succeq 0$ and $\mathbf{B} = [b_1\ b_2]^H[b_1\ b_2] \succeq 0$ with $\text{Rank}(\mathbf{A}) = \text{Rank}(\mathbf{B}) = 1$, then constraint (14a) becomes $\text{Tr}(\mathbf{\Xi}_i\mathbf{B}) = \alpha_i\sigma_r^2\xi_i$, and constraint (14e) becomes $\text{Tr}(\mathbf{G}^H\mathbf{G}\mathbf{A}) = p$ and $\text{Tr}(\mathbf{H}^H\mathbf{H}\mathbf{B}) = 1$. Furthermore, the constraint (14b) becomes $\beta_i\Big(\frac{\text{Tr}(\mathbf{\Phi}_i\mathbf{A})}{\theta_i} - \sigma_u^2\Big) \geq \sigma_z^2$, which can be cast as an LMI with Schur-Complement Lemma because it is a hyperbola [13].

Now we focus on (14c). After substituting the definition of $\mathbf{A}$ into it, (14c) becomes $\eta(1-\beta_i)\Big(\text{Tr}(\mathbf{\Phi}_i\mathbf{A}) + \sigma_u^2\Big) \geq \frac{1}{\xi_i} + 2p_c$, which is nonconvex in its current form. However, by applying two layers of slack variables, it can be transformed into a convex form. More specifically, the first-layer slack variable $\mu_i$ is introduced such that

$$\eta(1-\beta_i)\Big(\text{Tr}(\mathbf{\Phi}_i\mathbf{A}) + \sigma_u^2\Big) \geq \mu_i^2 \geq \frac{1}{\xi_i} + 2p_c, \ \forall i, \tag{15}$$

and the left inequality of (15) can be cast as

$$\begin{bmatrix} \text{Tr}(\mathbf{\Phi}_i\mathbf{A}) + \sigma_u^2 & \mu_i \\ \mu_i & \eta(1-\beta_i) \end{bmatrix} \succeq 0, \ \forall i. \tag{16}$$

The right inequality of (15) is nonconvex due to term $\mu_i^2$. To this end, the second-layer slack variable $\phi_i$ is introduced: $\mu_i^2 \geq \phi_i^2 + 2p_c \geq \frac{1}{\xi_i} + 2p_c$. It is now clear that the left inequality is an second-order cone (SOC) $\mu_i \geq ||[\phi_i\ \sqrt{2p_c}]||$ after taking square root on both sides, and the right inequality $\phi_i^2 \geq \frac{1}{\xi_i}$ is equivalent to $\xi_i \geq \frac{1}{\phi_i^2}$, which is obviously convex.

With the procedure presented above, we arrive at $\mathcal{P}3$

which is equivalent of $\mathcal{P}2$

$$\mathcal{P}3: \min_{\mathbf{A},\mathbf{B},\{\xi_i,\beta_i,\mu_i,\phi_i\}} \operatorname{Tr}(\mathbf{G}^H\mathbf{G}\mathbf{A}) + \frac{1}{\xi_1} + \frac{1}{\xi_2}$$

$$\text{s.t.} \quad \operatorname{Tr}(\mathbf{\Xi}_i\mathbf{B}) = \alpha_i\sigma_r^2\xi_i, \ \forall i \tag{17a}$$

$$\begin{bmatrix} \frac{1}{\theta_i}\operatorname{Tr}(\mathbf{\Phi}_i\mathbf{A}) - \sigma_u^2 & \sigma_z \\ \sigma_z & \beta_i \end{bmatrix} \succeq 0, \ \forall i \tag{17b}$$

$$\begin{bmatrix} \operatorname{Tr}(\mathbf{\Phi}_i\mathbf{A}) + \sigma_u^2 & \mu_i \\ \mu_i & \eta(1-\beta_i) \end{bmatrix} \succeq 0, \ \ \forall i \tag{17c}$$

$$\mu_i \geq ||[\phi_i \ \sqrt{2p_c}]||, \ \xi_i \geq \frac{1}{\phi_i^2}, \ \ \forall i \tag{17d}$$

$$\operatorname{Tr}(\mathbf{H}^H\mathbf{H}\mathbf{B}) = 1, \mathbf{A} \succeq 0, \mathbf{B} \succeq 0 \tag{17e}$$

$$\operatorname{Rank}(\mathbf{A}) = 1, \operatorname{Rank}(\mathbf{B}) = 1. \tag{17f}$$

Problem $\mathcal{P}3$ cannot be directly solved due to rank constraints (17f). However, by applying SDR in [14] to drop the rank constraints (17f), $\mathcal{P}3$ becomes a semidefinite programming (SDP) problem, which can be solved efficiently by CVX [15]. The following proposition shows that the relaxation does not affect the optimality.

**Proposition 1.** *The optimal rank-one solution $\mathbf{A}^*, \mathbf{B}^*$ to the relaxed problem of $\mathcal{P}3$ always exists.*

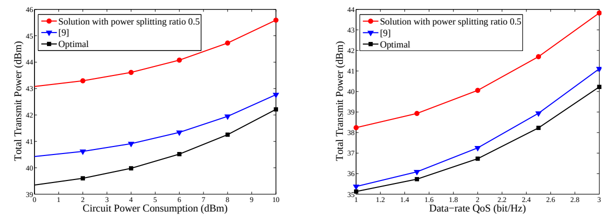
*Proof.* We address the proof for $\mathbf{A}$, and that for $\mathbf{B}$ is similar. First, since $\mathbf{A} \neq 0$, we have $\operatorname{Rank}(\mathbf{A}^*) \geq 1$. Next, supposing that the optimal $\{\beta_i^*, \mu_i^*\}$ is given, the constraint (17b) can be rearranged as $\operatorname{Tr}(\mathbf{\Phi}_i\mathbf{A}) \geq \frac{\sigma_z^2\theta_i}{\beta_i^*} + \sigma_u^2$, $\forall i = 1, 2$, and constraint (17d) is equivalent to $\operatorname{Tr}(\mathbf{\Phi}_i\mathbf{A}) \geq \frac{\mu_i^{*2}}{\eta(1-\beta_i^*)} - \sigma_u^2$, $\forall i = 1, 2$. Now we consider the following SDP problem

$$\min_{\mathbf{A}\succeq 0} \operatorname{Tr}(\mathbf{G}^H\mathbf{G}\mathbf{A}) \tag{18}$$

$$\text{s.t.} \operatorname{Tr}(\mathbf{\Phi}_i\mathbf{A}) \geq \max\Big[\frac{\sigma_z^2\theta_i}{\beta_i^*} + \sigma_u^2, \frac{\mu_i^{*2}}{\eta(1-\beta_i^*)} - \sigma_u^2\Big], \ \forall i,$$

where the constraint of problem (18) is obtained by taking the intersection of (17b) and (17d). Since problem (18) is equivalent to the SDR of $\mathcal{P}3$ with $\{\beta_i = \beta_i^*, \mu_i = \mu_i^*\}$, problem (18) and the SDR of $\mathcal{P}3$ must have the same optimal solution of $\mathbf{A}^*$. On the other hand, problem (18) has all together 2 constraints on $\mathbf{A}$, according to [16, Theorem 3.2], there exists $\mathbf{A}^*$ with $\operatorname{Rank}^2(\mathbf{A}^*) \leq 2$, which yields $\operatorname{Rank}(\mathbf{A}^*) \leq \sqrt{2}$, and $\operatorname{Rank}(\mathbf{A}^*) = 1$ holds. □

Using **Proposition 1** and the rank-reduction procedure in [16], the optimal rank-one solution $\mathbf{A}^*, \mathbf{B}^*$ can always be found. After $\mathbf{A}^*, \mathbf{B}^*$ are obtained and letting $p^* = \operatorname{Tr}(\mathbf{G}^H\mathbf{G}\mathbf{A}^*)$, the optimal $a_1^*, a_2^*, b_1^*, b_2^*$ can be found through eigendecomposition such that $\mathbf{A}^*/p^* = [a_1^* \ a_2^*]^H[a_1^* \ a_2^*]$, $\mathbf{B}^* = [b_1^* \ b_2^*]^H[b_1^* \ b_2^*]$. Then, the optimal $\mathbf{v}^*, \mathbf{w}^*, p^*, \{\beta_i^*, q_i^*\}$ of $\mathcal{P}1$ can be recovered accordingly. Notice that this problem has also been discussed in [9]. However, in contrast to the optimal solution here, the solution in [9] is sub-optimal.




(a) (b)

**Fig. 2**. Total transmit power for the case of $N = 8$ (a) Versus $p_c$ with $\overline{R} = 3\text{bit/Hz}$; (b) Versus $\overline{R}$ with $p_c = 5\text{dBm}$.

Since the problem dimension reduces from $N$ to 2 by using **Property 2**, solving the SDR problem of $\mathcal{P}3$ requires complexity $O(2N^2 + 2^7)$. On the other hand, the scheme in [9] requires $O(2tN^{3.5})$ ($t$ is the number of iterations). When $N = 2$, the two schemes have comparable complexities. But for $N \geq 4$, the proposed scheme has a lower complexity.

## 4. SIMULATION RESULTS AND DISCUSSIONS

This section provides simulation results to demonstrate the performance of the proposed scheme. In particular, each random channel is generated according to $\varrho \cdot \mathcal{CN}(\mathbf{0}, \mathbf{I})$, where the large-scale fading $\varrho = -30\text{dB}$ [2–7]. It is assumed that power conversion efficiency $\eta = 0.5$. The same data-rate targets $\overline{R}_i = \overline{R}$ in bit/Hz are requested by both users. Each point in the figures is obtained by averaging over 100 simulation runs, with independent channel in each run.

We consider the case of $N = 8$ and noise power $\sigma_r^2 = \sigma_u^2 = \sigma_z^2 = -30\text{dBm}$. Here, three schemes are compared: the optimal solution, the iterative solution from [9], and the solution with $\beta_1 = \beta_2 = 0.5$. As is shown in Fig. 2, the solution of this paper achieves the lowest transmit power over a wide range of circuit power $p_c$ in Fig. 2a and data-rate QoS $\overline{R}$ in Fig. 2b. Compared to the other two schemes, the proposed scheme has an advantage of $0.3 \sim 1\text{dB}$ and 3dB, respectively. It is also observed that fixing power splitting ratio leads to rather poor performances, which indicates that this design parameter should be jointly optimized with beamformers.

## 5. CONCLUSIONS

This paper studied a TWRC with harvest-then-transmit users. It was shown that the uplink data-rate constraints were active at the global optimum. Then we transformed the problem into an equivalent form by applying slack variables and LMIs. Finally, global optimality was obtained. Simulation results demonstrated that the proposed method outperformed existing methods.

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