LOW-COMPLEXITY ROBUST MULTI-CELL MISO DOWNLINK PRECODER DESIGN

Mostafa Medra *Timothy N. Davidson*

Department of Electrical and Computer Engineering,
McMaster University, Hamilton, Ontario, Canada

ABSTRACT

This paper develops low-complexity design techniques for robust linear precoders suitable for various multi-cell multiple-input single-output (MISO) downlink systems. The goal is to satisfy prespecified SINR requirements for users in multiple cells under some base station power constraints, in the absence of perfect channel state information (CSI). First, we consider the case of full cooperation between base stations and derive a simple iterative algorithm that achieves the required SINRs with high probability despite the presence of channel uncertainties. Then we consider the case of distributed coordination between base stations and develop a simple iterative algorithm that requires only very limited communication among the base stations. Our simulation results demonstrate that substantial robustness can be obtained at a low computational cost.

***Index Terms*—** Multi-cell, beamforming, MISO, Robust.

1. INTRODUCTION

Managing the interference among clusters of cells has the potential to provide significant performance gains over wireless networks that avoid interference between cells [1, 2]. One taxonomy of interference management schemes classifies them as being centralized or distributed, depending on where the design decisions are made, and as being cooperative or coordinating. In cooperative schemes multiple base stations (BSs) work together to transmit the same information to a receiver, whereas in coordinating schemes each receiver is assigned to a single BS, but the design of the transmissions from each BS is coordinated with that of the other BSs.

Among the many scenarios that could benefit from cooperation or coordination, we will focus on the multiple-input single-output (MISO) downlink case in which a cluster of BSs seek to cooperate or coordinate in the transmission of messages to multiple receivers each with a single antenna. We will focus on linear transmission schemes and hence the design variables are the shape of the beam transmitted to each receiver and the power allocated to that beam. Based on the assumption that sufficiently accurate channel state information (CSI) can be made available at the design nodes, a number of techniques for multi-cell downlink beamforming have been developed; e.g., [2–7]. More recently, efforts have been made to mitigate the sensitivity of those techniques to the uncertainties in the CSI that inevitably arise from the estimation and transmission of CSI; e.g., [8–14]. However, many such techniques require the solution of optimization problems that incur significantly greater computational cost than that of the original designs for the perfect CSI case (some of which are, themselves, quite computationally costly).

In this paper, we will develop low-complexity iterative algorithms for the beamforming directions and the power loading in both a centrally-design cooperative mutli-cell MISO downlink, and for a coordinated multi-cell MISO downlink that is designed in a distributed manner. Using insights from recent work on the isolated single-cell MISO downlink [15–17], we develop designs that provide substantial robustness to uncertainties and can be obtained using simple iterative algorithms. In our simulation experiments, these straightforward designs provide significantly lower outage rates than existing designs.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under grant RGPIN-2015-06631.

2. SYSTEM MODEL

The system that we consider consists of a total of K-users each with a single antenna served by a cluster of B BSs. User k is served by a subset of the BSs, $\mathcal{B}_k$, where $\mathcal{B}_k$ consists of only one BS index in the case of distributed precoding and contains all the BS indices in the case of full cooperation. We assume that each BS is equipped with N_t antennas and is provided with an imperfect version of the CSI of the users. We use the notation $\mathbf{h}_k^j \in \mathbb{C}^{N_t}$ to denote the channel between the BS j and user k. we let $\mathbf{w}_k^j$ denote the designed precoding vector for transmission from BS j to user k, and let s_k denote the intended normalized data symbol for that user. To unify the notations for the full cooperation and distributed cases, we define $\mathbf{h}_k = [\mathbf{h}_k^{1\,T}, \mathbf{h}_k^{2\,T}, \cdots, \mathbf{h}_k^{B\,T}]^T$ as the stacking of all the channel vectors to user k, and we define $\mathbf{w}_k$ analogously. We also let $\sqrt{\beta_k}$ denote the Euclidean norm of $\mathbf{w}_k$ and $\mathbf{u}_k$ denote its normalized direction; i.e., $\mathbf{w}_k = \sqrt{\beta_k}\mathbf{u}_k$. With that notation, we can write the received signal of user k in the following simplified form:

$$y_k = \mathbf{h}_k^H \mathbf{w}_k s_k + \textstyle\sum_{i \neq k} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k, \qquad (1)$$

where n_k is the zero mean circular Gaussian noise of variance σ_k^2 at user k. We will translate each user's quality-of-service (QoS) constraint into an SINR requirement $\text{SINR}_k \geq \gamma_k$. By defining $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$, we can write the SINR expression for user k as

$$\text{SINR}_k = \frac{\mathbf{h}_k^H \mathbf{W}_k \mathbf{h}_k}{\mathbf{h}_k^H (\sum_{i \neq k} \mathbf{W}_i) \mathbf{h}_k + \sigma_k^2}. \qquad (2)$$

For a BS or a cluster management centre to calculate the SINR at each user, it needs to know the actual channel vectors $\{\mathbf{h}_k\}$. However, in practice only an estimate of those vectors will be available. In this paper we will model the uncertainty additively; i.e., $\mathbf{h}_k^j = \mathbf{h}_{e_k}^j + \mathbf{e}_k^j$, and with $\mathbf{e}_k^j$ being an independent zero-mean circular Gaussian random variable of covariance $(\sigma_{e_k}^j)^2\mathbf{I}$. Among a number of scenarios, this model is appropriate in certain time division duplexing (TDD) systems in which channels are estimated during the uplink training phase.

In the following sections, we will first review an existing approach to precoding for intra-cell interference mitigation in an isolated cell. We will then show how that approach can be integrated with inter-cell interference mitigation techniques that capture different levels of coordinations among the BSs.

3. OFFSET-MAXIMIZATION FOR THE SINGLE CELL CASE

In the case of the downlink of an isolated cell, the task of satisfying the users' QoS constraints involves managing the interference imposed on one user due to simultaneous transmissions to other users in the same cell (i.e., managing intracell interference), and doing so in the presence of uncertainty in the BS's estimates of the channels to the users. Among many possible approaches to this problem, including [18–21], we will review the offset maximization approach to robust precoding, which was initially developed in [15], was extended to the case of per-antenna power constraints in [16] and enhanced with an alternative power loading in [17]. A key advantage of that approach and its enhanced power loading is its low computational cost.

For the single cell case, the offset maximization approach is based on the observation that under our additive model for the uncertainty in the CSI, the constraint $\text{SINR}_k > \gamma_k$ can be rewritten as

$$\mathbf{h}_{e_k}^H \mathbf{Q}_k \mathbf{h}_{e_k} - \sigma_k^2 + \mathbf{h}_{e_k}^H \mathbf{Q}_k \mathbf{e}_k + \mathbf{e}_k^H \mathbf{Q}_k \mathbf{h}_{e_k} + \mathbf{e}_k^H \mathbf{Q}_k \mathbf{e}_k \geq 0. \quad (3)$$

where $\mathbf{Q}_k = \mathbf{W}_k/\gamma_k - \sum_{j\neq k} \mathbf{W}_j$. This expression suggests that if we were to maximize the deterministic "offset" term in (3), we would obtain robustness against the terms that involve the error in the CSI. If we do so, with a total power constraint P_t, the semidefinite relaxation of the problem can be written as

$$r^\star = \max_{\mathbf{W}_k, r} \quad r \quad (4a)$$

$$\text{s.t. } \sum_{k=1}^K \text{tr}(\mathbf{W}_k) \leq P_t, \quad (4b)$$

$$\mathbf{h}_{e_k}^H \mathbf{Q}_k \mathbf{h}_{e_k} - \sigma_k^2 - r \geq 0, \quad (4c)$$

$$\mathbf{W}_k \succeq \mathbf{0}, \quad k = \{1, 2, ..., K\}, \quad (4d)$$

Although the problem in (4) is a semidefinite program and can be solved in a polynomial time, a closed-form solution was obtained in [15]. That solution also demonstrates that the semidefinite relaxation that led to (4) is tight. The derivation of the closed-form solution in [15] is based on the following problem

$$\min_{\mathbf{W}_k} \quad \sum_{k=1}^K \text{tr}(\mathbf{W}_k) \quad (5a)$$

$$\mathbf{h}_{e_k}^H \mathbf{Q}_k \mathbf{h}_{e_k} - \sigma_k^2 - r^\star \geq 0, \quad (5b)$$

$$\mathbf{W}_k \succeq \mathbf{0}, \quad k = \{1, 2, ..., K\}, \quad (5c)$$

where $r^\star$ is the optimal value of (4). Problems (4) and (5) are equivalent in the sense that the optimal value for (5) is P_t and that an optimal solution set $\{\mathbf{W}_k\}$ for one is also optimal for the other. The key to the derivation of the closed form is the similarity in structure between (5) and the power minimization problem for the perfect CSI case [22–24]. In particular, we first solve the following fixed point equations for the dual variables for the constraints in (5b)

$$\nu_k^{-1} = \mathbf{h}_{e_k}^H \Big(\mathbf{I} + \sum_j \nu_j \mathbf{h}_{e_j} \mathbf{h}_{e_j}^H\Big)^{-1} \mathbf{h}_{e_k} \big(1 + 1/\gamma_k\big). \quad (6)$$

Then we can find the beamforming directions using the eigen equation

$$\mathbf{u}_k = \big((\nu_k/\gamma_k)\mathbf{h}_{e_k}\mathbf{h}_{e_k}^H - \sum_{j\neq k} \nu_j \mathbf{h}_{e_j}\mathbf{h}_{e_j}^H\big)\mathbf{u}_k. \quad (7)$$

Having found those directions, we then determine the power loading. The original power loading method in [15] is based on solving the linear equations that arise from the fact that at optimality the constraints in (5b) hold with equality and that the optimal objective value is P_t; i.e., $\sum_i \beta_i = P_t$. This results in $K+1$ linear equations for $\{\beta_k\}_{k=1}^K$ and $r^\star$. That method gives the same "robustness" r to all users.

The enhanced power loading method in [17] is based on providing greater robustness to "weaker" users, and accordingly having comparable outage probabilities for all users. The notion of weakness is measured using the variance, $\sigma_{s_k}^2$, of $\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2$. The users with higher σ_{s_k} should be provided with more robustness, or offset, than other users with lower σ_{s_k}. Accordingly, the algorithm proposed in [17] is based on finding $\{\beta_k\}$ and $r^\star$ such that $\mathbb{E}(\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2) = \sigma_{s_k} r^\star$, and $\sum_k \beta_k = P_t$. With such a strategy, the power loading algorithm allocates power such that the mean value of the rearranged SINR expression in (3) is proportional to its standard deviation. Although $\mathbb{E}(\mathbf{h}_k^H \mathbf{Q}_k \mathbf{h}_k - \sigma_k^2)$ is linear in each β_k, σ_{s_k} is not, and thus complicates the problem. In [17] we adopted an iterative linearization technique in which σ_{s_k} was determined from the values of $\{\beta_k\}_{k=1}^K$ and $r^\star$ at the previous iteration. That algorithm converges with high probability [17]. The resulting power allocation provides similar outage performance for each user, and in the numerical experiments in [17] it provided improved overall outage performance.

4. NETWORK MIMO OFFSET-MAXIMIZATION

In the case of full cooperation, the BSs are all connected to a central processing unit, and all the CSI and the users' data are shared. In such a case, the system resembles a single BS with many distributed antennas and can be treated as a single cell but with different power constraints. Here we will consider the total power constraint and per-BS power constraints. If we define $\mathbf{\Lambda}_i$ to be a diagonal matrix with ones on the elements corresponding to the antennas of the ith BS and zeros elsewhere and P_i to be the power constraint on the ith BS, then the offset maximization precoding problem can be stated as

$$\max_{\mathbf{W}_k, r} \quad r \quad (8a)$$

$$\text{s.t. } \sum_{k=1}^K \mathbf{w}_k^H \mathbf{\Lambda}_i \mathbf{w}_k \leq P_i, \quad i = \{1, 2, ..., B\}, \quad (8b)$$

$$\sum_{k=1}^K \text{tr}(\mathbf{W}_k) \leq P_t, \quad (8c)$$

$$\mathbf{h}_{e_k}^H \mathbf{Q}_k \mathbf{h}_{e_k} - \sigma_k^2 - r \geq 0, \quad k = \{1, 2, ..., K\}. \quad (8d)$$

This problem can be solved with generic convex optimization techniques. Although those techniques are effective in that they produce an optimal solution in polynomial time, the computational cost can still be quite high. In the following sections we consider two special cases of the problem in (8) in which we can develop tailored algorithms to solve the problem more efficiently. The first special case arises when $P_t < P_i$, $\forall i$, in which case the condition in (8b) can never be active and can be removed. The second special case arises when $P_t > \sum_i P_i$, in which case the condition in (8c) cannot be active.

4.1. Dominant total power constraint

In the absence of (8b), the problem in (8) is in the same form as (4) and hence the existing techniques can be applied directly. The difference between the single-cell case and the multi-cell case lies in the fact that the channel vector $\mathbf{h}_k$ is the stacking of all the channel vectors from all the BSs to a certain user k, and, accordingly, those channel vectors have different error vectors with different error variances. In this case, we can derive the mean and variance of $\mathbf{h}_k^H\mathbf{Q}_k\mathbf{h}_k - \sigma_k^2$ as

$$\mathbb{E}(\mathbf{h}_k^H\mathbf{Q}_k\mathbf{h}_k - \sigma_k^2) \\ = \mathbf{h}_{e_k}^H\mathbf{Q}_k\mathbf{h}_{e_k} - \sigma_k^2 + \beta_k\mathbf{u}_k^H\mathbf{\Gamma}_e\mathbf{u}_k/\gamma_k - \sum_{j\neq k}\beta_j\mathbf{u}_j^H\mathbf{\Gamma}_e\mathbf{u}_j. \tag{9}$$

$$\sigma_{s_k}^2 = \text{var}\big(\mathbf{h}_{e_k}^H\mathbf{Q}_k\mathbf{h}_{e_k} + 2\text{Re}(\mathbf{e}_k^H\mathbf{Q}_k\mathbf{h}_{e_k}) + \mathbf{e}_k^H\mathbf{Q}_k\mathbf{e}_k - \sigma_k^2\big) \\ = 2\mathbf{h}_{e_k}^H\mathbf{Q}_k\mathbf{\Gamma}_e\mathbf{Q}_k\mathbf{h}_{e_k} + \text{tr}(\mathbf{\Gamma}_e^2\mathbf{Q}_k^2), \tag{10}$$

where $\mathbf{\Gamma}_e = \mathbb{E}(\mathbf{e}_i\mathbf{e}_i^H)$ and is diagonal by assumption. Since each $\mathbf{Q}_k$ is linear in $\boldsymbol{\beta}$, the expression for the mean in (9) is linear in $\boldsymbol{\beta}$. Therefore, we can adapt the iterative linearization technique that was developed for the single case [17] to produce the following algorithm

1. Calculate the beamforming directions $\mathbf{u}_k$ using (6) and (7).
2. Initialize $\sigma_{s_k} = 1$.
3. Update $r^\star$, and $\{\beta_k\}$ using $\mathbb{E}(\mathbf{h}_k^H\mathbf{Q}_k\mathbf{h}_k - \sigma_k^2) = \sigma_{s_k}r^\star$ and $\sum_k \beta_k = P_t$, where the expected value is given in (9).
4. Update each σ_{s_k} using (10).
5. Evaluate a termination criterion and return to 3 if not satisfied.

4.2. Dominant per-base station power constraints

When the constraint in (8c) is inactive, we can simplify the formulation in (8) to

$$r^\star = \max_{\mathbf{w}_k, r} \quad r \\ \text{s.t.} \ \sum_{k=1}^K \mathbf{w}_k^H\mathbf{\Lambda}_i\mathbf{w}_k \le P_i, \quad i = \{1,2,...,B\}, \\ \mathbf{h}_{e_k}^H\mathbf{Q}_k\mathbf{h}_{e_k} - \sigma_k^2 - r \ge 0, \quad k = \{1,2,...,K\}. \tag{11}$$

As we will see, dealing with all the BSs together as a one virtual BS then applying per-BS power constraints is analogous to dealing with one BS and applying per-antenna power constraints. Accordingly, the derivation for the closed-form solution for the problem in (11) will follow the same steps for the offset maximization algorithm with per-antenna power constraint [17, 25]. We will briefly summarize it here for completeness. Following what was done for the single case, we will consider the following equivalent problem

$$\min_{\mathbf{w}_k, \alpha} \quad \alpha \sum_{i=1}^B P_i \tag{12a}$$

$$\text{s.t.} \quad \sum_{k=1}^K \mathbf{w}_k^H\mathbf{\Lambda}_i\mathbf{w}_k \le \alpha P_i, \quad i = \{1,2,...,B\}, \tag{12b}$$

$$\mathbf{h}_{e_k}^H\mathbf{Q}_k\mathbf{h}_{e_k} - \sigma_k^2 - r^\star \ge 0, k = \{1,2,..,K\}. \tag{12c}$$

The equivalence here means that both problems share an optimal solution. This can be verified by observing that substituting the optimal solution of (11) in (12) will give us a value of $\alpha = 1$, and by observing that the optimal α can not be smaller than one, as that would mean that we could rescale the precoding vectors and have a larger $r^\star$ which contradicts the presumed optimality; i.e., problems (11) and (12) share the optimal solution with $\alpha = 1$. If we define q_i and ν_i to be the dual variables for the constraints in (12b) and (12c) respectively, and $\hat{\mathbf{Q}} = \sum_{i=1}^B q_i\mathbf{\Lambda}_i$, then we can write the Lagrangian of (12) as

$$\mathcal{L}(\mathbf{w}_k, \alpha, \nu_k, q_i) = \sum_{k=1}^K \nu_k(\sigma_k^2 + r^\star) + \alpha\Big(\sum_{i=1}^B P_i - \sum_{i=1}^B q_iP_i\Big) \\ + \sum_{k=1}^K \mathbf{w}_k^H\Big(\hat{\mathbf{Q}} + \sum_{j\neq k}\nu_j\mathbf{h}_{e_j}\mathbf{h}_{e_j}^H - \nu_k/\gamma_k\mathbf{h}_{e_k}\mathbf{h}_{e_k}^H\Big)\mathbf{w}_k.$$

The Lagrangian now has the same form as the Lagrangian in the case of one BS with per-antenna constraints presented in [16]. Accordingly, we suggest using an analogous iterative quasi-closed form solution. In the case of equal P_i that algorithm can be summarized as

1. Initialize $\hat{\mathbf{Q}}^0$ such that $\sum q_i = B$. Set $n = 0$.
2. Find ν_k^n using the fixed point equations $(\nu_k^n)^{-1} = \mathbf{h}_{e_k}^H\Big(\hat{\mathbf{Q}}^n + \sum_j \nu_j^n\mathbf{h}_{e_j}\mathbf{h}_{e_j}^H\Big)^{-1}\mathbf{h}_{e_k}\Big(1 + 1/\gamma_k\Big)$.
3. Solve for the directions $\mathbf{u}_k = \hat{\mathbf{w}}_k/\|\hat{\mathbf{w}}_k\|$, where $\hat{\mathbf{w}}_k = \Big(\hat{\mathbf{Q}}^n + \sum_j \nu_j^n\mathbf{h}_{e_j}\mathbf{h}_{e_j}^H\Big)^{-1}\mathbf{h}_{e_k}$.
4. Find $r^\star$, and $\{\beta_k\}$ by solving $\mathbb{E}(\mathbf{h}_k^H\mathbf{Q}_k\mathbf{h}_k - \sigma_k^2) = \sigma_{s_k}r^\star$ and $\sum_k \beta_k = \sum_i P_i$.
5. Update $\hat{\mathbf{Q}}^{n+1}$ using $q_i^{n+1} = q_i^n + t_n(\sum_{k=1}^K \mathbf{w}_k^H\mathbf{\Lambda}_i\mathbf{w}_k - P_i)$, where t_n is the step size used.
6. Increment n, check whether $\sum_{k=1}^K \mathbf{w}_k^H\mathbf{\Lambda}_i\mathbf{w}_k - P_i < \delta_i$, where δ_i is the maximum allowable violation of the power constraint for the ith BS. If the test fails, return to 2.

5. DISTRIBUTED ALGORITHM BASED ON VIRTUAL USERS

Implementing the centralized processing and data sharing system described in the previous section can be a challenging task no matter how the beamformers and the power allocation are determined. Therefore, there is considerable interest in distributed systems that coordinate their signals via limited backhaul communications [1]. In a distributed coordinated system each user is served by a single BS, and we will assume that that assignment has been made. We consider a system in which each BS obtains estimates of the channels to users that have been assigned to it, and also obtains estimates of the channels to users assigned to neighbouring BSs upon which the BS may impose significant interference. If we let $\mathbf{h}_{e_k}^b$ denote the estimate of the channel from BS b to a user k that is not assigned to that BS, then one way in which BS b could manage the interference it imposes on user k would be to enforce a constraint of the form $\|(\mathbf{h}_{e_k}^b)^H\sum_j \mathbf{w}_j^b\|^2 < \epsilon$. Such "soft-shaping" constraints (e.g., [26]) are convex and can be incorporated into a variety of precoder design formulations (e.g., [2, 26]) and effective beamforming vectors optimization can be obtained using generic convex tools. However, the structure of those constraints results in dual formulations that do not appear to be amendable to the analysis that we developed for the centralized case. In our quest for low-complexity algorithms for the distributed case, we will instead consider an alternative design approach that takes into consideration the interference imposed on users in other cells by treating them as virtual users when designing the normalized beamformers in the cell of interest [6, 27, 28]. This principle was implemented using a zero-forcing (ZF) approach in [27, 28], and the regularized zero-forcing approach in [6]. In this section we propose a scheme in which the normalized beamformers are designed using the closed-form solutions of the offset maximization approach in (6) and (7) and the power loading is designed using the simple iterative algorithm in [17].

To describe that approach, we let the set $\mathcal{K}_i$ denote the indices of the users assigned to BS i and let $\tilde{\mathcal{K}}_i$ denote the union of that set and the indices of the users to which BS i should mitigate its interference. We will discuss the selection of $\mathcal{K}_i$ and $\tilde{\mathcal{K}}_i$ below. The number of users in these sets are denoted by K_i and $\tilde{K}_i$, respectively. With the goal of computation efficiency in mind, each BS designs the normalized beamformers as if it were designing them for all users in $\tilde{\mathcal{K}}_i$. It then designs the power loading for the users in $\mathcal{K}_i$. That procedure is as follows

1. Find ν_k for all users in $\tilde{\mathcal{K}}_i$ using (6).
2. Solve for the directions for the users in $\mathcal{K}_i$ using (7).
3. Initialize $\sigma_{s_k} = 1$.
4. Find the power loading for the users in $\mathcal{K}_i$ and $r^\star$ using (9) and the power constraint $\sum_{k\in\mathcal{K}_i} \beta_k = P_i$.
5. Update σ_{s_k} using (10).
6. Return to 4 until an appropriate stopping criterion is satisfied.

Although the design of the beamforming directions and the power loading in the above algorithm is distributed, the BSs within a cluster coordinate their designs through the selection of the users in $\tilde{K}_i$. While many strategies are possible, one simple strategy that keeps the amount of information to be shared between the BSs in the cluster small is to first select $\mathcal{K}_i$ using a conventional BS assignment technique for cell-by-cell operation and then have the neighbouring BSs inform BS i of the users in their cells that are to have interference mitigated. BS i would acquire the CSI for these users as if they were assigned to the ith cell.

The above algorithm provides implicit control over the influence that the users in $\tilde{K}_i \backslash \mathcal{K}_i$ have on the beamforming directions in cell i. This is provided through the target SINR for those virtual users. As can be seen from (6), ν_k scales in an approximately inverse fashion with γ_k, and as can be seen in (7), ν_k controls the influence of the channel to user k on all the beamforming directions. This kind of flexibility is not present in the ZF and RZF techniques in [6, 27, 28].

6. SIMULATION RESULTS

To illustrate the performance of the proposed algorithms we consider a system consisting of two BSs separated by a distance of 2.4km, each with 4 antennas, serving a total of 4 users. Half of the users are uniformly distributed in a circle of radius 1.5km around the first BS, the other half are similarly distributed around the other BS. We assume a large scale fading model described with a path-loss exponent of 3.52 and log-normal shadow fading with 8dB standard deviation. The small scale fading is modelled using the standard i.i.d. Rayleigh model. We assume a TDD system with a channel estimation error variance $\sigma_e^2 = 0.04$, and an SINR target of $\gamma = 3$dB for all users (including the "virtual users" in the algorithms of Section 5). The per-BS power constraint is $P_i = P_t K_i / K$. We assume that each user has a signal sensitivity of -90dBm, and we will consider this power as the noise power. In Fig. 1, we plot the outage probability versus the total power constraint P_t for the cooperative algorithms with total power constraints (Section 4.1) and per-BS power constraints (Section 4.2). We also assess the performance of the distributed algorithms in Section 5; once with the users assigned to the BS for which they were generated and another time with BS selection according to the channel norm (i.e., the user is assigned to the BS that has a channel vector with bigger Euclidean norm).

We will compare our algorithms to two centralized algorithms and one distributed algorithm from the literature. The first is an adaptation of the robust centralized coordination algorithm in [10, Equation (5)] for the considered scenario. In the adaptation a binary search on the zero-outage region size is performed to find the largest "zero-outage" region for which a problem with a total power constraint is feasible. That problem is convex, but involves many linear matrix inequality constraints. The second comparison is with

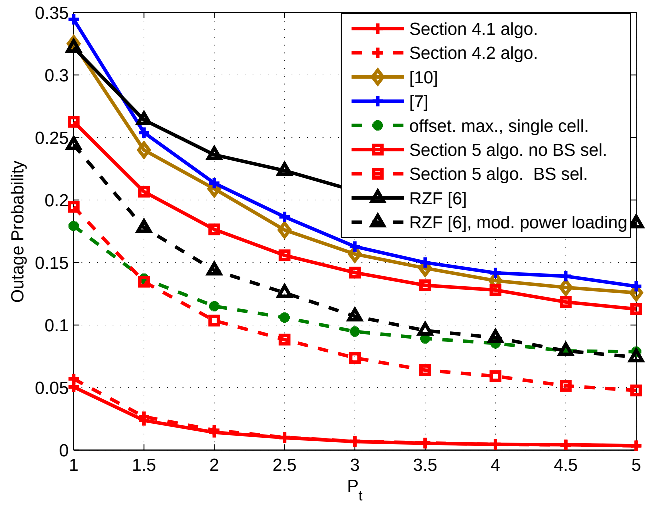

Fig. 1: Outage probability for 4 users, 4 antennas, γ=3dB, σ^2=-90dBm, $\sigma_h = 0.2$

the robust centralized coordination algorithm in [7, Equation (5)] with per-BS power constraint $P_i = P_t/2$. This algorithm involves repeated solutions of modified perfect-CSI problems. In these two cases, each BS is assigned to the users in its area (no BS selection), and the processing is done in a centralized manner.

From Fig. 1 we observe, as expected, that the centralized cooperative algorithms proposed in Sections 4.1 and 4.2 provide the best performance. Perhaps the more interesting observations from Fig. 1 are that the proposed distributed coordination algorithm provides better performance than the existing centralized coordination algorithms in [10] and [7]. This is despite the fact that the centralized algorithms in [10] and [7] incur significantly larger computational costs. A comparison of the distributed algorithms in Fig. 1 shows that the proposed distributed coordination algorithm outperforms the original versions of the distributed coordination RZF-based algorithm in [6] that uses the regularization factor described in [6, Equation (52)]. Furthermore, when the proposed algorithm is augmented with a simple BS selection scheme it also outperforms a variant of the algorithm in [6] that employs the power loading developed in [17]. (That variant significantly improves the performance of [6].) This is despite the fact that the proposed algorithm is based on a simple iterative algorithm. For reference, Fig. 1 includes the performance of the power loaded offset maximization approach [15] applied to each cell individually. In the low power regime this approach performs well, whereas at higher power levels, where the impact of the interference increases, its relative performance degrades.

7. CONCLUSION

In this paper we proposed multi-cell MISO downlink algorithms that can provide substantial robustness against channel uncertainties. We proposed a centralized cooperative algorithm that has significant performance gains compared to other algorithms in literature, and has lower computational complexity. We also provided a distributed coordination algorithm that needs very limited backhaul communication, and incurs an even lower computational cost, and yet can provide better performance than several existing methods.

8. REFERENCES

[1] D. Gesbert, S. Hanly, H. Huang, S. Shamai Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.

[2] M. Hong and Z.-Q. Luo, "Signal processing and optimal resource allocation for the interference channel," in *Academic Press Library in Signal Processing: Volume 2 Communications and Radar Signal Processing*, N. D. Sidiropoulos, F. Gini, R. Chellappa, and S. Theodoridis, Eds. Elsevier, 2014, vol. 2, ch. 8, pp. 409 – 469.

[3] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.

[4] A. Tolli, H. Pennanen, and P. Komulainen, "Decentralized minimum power multi-cell beamforming with limited backhaul signaling," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 570–580, February 2011.

[5] H. Pennanen, A. Tolli, and M. Latva-aho, "Decentralized coordinated downlink beamforming via primal decomposition," *IEEE Signal Process. Letters*, vol. 18, no. 11, pp. 647–650, Nov 2011.

[6] J. Mirza, P. J. Smith, P. A. Dmochowski, and M. Shafi, "Coordinated regularized zero-forcing precoding for multicell MISO systems with limited feedback," *arXiv preprint arXiv:1508.07333*, 2015.

[7] J. Qiu, R. Zhang, Z.-Q. Luo, and S. Cui, "Optimal distributed beamforming for MISO interference channels," in *Conf. Rec. 44th Asilomar Conf. Signals, Systems, Computers*, Nov. 2010, pp. 277–281.

[8] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, "Distributed robust multicell coordinated beamforming with imperfect CSI: An ADMM approach," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2988–3003, June 2012.

[9] A. Tajer, N. Prasad, and X. Wang, "Robust linear precoder design for multi-cell downlink transmission," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 235–251, Jan 2011.

[10] H. Pennanen, A. Tolli, and M. Latva-aho, "Decentralized robust beamforming for coordinated multi-cell MISO networks," *IEEE Signal Process. Letters*, vol. 21, no. 3, pp. 334–338, Mar. 2014.

[11] N. Vucic and H. Boche, "Robust transceiver optimization for multiuser MISO broadcast systems with MSE targets," in *Proc. 2nd IEEE Int. Wkshp on Computational Adv. in Multi-Sensor Adaptive Processing*, Dec. 2007, pp. 73–76.

[12] Q. Zhang, C. He, and L. Jiang, "Per-stream MSE based linear transceiver design for MIMO interference channels with CSI error," *IEEE Trans. Commun.*, vol. 63, no. 5, pp. 1676–1689, May 2015.

[13] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, "Chance-constrained robust beamforming for multi-cell coordinated downlink," in *Proc. IEEE Global Commun. Conf.*, Dec. 2012, pp. 4957–4962.

[14] M. Tshangini and M. R. Nakhai, "Second-order cone programming for robust downlink beamforming with imperfect CSI," in *Proc.IEEE Global Commun. Conf.*, Dec. 2013, pp. 3452–3457.

[15] M. Medra, W.-K. Ma, and T. N. Davidson, "Low-complexity robust MISO downlink precoder optimization for the limited feedback case," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Process.*, April 2015, pp. 3108–3112.

[16] M. Medra and T. N. Davidson, "Robust MISO downlink precoder design with per-antenna power constraints," in *Proc. IEEE Int. Wkshp Signal Process. Adv. Wireless Commun.*, Stockholm, June 2015, pp. 580–584.

[17] ——, "Per-user outage-constrained power loading technique for robust MISO downlink," to appear in *Conf. Rec. 49th Asilomar Conf. Signals, Systems, Computers*, 2015. [Online]. Available: http://goo.gl/UwslUZ

[18] G. Zheng, K.-K. Wong, and T.-S. Ng, "Robust linear MIMO in the downlink: A worst-case optimization with ellipsoidal uncertainty regions," *EURASIP J. Adv. Signal Process.*, vol. 2008, pp. 154:1–154:15, Jan. 2008.

[19] M. B. Shenouda and T. N. Davidson, "Convex conic formulations of robust downlink precoder designs with quality of service constraints," *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 4, pp. 714–724, Dec. 2007.

[20] ——, "Probabilistically-constrained approaches to the design of the multiple antenna downlink," in *Conf. Rec. 42nd Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, Oct. 2008, pp. 1120–1124.

[21] K.-Y. Wang, A.-C. So, T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "Outage constrained robust transmit optimization for multiuser MISO downlinks: Tractable approximations by conic optimization," *IEEE Trans. Signal Process.*, vol. 62, no. 21, pp. 5690–5705, Nov. 2014.

[22] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed. CRC Press, 2001, ch. 18.

[23] A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.

[24] E. Bjornson, M. Bengtsson, and B. Ottersten, "Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure," *IEEE Signal Process. Mag.*, vol. 31, no. 4, pp. 142–148, July 2014.

[25] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, June 2007.

[26] G. Scutari, D. P. Palomar, and S. Barbarossa, "Cognitive MIMO radio," *IEEE Signal Process. Mag.*, vol. 25, no. 6, pp. 46–59, Nov. 2008.

[27] S.-H. Park and I. Lee, "Analysis of degrees of freedom of interfering MISO broadcast channels," in *Proc. IEEE Global Telecommun. Conf.*, Nov. 2009, pp. 1–6.

[28] N. Lee and W. Shin, "Adaptive feedback scheme on K-cell MISO interfering broadcast channel with limited feedback," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 401–406, Feb. 2011.