COORDINATED UPLINK SCHEDULING AND BEAMFORMING FOR WIRELESS CELLULAR NETWORKS VIA SUM-OF-RATIO PROGRAMMING AND MATCHING

Kaiming Shen and Wei Yu

Electrical and Computer Engineering Department
University of Toronto, ON M5S 3G4, Canada
Email: {kshen,weiyu}@comm.utoronto.ca

ABSTRACT

This paper proposes a joint uplink user scheduling and beamforming algorithm for a multiple-antenna wireless cellular network. We show that coordinated optimization across the cells can significantly alleviate intercell interference, thereby improving the cell-edge rates in a multicell network. Unlike the downlink case, coordinating uplink transmission in a multicell network is significantly more challenging, because uplink interference depends strongly on the schedule and beamformers of neighboring cells. The main contribution of this paper is the recasting of the problem in terms of sum-of-ratio programming and a subsequent quadratic reformulation which allows scheduling and beamforming to be optimized through solving a matching problem. This problem reformulation also provides a new interpretation of the well-known weighted minimum mean square error (WMMSE) algorithm. Simulation results show that the proposed approach significantly outperforms both the WMMSE algorithm and the existing uncoordinated scheduling approach.

1. INTRODUCTION

Intercell interference has long been identified as a main bottleneck for wireless cellular networks. This paper focuses on the potential for alleviating interference in the *uplink* via the coordination of user scheduling and beamforming across the cells. The uplink transmit optimization problem is more challenging than the downlink, because the uplink interference pattern is a strong function of the scheduling of users and their respective beamformers, whereas downlink interference is a function of downlink beamformers alone. The coordinated optimization of uplink scheduling and beamforming is a mixed-integer nonconvex optimization problem, which is quite challenging to solve.

This paper focuses on the joint optimization of uplink scheduling and beamforming coordinated across multiple cells in a multiple-input multiple-output (MIMO) wireless cellular network. Our main contributions are a reformulation of the multicell weighted sum rate maximization problem as sum-of-ratio programming, as first proposed in [1] but now generalized in this paper for MIMO systems, and a new technique of weighted bipartite matching which allows a multicell coordinated joint optimization of uplink scheduling and beamforming. An interesting aspect of the algorithmic development of this paper is that the well-known weighted minimum mean square error (WMMSE) algorithm [2, 3] as applied to uplink scheduling and beamforming can be recovered as a variation of the proposed approach. But the proposed approach significantly outperforms WMMSE and the existing baseline as verified by simulation.

Although intercell interference can significantly affect the performance of wireless cellular networks, uplink scheduling schemes implemented in practice are often based on channel quality alone, or they assume worst-case interference [4]. Because of the difficulty in quantifying out-of-cell interference, most existing uplink scheduling algorithms in the literature are heuristic in nature. For example, [5] points out that a major challenge in uplink scheduling is interference prediction, and introduces several opportunistic heuristics. Likewise, the method proposed in [6] approximates the uplink signal-to-interference-and-noise ratio (SINR) by calculating a so-called signal-to-pollution ratio, where pollution refers to the total interference to all the other cells. A similar idea is proposed in [7]. The uplink scheduling method in [8] mimics the downlink, but doing so is not optimal. As explained in more details later, the uplink scheduling problem can also be solved using the WMMSE algorithm. However, WMMSE is primarily a beamforming algorithm; its complexity is often too high when large number of potential users are involved. The main objective of this paper is to show that by using a series of nontrivial problem reformulations, the uplink scheduling problem can be tackled rigorously, with significant system-level performance benefit.

2. PROBLEM STATEMENT

Consider the uplink of a MIMO cellular network with J base-stations (BSs), where each BS is equipped with M antennas and each user is equipped with N antennas. We denote $\mathcal{U}_i$ as the set of users that are associated with BS i. For ease of dis-

978-1-4799-9988-0/16/$31.00 ©2016 IEEE

cussion, we assume that only one single data stream is transmitted from each user. Since every BS has M antennas, we can equivalently think of each BS as having up to M simultaneous streams for the data transmission of uplink users. Let $s_{im} \in \mathcal{U}_i$ be the user to be scheduled in the mth stream at BS i. Let $\mathbf{v}_{s_{im}} \in \mathbb{C}^N$ be the transmit beamformer of the scheduled user s_{im}, and let $\mathbf{u}_{im} \in \mathbb{C}^M$ be the receive beamformer of BS i in its mth stream. A weighted sum rate objective is

$$f_o(\mathbf{s}, \mathbf{u}, \mathbf{v}) = \sum_{(i,m)} w_{s_{im}} \cdot \log\left(1 + \frac{|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)\neq(i,m)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}\right), \tag{1}$$

where $\mathbf{H}_{i,s_{im}} \in \mathbb{C}^{M\times N}$ is the uplink channel coefficient from the scheduled user s_{im} to BS i, $w_{s_{im}}$ is the priority of the user as determined by the upper layer, and σ^2 is the additive white Gaussian background noise power.

The joint uplink scheduling and beamforming problem is that of maximizing the above objective over the transmit and receive beamformers and over all possible user schedules s_{im} (i.e., the assignment of potential users to the data streams):

$$\begin{aligned} \underset{\mathbf{s},\mathbf{u},\mathbf{v}}{\text{maximize}} \quad & f_o(\mathbf{s}, \mathbf{u}, \mathbf{v}) & \text{(2a)} \\ \text{subject to} \quad & \|\mathbf{v}_{s_{im}}\|^2 \leq P & \text{(2b)} \\ & s_{im} \in \mathcal{U}_i & \text{(2c)} \end{aligned}$$

where P is the uplink transmit power constraint at the user side. Note that the interference at a BS depends on the users scheduled elsewhere in the network. In addition, the objective function is a nonconvex function of the beamformers. This makes the uplink scheduling and beamforming problem a difficult mixed-integer nonconvex optimization problem.

3. ALGORITHM

This paper proposes the idea of coordinating the scheduling of users across the cells together with their beamformers in the uplink in order to balance intercell interference and to maximize the overall system-level utility. The technical challenge here is the integer nature of the problem: when a different user is scheduled in a data stream, it produces a different interference pattern to the nearby BSs. The main contribution of this paper is a recasting of the original problem to a form amendable for coordinated optimization across multiple cells. This is a MIMO generalization to our early work on uplink scheduling for the single-input single-output (SISO) systems [1]. The scheduling problem in multiuser MIMO systems is more complicated, because of the need to introduce beamformers and to match users to the uplink data streams.

3.1. Lagrangian Reformulation

Our first goal is to move the variables $(\mathbf{s}, \mathbf{u}, \mathbf{v})$ to outside of the logarithm in the objective function by introducing a new variable γ_{im} denoting the SINR in the data stream (i, m), and rewriting (2) as

$$\begin{aligned} \underset{\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma}}{\text{maximize}} \quad & \sum_{(i,m)} w_{s_{im}} \log(1 + \gamma_{im}) \\ \text{subject to} \quad & \text{(2b), (2c)} \\ & \gamma_{im} = \frac{|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)\neq(i,m)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}. \end{aligned} \tag{3}$$

The Lagrangian with respect to the new equality constraint is

$$L(\mathbf{s}, \mathbf{u}, \mathbf{v}, \boldsymbol{\gamma}, \boldsymbol{\lambda}) = \sum_{(i,m)} w_{s_{im}} \log(1 + \gamma_{im}) - \sum_{(i,m)} \lambda_{im} \cdot \left(\gamma_{im} - \frac{|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)\neq(i,m)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}\right). \tag{4}$$

By setting $\partial L / \partial \gamma_{im}$ to zero for every (i, m), we arrive at

$$\gamma_{im} = \frac{w_{s_{im}}}{\lambda_{im}} - 1. \tag{5}$$

Since the optimal dual variable λ_{im} must satisfy the above, we can combine the above equation with the equality constraint on γ_{im} in (3) so as to express λ_{im} in $\mathbf{s}$, $\mathbf{u}$ and $\mathbf{v}$, i.e.,

$$\lambda_{im} = w_{s_{im}} - \frac{w_{s_{im}} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}. \tag{6}$$

Define a new function f_r as the Lagrangian function (4) with its variable λ_{im} substituted by the above, that is

$$f_r(\mathbf{s}, \mathbf{u}, \mathbf{v}, \boldsymbol{\gamma}) = \sum_{(i,m)} w_{s_{im}} \log(1 + \gamma_{im}) - \sum_{(i,m)} w_{s_{im}} \gamma_{im} + \sum_{(i,m)} \frac{w_{s_{im}} (1 + \gamma_{im}) |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}. \tag{7}$$

We arrive at the following equivalent reformulation of the original problem (2):

$$\begin{aligned} \underset{\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma}}{\text{maximize}} \quad & f_r(\mathbf{s}, \mathbf{u}, \mathbf{v}, \boldsymbol{\gamma}) \\ \text{subject to} \quad & \text{(2b), (2c).} \end{aligned} \tag{8}$$

The above reformulation is a generalization of our earlier result in [1] from SISO to MIMO with the inclusion of beamformers $\mathbf{u}$ and $\mathbf{v}$.

3.2. Quadratic Transform

The last term in the objective function (7) has a sum-of-ratio form, which is still difficult to deal with [9], but a further reformulation can be carried out based on the following fact, which is first used in [1] for joint uplink scheduling and power control in SISO systems, now applied to the joint uplink scheduling and beamforming of MIMO systems.

Proposition 1 ([1]). Given a constraint set $\mathcal{X}$ and two functions $A(\mathbf{x})$ and $B(\mathbf{x}) : \mathbb{R}^d \to \mathbb{R}^+$, the fractional programming problem

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & \frac{A(\mathbf{x})}{B(\mathbf{x})} && \text{(9a)} \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X} && \text{(9b)} \end{aligned}$$

is equivalent to

$$\begin{aligned} \underset{\mathbf{x},y}{\text{maximize}} \quad & 2y\sqrt{A(\mathbf{x})} - y^2 B(\mathbf{x}) && \text{(10a)} \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X}. && \text{(10b)} \end{aligned}$$

Proposition 1 "flattens" the ratio by decoupling its numerator and denominator. Applying this result to (8) gives the following equivalent reformulation:

$$\begin{aligned} \underset{\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma},\mathbf{y}}{\text{maximize}} \quad & f_q(\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma},\mathbf{y}) && \text{(11a)} \\ \text{subject to} \quad & \text{(2b), (2c),} && \text{(11b)} \end{aligned}$$

where the objective function is defined as

$$f_q(\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma},\mathbf{y}) = \sum_{(i,m)} \Bigg(w_{s_{im}} \log(1+\gamma_{im}) - w_{s_{im}}\gamma_{im} + 2y_{im}\sqrt{w_{s_{im}}(1+\gamma_{im})}|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}| - y_{im}^2 \sigma^2 \|\mathbf{u}_{im}\|^2 - \sum_{(j,n)} y_{jn}^2 |\mathbf{u}_{jn}^\dagger \mathbf{H}_{j,s_{im}} \mathbf{v}_{s_{im}}|^2 \Bigg). \quad (12)$$

3.3. Iterative Optimization with Matching

Following a similar iterative approach as in [1], we now optimize (12) iteratively over the variables $(\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma},\mathbf{y})$. The key new elements are the inclusion of beamformers $\mathbf{u}$ and $\mathbf{v}$, and the need to match the users to the beamformers.

Fixing other variables, the optimal $\mathbf{y}$ is just

$$y_{im}^* = \frac{\sqrt{w_{s_{im}}(1+\gamma_{im})}|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|}{\sum_{(j,n)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}. \quad (13)$$

After $\mathbf{y}$ is updated by the above, we find the optimal $\boldsymbol{\gamma}$ as

$$\gamma_{im}^* = \frac{|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)\neq(i,m)} |\mathbf{u}_{im}^\dagger \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}}|^2 + \sigma^2 \|\mathbf{u}_{im}\|^2}. \quad (14)$$

Likewise, the optimal $\mathbf{u}$ in the iterative optimization is

$$\mathbf{u}_{im}^* = \left(\sum_{(j,n)} \mathbf{H}_{i,s_{jn}} \mathbf{v}_{s_{jn}} \mathbf{v}_{s_{jn}}^\dagger \mathbf{H}_{i,s_{jn}}^\dagger + \sigma^2 \mathbf{I} \right)^{-1} \cdot \frac{\sqrt{w_{s_{im}}(1+\gamma_{im})}}{y_{im}} \mathbf{H}_{i,s_{im}} \mathbf{v}_{s_{im}}. \quad (15)$$

Note that the optimal $\boldsymbol{\gamma}$ is just an evaluation of SINR, and the optimal $\mathbf{u}$ is a scaled minimum mean square error equalizer.

We now present a new matching technique for the joint optimization of $\mathbf{s}$ and $\mathbf{v}$ while $\boldsymbol{\gamma}$, $\mathbf{y}$ and $\mathbf{u}$ are assumed fixed. The key observation is that the scheduling of user s_{im} and the choice of its transmit beamformer $\mathbf{v}_{s_{im}}$ in a particular data stream (i,m) contribute to the objective function (12) in a way that is *independent* of the scheduling and beamforming in other data streams. More specifically, when a user k is scheduled in the mth stream at BS i, i.e., $s_{im} = k$, the transmit beamformer for the user k that maximizes (12) is

$$\mathbf{v}_k^* = \left(\sum_{(j,n)} y_{jn}^2 \mathbf{H}_{j,k}^\dagger \mathbf{u}_{jn} \mathbf{u}_{jn}^\dagger \mathbf{H}_{j,k} + \eta_k^* \mathbf{I} \right)^{-1} \cdot y_{im}\sqrt{w_k(1+\gamma_{im})} \mathbf{H}_{i,k}^\dagger \mathbf{u}_{im} \quad (16)$$

where $\eta_k^* = \min\{\eta_k \geq 0 : \|\mathbf{v}_k^*\|^2 \leq P\}$ accounts for the power constraint. Therefore, the utility of scheduling a user to a data stream is completely determined by (12) and (16). This allows solving $\mathbf{s}$ and $\mathbf{v}$ by matching. To formalize the idea, we define the utility value of assigning user k to data stream (i,m) as (where $\mathbf{v}_k^*$ is computed by (16) for $s_{im} = k$)

$$\xi_{k,im} = 2y_{im}\sqrt{w_k(1+\gamma_{im})}|\mathbf{u}_{im}^\dagger \mathbf{H}_{i,k} \mathbf{v}_k^*| - w_k\gamma_{im} + w_k \log(1+\gamma_{im}) - \sum_{(j,n)} y_{jn}^2 |\mathbf{u}_{jn}^\dagger \mathbf{H}_{j,k} \mathbf{v}_k^*|^2. \quad (17)$$

Then maximizing f_q reduces to solving the following problem at each BS i individually:

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & \sum_{k\in\mathcal{U}_i} \sum_{m=1}^{M} \xi_{k,im} x_{k,im} && \text{(18a)} \\ \text{subject to} \quad & \sum_{k\in\mathcal{U}_i} x_{k,im} \leq 1 && \text{(18b)} \\ & \sum_{m=1}^{M} x_{k,im} \leq 1 && \text{(18c)} \\ & x_{k,im} \in \{0,1\}, && \text{(18d)} \end{aligned}$$

where the binary variable $x_{k,im}$ indicates whether or not user k is scheduled in the mth stream at its associated BS i. Observe that (18) is a weighted bipartite matching problem, which can be efficiently solved by the existing algorithms, e.g., auction algorithm [10] and Hungarian algorithm [11]. After finding the optimal $\mathbf{x}^*$, we recover $\mathbf{s}^*$ by

$$s_{im}^* = k, \text{ when } x_{k,im}^* = 1 \text{ for some } k \in \mathcal{U}_i. \quad (19)$$

We summarize the proposed algorithm below. Note that the algorithm is guaranteed to converge, because the weighted sum rate f_o is nondecreasing after each iteration.

Algorithm 1 Joint scheduling and beamforming algorithm

Initialization: Initialize $\mathbf{s}$, $\mathbf{u}$, $\mathbf{v}$ and $\boldsymbol{\gamma}$.
repeat
1) Update $\mathbf{y}$ by (13);
2) Update $\boldsymbol{\gamma}$ by (14);
3) Update $\mathbf{u}$ by (15);
4) Update $\mathbf{v}$ and $\mathbf{s}$ jointly by (16) and (19);
until Convergence

3.4. Proposed Algorithm vs. WMMSE

The well-known WMMSE algorithm [2, 3] can already be used for the uplink coordinated joint scheduling and beamforming problem. Assume that all the users in the network are scheduled at the beginning; run the WMMSE algorithm to design beamformers for all the users; then only schedule the users with positive transmit power levels at the end. Interestingly, there is a connection between the WMMSE algorithm and our sum-of-ratio optimization approach.

The WMMSE algorithm is originally derived based on a minimum mean square error analysis. In what follows, we give another derivation for WMMSE based on the quadratic transform in Section 3.2. Recall that in the derivation of the proposed algorithm in this paper, we use Proposition 1 to decouple the numerator and the denominator of the ratio $\frac{w_{s_{im}}(1+\gamma_{im})|\mathbf{u}_{im}^{\dagger}\mathbf{H}_{i,s_{im}}\mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)}|\mathbf{u}_{im}^{\dagger}\mathbf{H}_{i,s_{jn}}\mathbf{v}_{s_{jn}}|^2+\sigma^2\|\mathbf{u}_{im}\|^2}$ in f_r to derive f_q in (12). However, we could also have applied Proposition 1 to the ratio $\frac{|\mathbf{u}_{im}^{\dagger}\mathbf{H}_{i,s_{im}}\mathbf{v}_{s_{im}}|^2}{\sum_{(j,n)}|\mathbf{u}_{im}^{\dagger}\mathbf{H}_{i,s_{jn}}\mathbf{v}_{s_{jn}}|^2+\sigma^2\|\mathbf{u}_{im}\|^2}$ only without including the factor $w_{s_{im}}(1+\gamma_{im})$. In this case, we would have arrived at a different reformulation

$$\tilde{f}_q(\mathbf{s},\mathbf{u},\mathbf{v},\boldsymbol{\gamma},\mathbf{y}) = \sum_{(i,m)}\Bigg(w_{s_{im}}\log(1+\gamma_{im}) - w_{s_{im}}\gamma_{im} + w_{s_{im}}(1+\gamma_{im})\left(2y_{im}|\mathbf{u}_{im}^{\dagger}\mathbf{H}_{i,s_{im}}\mathbf{v}_{s_{im}}| - y_{im}^2\sigma^2\|\mathbf{u}_{im}\|^2\right) - \sum_{(j,n)} y_{jn}^2 w_{s_{jn}}(1+\gamma_{jn})|\mathbf{u}_{jn}^{\dagger}\mathbf{H}_{j,s_{im}}\mathbf{v}_{s_{im}}|^2\Bigg). \quad (20)$$

Now applying iterative optimization to this reformulation for the variables $\mathbf{u}$, $\mathbf{v}$, $\boldsymbol{\gamma}$ and $\mathbf{y}$ for fixed $\mathbf{s}$, we arrive at exactly the WMMSE algorithm with fixed user schedule.

The key difference between WMMSE and our proposed algorithm is that the scheduling variable $\mathbf{s}$ cannot be easily optimized in $\tilde{f}_q$, while this paper proposes to reformulate f_q in such a way so that $\mathbf{s}$ can be explicitly found by weighted bipartite matching. In contrast, the implicit scheduling in the WMMSE algorithm is both more computationally complex and has inferior performance as illustrated next.

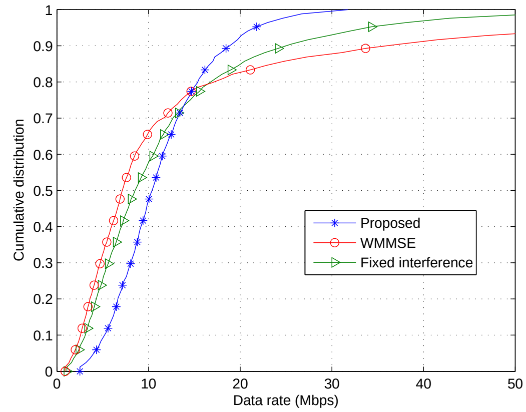

Fig. 1: User-throughput CDF of the different methods

4. SIMULATION RESULTS

Simulation is performed in a 7-cell wrapped-around topology with 4 antennas per BS and 2 antennas per user. The BS-to-BS distance is 800m. A total of 84 users are uniformly placed in the network; the users are associated with the strongest BS. The maximum transmit power spectrum density (PSD) of each user is -47dBm/Hz over 10MHz. The wireless channels are modeled with a path-loss exponent of 3.76 plus a shadowing and fading component. User priorities are set according to proportional fairness with weights updated as the reciprocals of long-term average user rates.

The proposed algorithm is compared with the aforementioned WMMSE algorithm [2, 3] and a per-cell scheduling baseline, where scheduling and beamforming are updated iteratively: in each data stream, the user with maximum weighted rate is scheduled, assuming fixed received interference from previous iteration; the WMMSE algorithm is applied subsequently for beamforming. Fig. 1 shows the significant performance gain of the proposed algorithm as compared to the WMMSE algorithm and the per-cell scheduling baseline assuming fixed interference. The gain is particularly large for low-rate users. For example, the rates of the 10th-percentile users nearly double under the proposed algorithm. These low-rate users are mostly located close to the cell edges, highlighting the important role of coordinated uplink scheduling and beamforming in interference mitigation.

5. CONCLUSION

This paper proposes a joint uplink user-scheduling, power control and beamforming algorithm coordinated across multiple cells. By taking advantage of a novel reformulation and a solution based on the matching algorithm, interference-aware coordination of uplink transmission is shown to be capable of significantly improving the cell-edge throughput.

6. REFERENCES

[1] K. Shen and W. Yu, "A coordinated uplink scheduling and power control algorithm for multicell networks," in *Asilomar Conf. Signals Syst. Comput.*, Oct. 2015.

[2] S. S. Christensen, R. Argawal, E. de Carvalho, and J. M. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 1–7, Dec. 2008.

[3] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, Sept. 2011.

[4] H. Safa and K. Tohme, "LTE uplink scheduling algorithms: Performance and challenges," in *19th Int. Conf. Telecommun. (ICT)*, Apr. 2012, pp. 1–6.

[5] G. Y. Li, J. Niu, D. Lee, and J. Fan, "Multi-cell coordinated scheduling and MIMO in LTE," *Commun. Surveys Tuts.*, vol. 16, no. 2, pp. 761–775, Mar. 2014.

[6] Y. Li, G. Zhu, and S. Zhang, "Transmitter centric scheduling in multi-cell MIMO uplink system," in *IEEE Int. Conf. Commun. (ICC)*, May 2010, pp. 1–6.

[7] B. O. Lee, O.-S. Shin, and K. B. Lee, "Uplink user selection scheme for multiuser MIMO systems in a multicell environment," in *IEEE Int. Conf. Commun. (ICC)*, May 2010, pp. 1–6.

[8] W. Yu, T. Kwon, and C. Shin, "Multicell coordination via joint scheduling, beamforming and power spectrum adaptation," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 1–14, June 2013.

[9] S. Schaible and J. Shi, "Fractional programming: The sum-of-ratios case," *Optimization Methods and Software*, vol. 18, no. 2, pp. 219–229, 2003.

[10] D. P. Bertsekas, "The auction algorithm: A distributed relaxation method for the assignment problem," *Ann. Operations Research*, vol. 14, no. 1, pp. 105–123, Dec. 1988.

[11] H. W. Kuhn, "The Hungarian method for the assignment problem," *Naval Research Logistics Quarterly*, vol. 2, no. 1, pp. 83–97, Mar. 1955.