RESILIENT DECENTRALIZED CONSENSUS-BASED STATE ESTIMATION FOR SMART GRID IN PRESENCE OF FALSE DATA

Reza Hassani Etemad[+] *and Farshad Lahouti*[+*]

[+]School of Electrical and Computer Engineering University of Tehran,
[*]Electrical Engineering Department, California Institute of Technology

ABSTRACT

State estimation is an essential part of energy management system in smart grid as it is a basis for many of the associated management and control processes. In this paper, we present a decentralized state estimation approach, based on consensus optimization and the alternating direction method of multipliers, that is robust against certain harsh class of false data injection schemes. The proposed scheme provides a reliable estimate of the global system state in a distributed manner even if the system is regionally unobservable to some regional controllers, but globally observable across regions. The scheme also accommodates different communication network topologies for a given power network. We assess the performance of the presented schemes on IEEE 14 and 118 bus test systems.

Index Terms— State Estimation, Smart Grid, ADMM

1. INTRODUCTION

Smart grid as the next generation of power grid utilizes sensed grid data to automatically improve the stability, efficiency, economics, and reliability of production and distribution of electricity. The smart grid uses communications capability to collect data and learn about the behavior of consumers and suppliers [1]. State estimation enables most processes in an energy management system of a smart grid. As such, it is instrumental in preserving sustainability and increasing the efficiency of power system operation. Owing to communication deficiencies, difficulty in computation and reliability issues, decentralized state estimation is highly essential [2]. False data injection in smart grids by potentially malicious users would lead to incorrect state estimation [3]. Specifically, it may result in energy theft from consumers, device malfunction during power generation and incorrect dispatching for the distribution procedure [4]. Therefore, a decentralized state estimation that is reliable and robust against false data injection is instrumental in the smart grid; and that is the research problem we here investigate.

Several approaches have been proposed for distributed state estimation in power networks, that may be categorized as partially distributed, hierarchical or fully distributed. A survey on hierarchical state estimation is presented in [5]. In [6] and [7] two-level state estimation for multi-area power systems is studied. In [8], schemes for two-level and decentralized state estimation are surveyed. These hierarchical methods need a coordinating center for information aggregation, and local observability for state estimation in each region [9]. In [10] a fully distributed state estimation scheme based on first order diffusion algorithms is proposed. The algorithm estimates the entire state of the network in each region with low computational complexity per iteration. Nonetheless, it requires a large number of iterations to achieve a desired quality estimation. In [11], another distributed algorithm is proposed, which in each region only estimates the corresponding state variables. The scheme is developed for robust performance in presence of sparse false data. This algorithm converges in a smaller number of iterations in comparison with [10], however in the assumed setting, the communication network topology is dictated by the power network topology.

The false data injection security problem in smart grids is presented in [3] and classic approaches to handle it is reviewed in [1]. In [12] distributed joint anomaly detection and state recovery based on consensus and innovation is proposed. The security problem of manipulating the information exchanged between regions in order to hinder the convergence of the distributed algorithm is studied in [13]. In [14], the problem of constructing sparse vectors for an unobservable anomaly in centralized and distributed form has been formulated.

In this paper, we consider the state estimation problem in smart grid as a consensus optimization problem and then present a fully decentralized solution based on Alternating Direction Method of Multipliers (ADMM). The solution, dubbed Decentralized Consensus-based State Estimation algorithm (DCSE) estimates global state variables in each region without the need for local observability. We then consider a malicious third party who injects false data into the measurements. We first model the anomaly due to false measurements in smart grid and then present an anomaly resilient scheme referred to as the Resilient Decentralized Consensus-based State Estimation (RDCSE). The proposed scheme jointly detects the anomaly and estimates the states in a decentralized manner. Finally we validate the presented schemes by simulation on IEEE 14 and 118 bus systems and comparison with [10].

2. CONSENSUS-BASED STATE ESTIMATION

In an electric power grid, assuming DC power flow, the control center needs to monitor the voltage phase angles of all buses to manage the network. In this regard the control center collects the readings from remote electric meters to estimate the system operation state. In Supervisory Control and Data Acquisition (SCADA) systems, specific measurement data include branch active power flows and bus active power injection [15]. On the other hand Phasor Measurement Units (PMU) provide accurate synchronous phasor measurements for geographically dispersed nodes in power networks [16].

PMU measurements have a linear relation with the state variables. State variables in this case are voltage magnitude and angle of all buses. State estimation by DC power flow, in SCADA systems or using PMUs, is a convex optimization problem. In the following, we first review the power system state estimation problem in centralized form, and then present a decentralized solution.

2.1. Centralized State Estimation

State estimation is the process of estimating unknown state variables from measurement variables. Let $\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_N)$, $\mathbf{x} \in \mathbf{R}^N$ and $\mathbf{z} = (\mathbf{z}_1, ..., \mathbf{z}_M), \mathbf{z} \in \mathbf{R}^M$ be state and measurement variables, respectively. We have

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \tag{1}$$

where $\mathbf{h}(\mathbf{x}) = (\mathbf{h}_1(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N), ..., \mathbf{h}_M(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N))$ and $\mathbf{e}$ is the measurement noise. The function $\mathbf{h}(\mathbf{x})$ is in general nonlinear, but assuming DC power flow or using PMU measurements for state estimation, it becomes linear and as such we can write (1) as follows

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{e}, \tag{2}$$

where $\mathbf{H}$ is the Jacobean matrix of $\mathbf{h}(\mathbf{x})$. State variables can be estimated using the following optimization problem

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{z} - \mathbf{H}\mathbf{x}\|_2^2. \tag{3}$$

2.2. Decentralized State Estimation

In this setting, the measurements are assumed to be distributed in R regions, and the purpose is to estimate the complete set of state variables in a decentralized manner. This is while there is no centralized controller and the estimation is to be accomplished by collaboration of regional controllers. We can rewrite (2) as follows

$$\begin{array}{l} [\ \mathbf{z}_1 \quad \mathbf{z}_2 \quad ... \quad \mathbf{z}_R\ ]^T = [\ \mathbf{H}_1 \quad \mathbf{H}_2 \quad ... \quad \mathbf{H}_R\ ]^T\mathbf{x} + \\ [\ \mathbf{e}_1 \quad \mathbf{e}_2 \quad ... \quad \mathbf{e}_R\ ]^T \end{array} \tag{4}$$

where $\mathbf{z}_i$, $\mathbf{H}_i$ and $\mathbf{e}_i$ are the measurement vector, Jacobean matrix and the measurement error corresponding to region i. Moreover, we assume that power system is globally observable. If we consider $\mathbf{f}_i = \frac{1}{2}\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}\|_2^2$ as the objective function of region $i \in \{1, ..., R\}$, the centralized objective function is $\sum_{i=1}^{R} \mathbf{f}_i = \sum_{i=1}^{R} \frac{1}{2}\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}\|_2^2 = \frac{1}{2}\|\mathbf{z} - \mathbf{H}\mathbf{x}\|_2^2$. Hence, the desired optimization problem is expressed as follows

$$\min_{\mathbf{x}} \sum_{i=1}^{R} \frac{1}{2}\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}\|_2^2. \tag{5}$$

We aim to formulate the state estimation problem in the form of a consensus optimization in which several agents try to minimize the sum of their local objective functions with regard to a variable set which is common between agents. As we shall show in the sequel, the proposed solution is fully decentralized without a central coordinating node, and with high accuracy and fast rate of convergence. There are several methods for decentralized consensus optimization, such as distributed sub-gradient descent algorithms, dual averaging methods and the ADMM. Among these, for closed proper and convex objective functions, the ADMM has fast convergence in many applications [17] and converges to the centralized solution [18]. As we assumed global observability, the ADMM solution for the decentralized problem is unique. Hence, there is no constraint on local observability for convergence.

In the following, we convert the problem in (5) to the ADMM format and solve it in a distributed manner in line with [17]. We have

$$\begin{array}{ll} \min\limits_{\{\mathbf{x}_i, \mathbf{y}_{ij}\}} & \sum_{i=1}^{R} \frac{1}{2}\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}_i\|_2^2 \\ s.t. & \mathbf{x}_i = \mathbf{y}_{ij}, \quad \mathbf{x}_j = \mathbf{y}_{ij}\ , \ \forall (i,j) \in E \end{array} \tag{6}$$

where E is the graph of collaboration network between regional controllers and $\mathbf{y}_{ij}$'s are auxiliary variables, which impose the constraints for neighboring regions. The collaboration network identifies the nodes cooperating nodes in each iteration. The problem (6) can now be rewritten in the standard ADMM format as follows

$$\min_{\mathbf{x},\mathbf{y}} \quad \mathbf{f}(\mathbf{x}) + \mathbf{p}(\mathbf{y}) \ \ s.t \quad \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} = \mathbf{0} \tag{7}$$

where $\mathbf{f}(\mathbf{x})$ is equal to $\sum_{i=1}^{R} \mathbf{f}_i$ and $\mathbf{p}(\mathbf{y}) = 0$, $\mathbf{x}$ and $\mathbf{y}$ are concatenation of $\mathbf{x}_i$ and $\mathbf{y}_{ij}$, respectively. $\mathbf{A}$ is a matrix based on topology of collaboration network and it consist of two parts $[\mathbf{A}_1; \mathbf{A}_2]$ such that $\mathbf{A}_1 + \mathbf{A}_2$ is the extended unoriented incidence matrix. $\mathbf{B}$ is a matrix consisting of two identical parts $[-I; -I]$, where I is identity matrix. The augmented Lagrangian of (7) is expressed as follows

$$L_{\mathbf{x},\mathbf{y},\boldsymbol{\lambda}} = \sum_{i=1}^{R} \frac{1}{2}\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}\|_2^2 + \boldsymbol{\lambda}^T(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y}) + \frac{\mu}{2}\|\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y}\|_2^2 \tag{8}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier and μ is a constant parameter. Thus, the ADMM steps based on the augmented Lagrangian are as follows

$$\begin{array}{l} \mathbf{x}^{k+1} = \arg\min\limits_{\mathbf{x}} \sum_{i=1}^{R} \frac{1}{2}\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}\|_2^2 + \boldsymbol{\lambda}^{k^T}(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y}^k) + \\ \frac{\mu}{2}\left\|\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y}^k\right\|_2^2 \\ \mathbf{y}^{k+1} = \arg\min\limits_{\mathbf{y}} \sum_{i=1}^{R} \frac{1}{2}\left\|\mathbf{z}_i - \mathbf{H}_i\mathbf{x}^{k+1}\right\|_2^2 + \boldsymbol{\lambda}^{k^T}(\mathbf{A}\mathbf{x}^{k+1} + \mathbf{B}\mathbf{y}) + \\ \frac{\mu}{2}\left\|\mathbf{A}\mathbf{x}^{k+1} + \mathbf{B}\mathbf{y}\right\|_2^2 \\ \\ \boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \mu(\mathbf{A}\mathbf{x}^{k+1} + \mathbf{B}\mathbf{y}^{k+1}) \end{array} \tag{9}$$

The above steps can be simplified [17] into the following two steps

$$\begin{array}{l} \mathbf{x}_i^{k+1} = \left(\mathbf{H}_i^T\mathbf{H}_i + 2\mu\,|N_i|\,\mathbf{I}\right)^{-1} \\ \left[-\boldsymbol{\sigma}_i^k + \mathbf{H}_i^T\mathbf{z}_i + \mu\left(|N_i|\,\mathbf{x}_i^k + \sum\limits_{j\in N_i} \mathbf{x}_j^k\right)\right] \\ \boldsymbol{\sigma}_i^{k+1} = \boldsymbol{\sigma}_i^k + \mu\left(|N_i|\,\mathbf{x}_i^{k+1} - \sum\limits_{j\in N_i} \mathbf{x}_j^{k+1}\right). \end{array} \tag{10}$$

In the above, N_i denotes the set of neighboring nodes of node i, $|N_i|$ is its cardinality and σ_i is the algorithm parameter for node i which is initially set to zero. $\mathbf{x}_i^{k+1}$ is the global state variables in node i at the step $k+1$ which in DCSE can be obtained from (10).

In this Section, we presented DCSE a distributed algorithm for state estimation based on decentralized consensus optimization. Next we extend this algorithm for robustness against false data injection.

3. RESILIENT DECENTRALIZED CONSENSUS-BASED STATE ESTIMATION

In this Section, we assume that there is a malicious party, Oscar, who intends to disturb the state estimation process by injecting false data into the measurements. Thus, we aim to reliably estimate the state variables from the unreliable measurements. We consider the following optimization problem

$$\begin{aligned}&\min_{\mathbf{x,c}} \tfrac{1}{2}\left\|\mathbf{z}-\mathbf{Hx}-\mathbf{a}\right\|_2^2\\ &s.t.\ \ \left\|\mathbf{a}_i\right\|_0<\tau_1, \quad \left\|\mathbf{a}_i\right\|_2^2<\tau_2, \quad \mathbf{a}_i(j)\le 0\,\forall j;\\ &\qquad \mathbf{a}_i=\mathbf{H}_i\mathbf{c} \quad \text{for } \ i=1,...,R\end{aligned} \tag{11}$$

where we also jointly estimate the false data or the anomaly vector, $\mathbf{a}$, in the process. In formulating (11), we have made the following assumptions about $\mathbf{a}$: (i) As in [3], the false data is considered additive and a linear combination of the column vectors of $\mathbf{H}$, i.e., $\mathbf{a}=\mathbf{Hc}$ or $[\mathbf{a}_1\ \mathbf{a}_2\ ...\ \mathbf{a}_R]^T=[\mathbf{H}_1\ \mathbf{H}_2\ ...\ \mathbf{H}_R]^T\mathbf{c}$; (ii) the anomaly vector is sparse as Oscar can only alter a limited number of measurements in each region, and (iii) Oscar has limited energy for perturbing the state estimation and try disturb just by reducing the amount of measurements in each region because of some reliability and economical reasons.

Solving (11) with l_0 norm constraint is NP-hard, and as such we relax the l_0 norm to the l_1 norm constraint. We have

$$\begin{aligned}&\min_{\mathbf{x,c}} \tfrac{1}{2}\left\|\mathbf{z}-\mathbf{Hx}-\mathbf{a}\right\|_2^2\\ &s.t.\ \ \left\|\mathbf{H}_i\mathbf{c}\right\|_1<\tau_1^{'}, \quad \left\|\mathbf{H}_i\mathbf{c}\right\|_2^2<\tau_2, \quad \mathbf{H}_i\mathbf{c}\le\mathbf{0};\\ &\qquad \text{for } \ i=1,...,R\end{aligned} \tag{12}$$

Considering $\mathbf{H}_i=[\mathbf{h}_{1i};\ \mathbf{h}_{2i};\ ...\ ;\mathbf{h}_{Mi}]$, where $\mathbf{h}_{ji}$ is the j'th row of matrix $\mathbf{H}_i$, we have $\left\|\mathbf{H}_i\mathbf{c}\right\|_1=|\mathbf{h}_{1i}\mathbf{c}|+|\mathbf{h}_{2i}\mathbf{c}|+...+|\mathbf{h}_{Mi}\mathbf{c}|$ and the first constraint in (12) may be rewritten as

$$\left\|\mathbf{H}_i\mathbf{c}\right\|_1=-(\mathbf{h}_{1i}+\mathbf{h}_{2i}+\mathbf{h}_{3i}+...)\mathbf{c}=\mathbf{g}_i\mathbf{c} \tag{13}$$

where $\mathbf{g}_i$ is the negative sum of the rows of the matrix $\mathbf{H}_i$. Using strong Lagrange duality and the last constraint in (12), the objective function in (11), $\mathbf{f}(\mathbf{x},\mathbf{c})$, may be rewritten as $\sum_{i=1}^{R}\mathbf{f}_i(\mathbf{x},\mathbf{c})$, where $\mathbf{f}_i(\mathbf{x},\mathbf{c})$ or the objective function in region i is given by

$$\mathbf{f}_i(\mathbf{x},\mathbf{c})=\frac{1}{2}\left\|\mathbf{z}_i-\mathbf{H}_i\mathbf{x}-\mathbf{H}_i\mathbf{c}\right\|_2^2+\lambda_1\mathbf{g}_i\mathbf{c}+\lambda_2\left\|\mathbf{H}_i\mathbf{c}\right\|_2^2. \tag{14}$$

Since our purpose is to solve the above problem in a distributed manner we can rewrite (12) in the following form

$$\min_{\mathbf{x,c,y}} \ \mathbf{f}(\mathbf{x},\mathbf{c})+\mathbf{p}(\mathbf{y}) \quad s.t\ \ \mathbf{Ax}+\mathbf{Dy}=\mathbf{0} \tag{15}$$

where $\mathbf{p}(\mathbf{y})=0$. For solving the above problem via ADMM we first should write the augmented Lagrangian of (15).

$$L(\mathbf{x},\mathbf{c},\mathbf{y},\mathbf{v})=\mathbf{f}(\mathbf{x},\mathbf{c})+\mathbf{v}^T(\mathbf{Ax}+\mathbf{Dy})+\frac{\mu_1}{2}\left\|\mathbf{Ax}+\mathbf{Dy}\right\|_2^2 \tag{16}$$

where v is the Lagrange multiplier and μ_1 is a constant parameter for the algorithm. The update steps of ADMM are as below. In the following steps the $\mathbf{c}$ update could be jointly done by either the $\mathbf{x}$ step or the $\mathbf{y}$ step. As the augmented Lagrangian is decomposable with $\mathbf{c}$ and $\mathbf{y}$, we prefer to update $\mathbf{c}$ with $\mathbf{y}$.

Algorithm 1 RDCSE

Initialize $\mathbf{x}_i$ and $\mathbf{c}_i$ with zero
for $i=1,...,R$ **do**
  At first update $\mathbf{x}_i$ as (25) for each region
  Update $\boldsymbol{\eta}_i$ for each region as (25)
  Update $\mathbf{c}_i$ for each region as (29)
  Update $\boldsymbol{\beta}_i$ for each region as (29)
end for

$$\begin{aligned}\mathbf{x}^{k+1}&=\operatorname*{argmin}_{\mathbf{x}} L\left(\mathbf{x},\mathbf{c}^k,\mathbf{y}^k,\mathbf{v}^k\right)\\ \mathbf{c}^{k+1},\mathbf{y}^{k+1}&=\operatorname*{argmin}_{\mathbf{c,y}} L\left(\mathbf{x}^k,\mathbf{c},\mathbf{y},\mathbf{v}^k\right)\\ \mathbf{v}^{k+1}&=\mathbf{v}^k+\mu_1\left(\mathbf{Ax}^{k+1}+\mathbf{Dy}^{k+1}\right)\end{aligned} \tag{17}$$

We can find simpler steps for updating $\mathbf{x}$ similar to what we did for DCSE.

$$\begin{aligned}&\mathbf{x}_i^{k+1}=\left(\mathbf{H}_i^T\mathbf{H}_i+2\mu_1\left|N_i\right|\mathbf{I}\right)^{-1}\\ &\left[\mathbf{H}_i^T(\mathbf{z}_i-\mathbf{H}_i{\mathbf{c}_i}^k)-\boldsymbol{\eta}_i^k+\mu_1\left(\left|N_i\right|\mathbf{x}_i^k+\sum_{j\in N_j}\mathbf{x}_j^k\right)\right]\\ &\boldsymbol{\eta}_i^{k+1}=\ \boldsymbol{\eta}_i^k+\mu_1\left(\left|N_i\right|\mathbf{x}_i^{k+1}-\sum_{j\in N_j}\mathbf{x}_j^{k+1}\right)\end{aligned} \tag{18}$$

For $\mathbf{c}$ update we can follow a similar approach, i.e.,

$$\mathbf{c}^{k+1}=\arg\min_{\mathbf{c}}\mathbf{f}(\mathbf{x}^k,\mathbf{c}) \tag{19}$$

We can solve the problem by formulating to ADMM form.

$$\min_{\mathbf{c,y}} \ \mathbf{f}(\mathbf{x}^{k+1},\mathbf{c})+\mathbf{q}(\mathbf{y}) \quad s.t\ \ \mathbf{Ac}+\mathbf{By}=\mathbf{0} \tag{20}$$

where $\mathbf{q}(\mathbf{y})=0$,then the augmented Lagrangian of the (20) is as follows.

$$L(\mathbf{c},\mathbf{y},\mathbf{v})=\mathbf{f}(\mathbf{x}^{k+1},\mathbf{c})+\mathbf{v}^T(\mathbf{Ac}+\mathbf{By})+\frac{\mu_2}{2}\left\|\mathbf{Ac}+\mathbf{By}\right\|_2^2 \tag{21}$$

where v is the Lagrange multiplier and μ_2 is a constant parameter for the algorithm. Similar to the $\mathbf{x}$ update steps we can find the $\mathbf{c}$ update steps as follows

$$\begin{aligned}&\mathbf{c}_i^{k+1}=\left(\mathbf{H}_i^T\mathbf{H}_i+2\boldsymbol{\lambda}_2\mathbf{H}_i^T\mathbf{H}_i+2\mu_2\left|N_i\right|\mathbf{I}\right)^{-1}\\ &\left[\mathbf{H}_i^T(\mathbf{z}_i-\mathbf{H}_i\mathbf{x}_i^k)-\lambda_1\mathbf{g}_i^T-\boldsymbol{\beta}_i^k+\mu_2\left(\left|N_i\right|\mathbf{c}_i^k+\sum_{j\in N_j}\mathbf{c}_j^k\right)\right]\\ &\boldsymbol{\beta}_i^{k+1}=\ \boldsymbol{\beta}_i^k+\mu_2\left(\left|N_i\right|\mathbf{c}_i^{k+1}-\sum_{j\in N_j}\mathbf{c}_j^{k+1}\right)\end{aligned} \tag{22}$$

Finally, we summarize the presented scheme, henceforth referred to as the Resilient Decentralized Consensus-based State Estimation (RDCSE), in Algorithm 1. In the next Section, we will validate the performance of the proposed schemes.

4. PERFORMANCE EVALUATION

For simulation we test DCSE and RDCSE on IEEE 14 and IEEE 118 bus test systems based on the data from MATPOWER [19]. Similar to [11], the two systems are partitioned to 4 and 3 regions, respectively. We consider PMU measurements (voltage and line currents) for state estimation. In IEEE 14 and 118 bus systems, there are 46 and

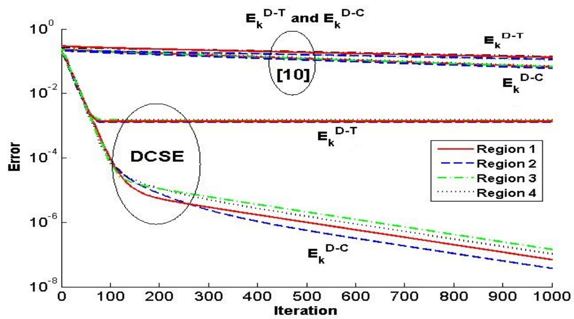

Fig. 1. $\mathcal{E}_{i,k}^{DT}$ and $\mathcal{E}_{i,k}^{DC}$ for DCSE and [10] for IEEE 14 bus

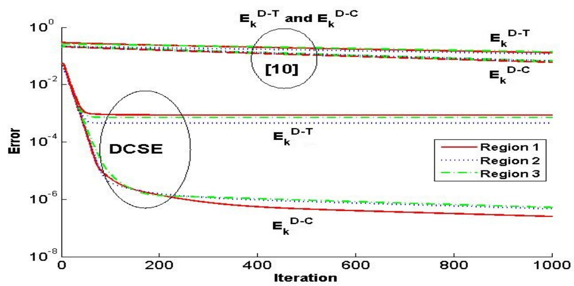

Fig. 2. $\mathcal{E}_{i,k}^{DT}$ and $\mathcal{E}_{i,k}^{DC}$ for DCSE and [10] for IEEE 118 bus

564 measurements, and 28 and 236 state variables, receptively. Moreover, in all of the test cases we consider a line topology for communication network. As a benchmark for comparisons, we consider the approach proposed in [10] that similarly aims to estimate the global system state in each region. In the sequel, the superscripts T, C and D for variables indicate "true values", "centralized estimation" and "decentralized estimation", respectively. For example, $\mathbf{x}_{i,k}^{D}$ is the distributed estimate of states region i at iteration k. We consider the following normalized errors as performance metrics $\mathcal{E}_{i,k}^{DT} = L_i^{-1}\left\|\mathbf{x}_{i,k}^{D} - \mathbf{x}_i^{T}\right\|_2$, $\mathcal{E}_{i,k}^{DC} = L_i^{-1}\left\|\mathbf{x}_{i,k}^{D} - \mathbf{x}_i^{C}\right\|_2$ in which L_i is the number of state variables in region i.

Figures 1 and 2 show $\mathcal{E}_{i,k}^{DT}$ and $\mathcal{E}_{i,k}^{DC}$ for DCSE and the scheme in [10] for each region $i \in \{1, ..., R\}$ as a function of the iteration number k for the IEEE 14 and 118 bus system, respectively. As evident in these figures the proposed DCSE scheme noticeably outperforms the scheme of [10] in convergence rate and accuracy over both bus systems and in terms of both performance metrics. For DCSE experiments, we empirically set the constant parameter μ to 7000, and consider 1000 tests on average. The measurement noise is an independent zero mean Guassian noise with standard deviation 0.01.

To validate that the proposed DCSE does not need local observability in each region, we consider the IEEE 14 bus system with 44 measurements (as opposed to the original 46) such that one of the regions is locally unobservable, but the system is still globally observable.The simulation results, depicted in Figure 3, show that in this setting the proposed

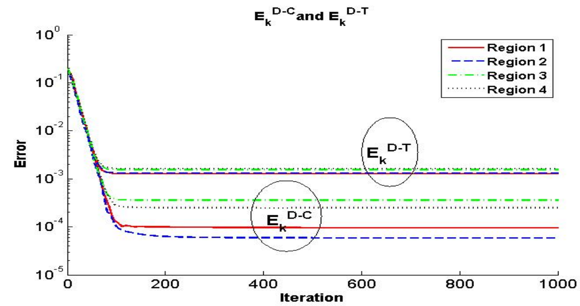

Fig. 3. $\mathcal{E}_{i,k}^{DT}$, $\mathcal{E}_{i,k}^{DC}$ for IEEE 14 bus without local observability

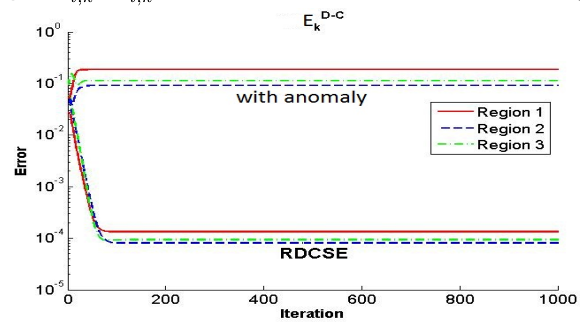

Fig. 4. $\mathcal{E}_{i,k}^{DC}$ with anomaly vector 1 and RDCSE in IEEE 118

scheme still arrives at a reliable estimate with a reasonable accuracy. We next examine the performance of the proposed RDCSE scheme in presence of false data. Table 1 shows $\mathcal{E}_{i,k}^{DC}$ for 3 different anomaly vectors over the IEEE 118 bus system at iteration $k = 1000$. As evident, the proposed RDCSE scheme decreases $\mathcal{E}_{i,k}^{DC}$ significantly and achieves very good accuracy. Figure 4 shows $\mathcal{E}_{i,k}^{DC}$ for one of the anomaly vectors considered in IEEE 118 bus system. It is evident that RDCSE cancels most of the disturbance due to anomaly and provides a performance that is closer to that of the DCSE. In these experiments, we empirically set parameters μ_1 and μ_2 to 7000, and λ_1 and λ_2 to 5 and 500 and considered 100 different test cases.

Anomaly vector/Region	$i=1$	$i=2$	$i=3$
Anomaly vector 1	2	1	0.9
Anomaly vector 2	0.7	0.5	0.6
Anomaly vector 3	1	0.9	0.8

Table 1. $\mathcal{E}_{i,k}^{DC} \times 10^{-4}$ for RDCSE at $k = 1000$ in IEEE 118

5. CONCLUSION

Considering a smart grid that relies on PMU measurements for state estimation and is subject to false data injection anomaly, a resilient decentralized state estimation scheme based on consensus optimization and ADMM algorithm was presented. The performance results on IEEE 14 and 118 bus systems demonstrated that the proposed scheme substantially suppresses the effect of anomalies within a certain harsh class and reliably estimates the system state in a decentralized manner.

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