STOCHASTIC ONLINE CONTROL FOR SMART-GRID POWERED MIMO DOWNLINK TRANSMISSIONS

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ABSTRACT

An infinite time-horizon resource allocation problem is formulated to maximize the time-averaged multi-input multi-output (MIMO) downlink throughput, subject to a time-averaged energy cost budget. By using the advanced time decoupling technique, a novel stochastic subgradient based online control (SGOC) approach is developed for the resultant smart-grid powered communication system. It is analytically established that even without a-priori knowledge of the underlying random processes, the proposed online algorithm is capable of yielding a feasible and asymptotically optimal solution.

Index Terms— MIMO broadcast channels, smart grids, high-penetration renewables, stochastic optimization.

1. INTRODUCTION

Downlink communications from the base station (BS) to mobile users in wireless cellular systems is usually analyzed as a Gaussian broadcast (BC) channel in information-theoretic approaches. Shannon's capacity for both single-input-single-output (SISO) and multi-input multi-output (MIMO) BC channels has been well documented [1–4], when the transmitters (here BSs) are powered by persistent energy sources of the conventional electricity grid. However, the current grid infrastructure is on the verge of a major paradigm shift, migrating from the aging grid to a "smart" one.

While integration of smart-grid technologies into resource allocation clearly holds the key to fully exploiting the potential of future downlink communications [5], only a few works are available in this direction. Leveraging limited smart-grid capabilities in simplified smart-grid models, recent works [5,6] addressed energy-efficient resource allocation for coordinated cellular downlink transmissions. Building on realistic smart-grid models, our recent works in [7,8] developed energy management to minimize the energy transaction cost subject to user quality-of-service (QoS) guarantees of coordinated cellular downlink settings. None of these works though touched on the impact of advanced smart-grid capabilities on the fundamentally achievable rate limits for the BC channels in cellular networks.

As MIMO techniques have been well adopted by wireless standards, we study here the optimal resource allocation for smart-grid powered MIMO downlink transmissions to approach the fundamental rate limits in future cellular networks. Specifically, we develop an *online* resource allocation approach, which dynamically makes *instantaneous* decisions without a-priori knowledge of any statistics of the underlying random channel, renewables, and electricity price processes. To this end, the intended task is formulated as an infinite horizon optimization problem aiming to maximize the time-averaged (weighted) downlink throughput subject to a time-averaged energy cost budget. Targeting a low-complexity solution, we adopt the relaxation techniques in [8, 9] to decouple the decision variables across time. Leveraging the stochastic dual-subgradient method, we propose a novel online control algorithm. To analyze our scheme, we generalize the framework in [8, 9] to characterize the two coupled "virtual" queues involved in our online control. We then establish analytically that the proposed algorithm can yield a feasible and asymptotically optimal strategy for the original problem.

The rest of the paper is organized as follows. The system models are described in Section 2. The proposed dynamic resource allocation scheme is developed and analyzed in Section 3. Numerical results are provided in Section 4, followed by the conclusions.

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2. SYSTEM MODELING

Consider a MIMO BC downlink where a BS with N_t antennas communicates to K mobile users, each having N_r antennas. Powered by a smart microgrid, the BS is equipped with one or more energy harvesting devices (solar panels and/or wind turbines), and can perform two-way energy trading with the main grid. In addition, the BS has a battery so that it can store part of the harvested energy for later use.

2.1. MIMO Downlink Channels

Consider an infinite scheduling horizon, indexed by the set $\mathcal{T} := \{0, 1, 2, \ldots\}$. Per slot $t \in \mathcal{T}$, let $\boldsymbol{H}_{k,t} \in \mathbb{C}^{N_r \times N_t}$ denote the channel coefficient matrix from the BS to user $k = 1, \ldots, K$, and $\mathcal{H}_t := \{\boldsymbol{H}_{1,t}, \ldots, \boldsymbol{H}_{K,t}\}$. Let $\boldsymbol{x}(t) \in \mathbb{C}^{N_t \times 1}$ denote the transmitted vector signal, which is the superposition of those transmitted to individual users: $\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{x}_k(t)$. The complex-baseband signal received at user k is then

$$\boldsymbol{y}_k(t) = \boldsymbol{H}_{k,t}\boldsymbol{x}(t) + \boldsymbol{z}_k(t) \tag{1}$$

where $\boldsymbol{z}_k(t)$ is additive complex-Gaussian noise with zero mean and covariance matrix $\boldsymbol{I}$ (the identity matrix of size N_r).

The MIMO BC capacity is known to be achievable by dirty paper coding [10]. For the codeword $\boldsymbol{x}_k(t)$, the transmit covariance matrix of user k is $\boldsymbol{S}_{k,t} := \mathbb{E}[\boldsymbol{x}_k(t)\boldsymbol{x}_k^\dagger(t)]$. With $P_{x,t}$ denoting the transmit-power budget at the BS, it holds that $\sum_{k=1}^{K} \mathrm{tr}(\boldsymbol{S}_{k,t}) \leq P_{x,t}$. With r_k denoting the achievable transmission rate for user k, $\pi(k)$ the kth user of a certain permutation π of $\{1, 2, \ldots, K\}$, and $|\cdot|$ the determinant operator, the rate K-tuple corresponding to permutation π, is

$$\mathcal{R}_\pi(P_{x,t};\mathcal{H}_t)=\cup_{\{\boldsymbol{S}_{k,t}:\ \sum_{k=1}^K \mathrm{tr}(\boldsymbol{S}_{k,t})\le P_{x,t}\}}\{(r_1,\dots,r_K): r_{\pi(k)}\le \log\frac{\left|\boldsymbol{I}+\sum_{u=1}^{k}\boldsymbol{H}_{\pi(u),t}\boldsymbol{S}_{\pi(u),t}\boldsymbol{H}^\dagger_{\pi(u),t}\right|}{\left|\boldsymbol{I}+\sum_{u=1}^{k-1}\boldsymbol{H}_{\pi(u),t}\boldsymbol{S}_{\pi(u),t}\boldsymbol{H}^\dagger_{\pi(u),t}\right|},\ \forall k\} \quad (2)$$

and the BC capacity region per slot is

$$\mathcal{C}_{\mathrm{BC}}(P_{x,t};\mathcal{H}_t)=\mathcal{C}o\left(\cup_\pi\mathcal{R}_\pi(P_{x,t};\mathcal{H}_t)\right) \quad (3)$$

where $\mathcal{C}o(\cdot)$ denotes the convex hull of the union over all permutations π of $\{1,2,\dots,K\}$.

2.2. Smart Grid Operations

Let E_t denote the BS energy harvested at the beginning of slot t. Further, let C_0 denote the initial energy, and C_t the state of charge (SoC) in the battery at the beginning of slot t. With $C^{\max}$ and $C^{\min}$ bounding the capacity of battery, we have $C^{\min}\le C_t\le C^{\max},\forall t$. With $P_{b,t}$ denoting the power delivered to or drawn from the battery at slot t, the stored energy obeys $C_{t+1}=C_t+P_{b,t}$, where the power (dis)charged is bounded by $P_b^{\min}\le P_{b,t}\le P_b^{\max}$. Per slot t, the total BS energy consumption $P_{g,t}$ includes the transmission-related power $P_{x,t}$, and a constant power $P_c>0$ due to other components such as the data processor, and circuits; hence, $P_{g,t}=P_c+P_{x,t}$, where it is further assumed that $P_{g,t}\le P_g^{\max}$.

When the renewable harvested energy is insufficient, the main grid can supply the needed $P_{g,t}$ to the BS with an amount $[P_{g,t}-E_t+P_{b,t}]^+$; with a two-way energy trading mechanism present, the BS can also sell its surplus energy $[P_{g,t}-E_t+P_{b,t}]^-$ to the grid at a fair price in order to reduce operational costs, where $[a]^+:=\max\{a,0\}$, and $[a]^-:=\max\{-a,0\}$. Suppose that the energy can be purchased from the grid at price $\alpha_t\in[\alpha^{\min},\alpha^{\max}]$, while the energy is sold to the grid at price $\beta_t\in[\beta^{\min},\beta^{\max}]$, with $\alpha_t>\beta_t$ per slot t. Per slot t, the transaction cost for the BS is given by $G(P_{g,t},P_{b,t})=\alpha_t[P_{g,t}-E_t+P_{b,t}]^+-\beta_t[P_{g,t}-E_t+P_{b,t}]^-$.

3. DYNAMIC RESOURCE ALLOCATION

Let w_k denote the priority weight for user k, $\mathcal{S}_t:=\{\boldsymbol{S}_{1,t},\dots,\boldsymbol{S}_{K,t}\}$, and $G^{\max}$ the maximum allowable power cost at the BS. Let $r_k^B(\mathcal{S}_t)$ denote the achievable transmission rate for user k, and $\boldsymbol{r}^B(\mathcal{S}_t):=[r_1^B(\mathcal{S}_t),\dots,r_K^B(\mathcal{S}_t)]$. Over the scheduling horizon $\mathcal{T}$, the central controller at the BS determines the optimal transmit covariance matrices $\{\mathcal{S}_t,\forall t\}$, transmit-power $\{P_{x,t},\forall t\}$, and battery charging energy $\{P_{b,t},\forall t\}$, in order to maximize the limiting average (weighted) total throughput, subject to average energy cost constraint. For notational brevity, we introduce the auxiliary variable $P_t:=P_{g,t}+P_{b,t}$, and express the variables $\{P_{b,t}\}$ in terms of $\{P_t,P_{x,t}\}$. In sum, we wish to solve

$$\max_{\{\mathcal{S}_t,C_t,P_t,P_{x,t}\}}\ \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{K}[w_k\sum_{t=0}^{T-1}(r_k^B(\mathcal{S}_t))] \quad (4a)$$

$$\text{s. t.}\ \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}G(P_t)\le G^{\max} \quad (4b)$$

$$0\le P_{x,t}\le P_g^{\max}-P_c \quad (4c)$$

$$P_b^{\min}\le P_t-P_{x,t}-P_c\le P_b^{\max} \quad (4d)$$

$$C_{t+1}=C_t+P_t-P_{x,t}-P_c \quad (4e)$$

$$C^{\min}\le C_t\le C^{\max} \quad (4f)$$

$$\boldsymbol{r}^B(\mathcal{S}_t)\in\mathcal{C}_{\mathrm{BC}}(P_{x,t};\mathcal{H}_t),\ \forall t. \quad (4g)$$

3.1. Reformulation and Relaxation

With $\psi_t:=(\alpha_t-\beta_t)/2$ and $\phi_t:=(\alpha_t+\beta_t)/2$, it follows that $G(P_t)=\psi_t|P_t-E_t|+\phi_t(P_t-E_t)$. Since $\alpha_t>\beta_t>0$, we have $\phi_t>\psi_t>0$; hence, $G(P_t)$ is a convex function of P_t.

Now let us convexify the rate functions $r_k^B(\mathcal{S}_t)$. By the uplink-downlink duality [11–13], the BC capacity region $\mathcal{C}_{\mathrm{BC}}(P_{x,t};\mathcal{H}_t)$ can be alternatively characterized by the capacity regions of a set of "dual" multi-access channels (MACs). In the dual MAC, the received signal is $\boldsymbol{y}(t)=\sum_{k=1}^K\boldsymbol{H}^\dagger_{k,t}\boldsymbol{x}_k(t)+\boldsymbol{z}(t)$, where $\boldsymbol{x}_k(t)$ is the signal transmitted by user k, and $\boldsymbol{z}(t)$ is additive complex-Gaussian noise with zero mean and covariance matrix $\boldsymbol{I}$. Let $\boldsymbol{Q}_k:=\mathbb{E}[\boldsymbol{x}_k\boldsymbol{x}_k^\dagger]\succeq\boldsymbol{0}$ denote the transmit covariance matrix of user k, and let $\boldsymbol{p}:=[P_1,\dots,P_K]^\top$ collect the transmit-power budgets of all users. The uplink-downlink duality dictates that the BC capacity region (3) equals the union of the MAC capacity regions corresponding to all power vectors $\boldsymbol{p}$ satisfying $\sum_{k=1}^K P_k\le P_{x,t}$; that is,

$$\mathcal{C}_{\mathrm{BC}}(\boldsymbol{p};\mathcal{H}_t)=\cup_{\{\boldsymbol{p}:\ \sum_{k=1}^K P_k\le P_{x,t}\}}\mathcal{C}_{\mathrm{MAC}}(\boldsymbol{p};\mathcal{H}_t^\dagger). \quad (5)$$

With $R_t(P_{x,t}):=\max_{\boldsymbol{r}^B(\mathcal{S}_t)\in\mathcal{C}_{\mathrm{BC}}(P_{x,t};\mathcal{H}_t)}\sum_{k=1}^K w_k r_k^B(\mathcal{S}_t)$, [14, Lemma 1] has established the following result.

Lemma 1 *Function $R_t(P_{x,t})$ can be obtained by the optimal value of*

$$\max_{\boldsymbol{Q}_k\succeq\boldsymbol{0}}\ \sum_{k=1}^K(w_{\pi(k)}-w_{\pi(k+1)})\log\left|\boldsymbol{I}+\sum_{u=1}^k\boldsymbol{H}^\dagger_{\pi(u),t}\boldsymbol{Q}_{\pi(u)}\boldsymbol{H}_{\pi(u),t}\right|$$
$$\text{s. t.}\ \sum_{k=1}^K\mathrm{tr}(\boldsymbol{Q}_k)=P_{x,t} \quad (6)$$

where π is the permutation of user indices $\{1,\dots,K\}$ such that $w_{\pi(1)}\ge\cdots\ge w_{\pi(K)}$, and $w_{\pi(K+1)}=0$. In addition, $R_t(P_{x,t})$ is a strictly concave and increasing function of $P_{x,t}$.

Using $R_t(P_{x,t})$, the optimal BC problem can be converted into the optimal sum-power allocation for an equivalent "point-to-point" link, as follows

$$R^*:=\max_{\{C_t,P_t,P_{x,t}\}}\ \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}[R_t(P_{x,t})],\ \text{s. to (4b)}-\text{(4f)}. \quad (7)$$

The convexity of constraint (4b) has been already clarified, while constraints (4c)-(4f) are linear. As $R_t(P_{x,t})$ is a concave function of $P_{x,t}$ per Lemma 1, problem (7) is a convex program. Although (7) becomes convex after judicious reformulation, it is still difficult to solve since it entails maximization of the average total throughput over an infinite time horizon. Furthermore, the battery energy level relations in (4e) couple the optimization variables over the infinite time horizon, which renders the problem intractable for traditional solvers such as dynamic programming.

By recognizing that (4e) can be viewed as an energy queue recursion, we next apply the time decoupling technique to turn (7) into a tractable form [9]. For the queue of C_t, the arrival and departure are P_t and $P_{x,t}+P_c$, respectively, per slot t. For random variables $\{\mathcal{H}_t,E_t,\alpha_t,\beta_t\}$, we assume that mean ergodicity holds almost surely (as), meaning

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R_t(P_{x,t})\overset{\text{as}}{=}\mathbb{E}[R_t(P_{x,t})] \quad (8)$$

$$\max_{\boldsymbol{Q}_k\succeq\boldsymbol{0},P_t\geq 0}\ \sum_{k=1}^{K}(w_{\pi(k)}-w_{\pi(k+1)})\log\left|\boldsymbol{I}+\sum_{u=1}^{k}\boldsymbol{H}^{\dagger}_{\pi(u),t}\boldsymbol{Q}_{\pi(u)}\boldsymbol{H}_{\pi(u),t}\right|+\lambda_2(j)\sum_{k=1}^{K}\mathrm{tr}(\boldsymbol{Q}_k)-\lambda_2(j)P_t-\lambda_1(j)G(P_t)$$
$$\text{s. t. }\ 0\leq\sum_{k=1}^{K}\mathrm{tr}(\boldsymbol{Q}_k)\leq P_g^{\max}-P_c,\quad P_b^{\min}\leq P_t-\sum_{k=1}^{K}\mathrm{tr}(\boldsymbol{Q}_k)-P_c\leq P_b^{\max} \tag{13}$$

and likewise for $\mathbb{E}[G(P_t)]$, $\mathbb{E}[P_t]$, and $\mathbb{E}[P_{x,t}]$, where the expectation is taken over all sources of randomness. Now simply remove the variables $\{C_t\}$, and consider the following problem

$$\begin{aligned}\tilde{R}^* :=& \max_{\{P_t,P_{x,t}\}}\mathbb{E}[R_t(P_{x,t})]\\ \text{s. t. }& \mathbb{E}[G(P_t)]\leq G^{\max},\quad \mathbb{E}[P_t]=P_c+\mathbb{E}[P_{x,t}]\\ & \text{(4c)}-\text{(4d)}.\end{aligned} \tag{9}$$

It can be shown that (9) is a relaxed version of (7). Specifically, any feasible solution of (7) also satisfies the constraints in (9) [15]. As a result, the optimal value of (9) is not less than that of (7); that is, $\tilde{R}^*\geq R^*$. Note that the time coupling constraint (4e) has been relaxed in problem (9), which then becomes easier to solve.

We next develop a stochastic dual subgradient solver for (9), which under proper initialization can provide an asymptotically optimal solution of the original resource allocation problem (4).

3.2. Dual Subgradient Approach

Let $\mathcal{F}_t$ denote the set of $\{P_t,P_{x,t}\}$ satisfying constraints (4c)–(4d) per t, and $\boldsymbol{\lambda}:=[\lambda_1,\lambda_2]$ collect the Lagrange multipliers associated with the two average constraints. With the convenient notation $\boldsymbol{X}_t:=\{P_t,P_{x,t}\}$ and $\boldsymbol{X}:=\{\boldsymbol{X}_t,\forall t\}$, the partial Lagrangian function of (9) is

$$\begin{aligned}L(\boldsymbol{X},\boldsymbol{\lambda}):=&\mathbb{E}[R_t(P_{x,t})]-\lambda_1(\mathbb{E}[G(P_t)]-G^{\max})\\ &-\lambda_2(\mathbb{E}[P_t]-P_c-\mathbb{E}[P_{x,t}])\end{aligned} \tag{10}$$

while the Lagrange dual function is given by $D(\boldsymbol{\lambda}):=\max_{\{\boldsymbol{X}^t\in\mathcal{F}^t\}_t} L(\boldsymbol{X},\boldsymbol{\lambda})$, and the dual problem of (9) is: $\min_{\lambda_1\geq 0,\lambda_2}\ D(\boldsymbol{\lambda})$.

For the dual problem, we can resort to a standard subgradient method to obtain $\boldsymbol{\lambda}^*$. This amounts to running the iterations

$$\begin{aligned}\lambda_1(j+1)&=[\lambda_1(j)-\mu(G^{\max}-\mathbb{E}[G(P_t(j))])]^+\\ \lambda_2(j+1)&=\lambda_2(j)-\mu(P_c+\mathbb{E}[P_{x,t}(j)]-\mathbb{E}[P_t(j)])\end{aligned} \tag{11}$$

where j is the iteration index, and $\mu>0$ is an appropriate stepsize; while primal variables $P_t(j)$ and $P_{x,t}(j)$ are given by

$$\begin{aligned}\{P_t(j),P_{x,t}(j)\}\in\arg&\max_{\{P_t,P_{x,t}\}\in\mathcal{F}_t}[R_t(P_{x,t})\\ &-\lambda_1(j)G(P_t)-\lambda_2(j)(P_t-P_c-P_{x,t})].\end{aligned} \tag{12}$$

By Lemma 1, the convex maximization problem (12) can be transformed into (13) at the top of the page, which can be efficiently solved by an off-the-shelf solver in polynomial time. With $\{P_t(\boldsymbol{\lambda}(j)),\boldsymbol{Q}_k(\boldsymbol{\lambda}(j)),\forall k\}$ denoting the optimal solution of (13), one can subsequently determine $P_t(j)=P_t(\boldsymbol{\lambda}(j))$, and $P_{x,t}(j)=\sum_{k=1}^{K}\mathrm{tr}(\boldsymbol{Q}_k(\boldsymbol{\lambda}(j)))$.

3.3. Stochastic Subgradient Online Control

A challenge associated with the subgradient iterations (11) is computing $\mathbb{E}[P_t(j)]$, $\mathbb{E}[P_{x,t}(j)]$, and $\mathbb{E}[G(P_t(j))]$ per iterate. This amounts to performing (high-dimensional) integration over unknown joint distribution functions; or approximately, computing the corresponding time-averages over an infinite time horizon. To overcome this impractical requirement, we resort to a stochastic subgradient approach. Dropping $\mathbb{E}$ from (11), consider the iterations

$$\begin{aligned}\hat{\lambda}_1^{t+1}&=[\hat{\lambda}_1^t-\mu(G^{\max}-G(P_t(\hat{\boldsymbol{\lambda}}^t)))]^+\\ \hat{\lambda}_2^{t+1}&=\hat{\lambda}_2^t-\mu(P_c+P_{x,t}(\hat{\boldsymbol{\lambda}}^t)-P_t(\hat{\boldsymbol{\lambda}}^t))\end{aligned} \tag{14}$$

where $\{\hat{\lambda}_1^t,\hat{\lambda}_2^t\}$ are stochastic estimates of those in (11), and $P_t(\hat{\boldsymbol{\lambda}}^t)$, $P_{x,t}(\hat{\boldsymbol{\lambda}}^t)$ are obtained by solving (12) with $\boldsymbol{\lambda}(j)$ replaced by $\hat{\boldsymbol{\lambda}}^t$.

The update (14) is in fact an *online* approximation algorithm based on the *instantaneous* decisions $\{P_t(\hat{\boldsymbol{\lambda}}^t),P_{x,t}(\hat{\boldsymbol{\lambda}}^t)\}$ per slot t. Based on (14), we will develop next a stochastic subgradient based online control (SGOC) algorithm for the original problem (4). The algorithm is implemented at the BS as follows.

SGOC: Initialize with a proper $\hat{\boldsymbol{\lambda}}^0:=[\hat{\lambda}_1^0,\hat{\lambda}_2^0]$. At every time slot t, observe $\hat{\boldsymbol{\lambda}}^t,\mathcal{H}_t,E_t,\alpha_t,\beta_t$, and then do:

Energy management. Obtain $\{P_t(\hat{\boldsymbol{\lambda}}^t),P_{x,t}(\hat{\boldsymbol{\lambda}}^t)\}$ by solving (12). Perform energy transaction with the main grid; and (dis)charge the battery with the amount $P_{b,t}=P_t(\hat{\boldsymbol{\lambda}}^t)-P_{x,t}(\hat{\boldsymbol{\lambda}}^t)-P_c$.

Broadcast schedule. Given $P_{x,t}(\hat{\boldsymbol{\lambda}}^t)$ at the BS, solve the convex problem (6) to obtain the optimal "dual" MAC transmit-covariance matrices $\{\boldsymbol{Q}_k(P_{x,t}(\hat{\boldsymbol{\lambda}}^t)),\forall k\}$. Define for $k=1,\ldots,K$,

$$\begin{aligned}\boldsymbol{A}_k&:=\boldsymbol{I}+\boldsymbol{H}_{\pi(k)}\left(\textstyle\sum_{u=1}^{k-1}\boldsymbol{S}_{\pi(u),t-1}\right)\boldsymbol{H}^{\dagger}_{\pi(k)},\\ \boldsymbol{B}_k&:=\boldsymbol{I}+\textstyle\sum_{u=k+1}^{K}\left(\boldsymbol{H}^{\dagger}_{\pi(u)}\boldsymbol{Q}_{\pi(u)}(P_{x,t}(\hat{\boldsymbol{\lambda}}^t))\boldsymbol{H}_{\pi(u)}\right)\end{aligned}$$

and use them to find the covariance matrices

$$\boldsymbol{S}_{\pi(k),t}=\boldsymbol{B}_k^{-\frac{1}{2}}\boldsymbol{F}_k\boldsymbol{G}_k^{\dagger}\boldsymbol{A}_k^{\frac{1}{2}}\boldsymbol{Q}_{\pi(k)}(P_{x,t}(\hat{\boldsymbol{\lambda}}^t))\boldsymbol{A}_k^{\frac{1}{2}}\boldsymbol{G}_k\boldsymbol{F}_k^{\dagger}\boldsymbol{B}_k^{-\frac{1}{2}}$$

where $\boldsymbol{F}_k$ and $\boldsymbol{G}_k$ could be obtained via singular value decomposition of the effective channel $\boldsymbol{H}_{\pi(k)}$ given by $\boldsymbol{B}_k^{-\frac{1}{2}}\boldsymbol{H}^{\dagger}_{\pi(k)}\boldsymbol{A}_k^{-\frac{1}{2}}=\boldsymbol{F}_k\boldsymbol{Z}\boldsymbol{G}_k^{\dagger}$ with a square and diagonal matrix $\boldsymbol{Z}$ [12]. Perform MIMO broadcast with covariance matrix $\boldsymbol{S}_{k,t}$ per user k.

Lagrange multiplier updates. With $P_t(\hat{\boldsymbol{\lambda}}^t)$, $P_{x,t}(\hat{\boldsymbol{\lambda}}^t)$ available, update Lagrange multipliers $\hat{\boldsymbol{\lambda}}^{t+1}$ via (14).

3.4. Performance Guarantees

Next, we will rigorously establish that the proposed algorithm asymptotically yields a feasible and optimal solution of (4) under proper initialization. To this end, we first assert the asymptotic optimality of the proposed SGOC algorithm in the following sense.[1]

Lemma 2 *If $\{\mathcal{H}_t,E_t,\alpha_t,\beta_t\}$ are i.i.d. over slots, then the time-averaging throughput under the proposed SGOC algorithm satisfies*

$$R^*\geq\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}[R_t(P_{x,t}(\hat{\boldsymbol{\lambda}}^t))]\geq R^*-\mu M$$

where $M:=\frac{1}{2}[(\max\{P_b^{\max},-P_b^{\min}\})^2+(G^{\max})^2+(\max\{\alpha^{\max}(P_g^{\max}+P_b^{\max}),\beta^{\max}(E^{\max}-P_b^{\min})\})^2]$.

[1]The proofs of all lemmas and the theorem are omitted due to limited space, but can be found in the journal version of this paper [15].

Lemma 2 asserts that the proposed SGOC algorithm converges to a region with optimality gap smaller than μM, which vanishes as the stepsize $\mu \to 0$. However, since the proposed algorithm is based on a solver for the relaxed (9), it is not guaranteed that the resultant dynamic control policy is a feasible one also for (7). In the sequel, we will establish that the SGOC in fact can yield a feasible policy for (7), when it is properly initialized.

Let $R'_t(P_{x,t})$ denote the left (or right) derivative of $R_t(P_{x,t})$. Define $R'(0) := \max\{R'_t(0), \forall t\}$, and assume $R'(0) < \infty$. Using the compact notation $\delta_{\lambda_1} := \max\{0, \alpha^{\max}(P_g^{\max} + P_b^{\max}) - G^{\max}\}$, we have established the following.

Lemma 3 *If the stepsize satisfies $\mu \geq \underline{\mu}$, where*

$$\underline{\mu} := \frac{\alpha^{\max} R'(0)}{\beta^{\min}(C^{\max} - C^{\min} + P_b^{\min} - P_b^{\max} - \delta_{\lambda_1})} \tag{15}$$

the SGOC iterates satisfy $\hat{\lambda}_2^t \in [-\alpha^{\max}(\frac{R'(0)}{\beta^{\min}} + \mu\delta_{\lambda_1}) + \mu P_b^{\min}, \mu C^{\max} - \mu C^{\min} - \alpha^{\max}(\frac{R'(0)}{\beta^{\min}} + \mu\delta_{\lambda_1}) + \mu P_b^{\min}]$.

Note that $\hat{\lambda}_1^t$ and $\hat{\lambda}_2^t$ can be seen as two "virtual queues," and the evolution of $\hat{\lambda}_2^t$ in (14) in fact depends on the value of $\hat{\lambda}_1^t$; in other words, the two "virtual queues" are coupled. This coupling of "virtual queues" complicates the analysis, and it is clearly different from [8,9], where the "virtual queues" evolve independently. Yet, by exploiting the revealed characteristics of our SGOC policy, we can first upper and lower bound $\hat{\lambda}_1^t$. Then capitalizing on the specific coupling of the two "queues," we further derive the stepsize lower bound $\underline{\mu}$ to ensure the bounds in Lemma 3 for $\hat{\lambda}_2^t$.

We consider now the mapping between the real and virtual energy queues: $C_t = \frac{\hat{\lambda}_2^t}{\mu} + \frac{\alpha^{\max} R'(0)}{\mu\beta^{\min}} + \alpha^{\max}\delta_{\lambda_1} + C^{\min} - P_b^{\min}$. It can be readily inferred from Lemma 3 that $C^{\min} \leq C_t \leq C^{\max}$ holds, $\forall t$; hence, (4f) is always satisfied under the SGOC. Based on Lemmas 2 and 3, we arrive at the main result.

Theorem 1 *If we initialize with $\hat{\lambda}_2^0 = \mu C_0 - \mu C^{\min} + \mu P_b^{\min} - \alpha^{\max}(\frac{R'(0)}{\beta^{\min}} + \mu\delta_{\lambda_1})$, and select $\mu \geq \underline{\mu}$, then the proposed SGOC yields a feasible dynamic control for* (7)*, which is asymptotically optimal in the sense* $\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}[R_t(P_{x,t}(\hat{\boldsymbol{\lambda}}^t))] \geq R^* - \mu M$.

Clearly, the minimum optimality gap between the SGOC and the offline scheduling is given by $\underline{\mu} M$. The asymptotically optimal solution can be attained if we have very small power purchase prices α_t, or, very large battery capacities $C^{\max}$, so that $\underline{\mu} \to 0$. This makes sense intuitively because when the BS battery has a large capacity, the upper bound in (4f) is loose. In this case, with proper initialization, the SGOC using any μ will be feasible for (7), or, (4).

4. NUMERICAL RESULTS

The considered MIMO downlink has a BS with $N_t = 2$ antennas, communicating to $K = 10$ mobile users equipped with $N_r = 2$ antennas each. The system bandwidth is 1 MHz, and each entry of $\boldsymbol{H}_{k,t}$ is a zero-mean complex-Gaussian random variable with unit variance. The maximum/minimum SoCs are set to $C^{\max} = 50$, $C^{\min} = 0$ kWh, while the (dis-)charging rates are set to $P_b^{\min} = -5$ and $P_b^{\max} = 5$ kWh/slot. The energy purchase price α_t is uniformly distributed over [0.1, 1], while the selling price is set as $\beta_t = r\alpha_t$ with $r = 0.9$. The stepsize is chosen as $\mu = \underline{\mu}$. Two baseline schemes are introduced in this setup, where ALG 1 is a "greedy" scheme that maximizes the instantaneous throughput in (7) per slot

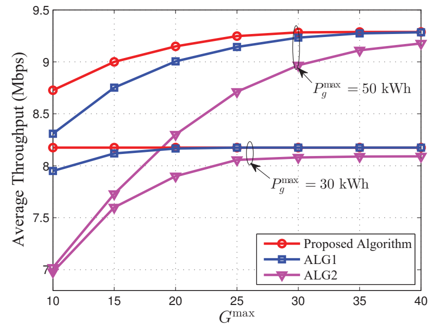

Fig. 1. Average throughput versus $G^{\max}$.

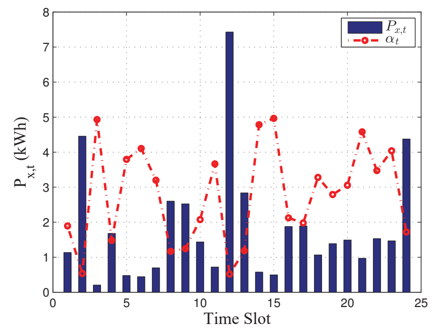

Fig. 2. SGOC schedule of $P_{x,t}$ ($G^{\max} = 15$ and $P_g^{\max} = 10$ kWh).

without leveraging the battery. ALG 2 is similar to the proposed one in the sense that it uses the stochastic dual subgradient to iteratively approximate the primal solution; yet, neither renewable energy nor battery is taken into account.

The average throughputs of the SGOC and ALGs 1-2 are compared with respect to the growth of $G^{\max}$ in Fig. 1. Clearly, the throughputs of all three algorithms improve as $G^{\max}$ or $P_g^{\max}$ increases since larger energy cost or looser maximum energy consumption limit will allow more energy purchases from the smart grid and larger energy consumption, leading to the increase of average throughputs. In both cases, the proposed algorithm performs better than ALGs 1-2. Specifically, when $G^{\max} = 10$ and $P_g^{\max} = 50$ kWh, the proposed scheme has 5.0% and 24.3% gains in average throughput over ALGs 1 and 2, respectively. Intuitively speaking, this is because the proposed algorithm intelligently leverages the renewable energy and energy storage device to hedge against future losses, which cannot be fully exploited by ALGs 1-2.

Fig. 2 depicts the power schedule $P_{x,t}$ of the proposed SGOC over time, and the fluctuation of energy purchase prices α_t. It can be clearly observed that the power consumption highly depends on the instantaneous price α_t. In particular, the proposed scheme tends to consume more power when α_t is lower (e.g., $t = 2, 12, 24$), and tends to consume less power when α_t is higher (e.g., $t = 3, 14, 15$).

5. CONCLUSIONS

Real-time resource allocation was developed for smart-grid powered MIMO downlinks. Relying on the stochastic subgradient method, an novel online algorithm was proposed to obtain a feasible and asymptotically optimal solution without knowing the distribution of the underlying stochastic processes. Our generalized performance analysis framework has fairly broad applicability for online control of wireless networks with coupled "real" or "virtual" queues.

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