ENERGY DETECTION IN ISI CHANNELS USING LARGE-SCALE RECEIVER ARRAYS

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ABSTRACT

We investigate the performance of an energy detection (ED) system with a large number of receiver antennas in multi-path propagation channels. Contrast to existing works focusing on inter-symbol-interference (ISI) free scenarios, in multi-path environments with resolvable path components, ISI jeopardizes the performance of the ED receiver. Asymptotically, when the number of receiver antennas is sufficiently large, we show that a zero-forcing equalizer, requiring only the energy of channel taps to compute the filter coefficients, can be employed to effectively remove the ISI. This significantly reduces the burden on acquiring channel state information (CSI), since only the energy of each channel taps is demanded instead of complete CSI. As a result of asymptotic properties, to decode symbols at the output of the equalizer, decision threshold values take rather simple forms, reducing the computational complexity. Monte Carlo simulation results show that compared with using instantaneous channel energies for channel equalization, employing average channel energy leads to resembling symbol error rate, but demanding a lower filter coefficients updating frequency.

Index Terms— Energy Detection, ISI channel, Massive MIMO, mm-wave Signal Processing.

1. INTRODUCTION

Employing a large number of antennas at base-stations or user terminals is seen as a promising solution to increase system throughput and energy efficiency [1, 2, 3]. To deliver these merits, accurate channel state information (CSI) is required to facilitate coherent processing at the receiver. With a reliable CSI, simple linear processing can be employed to maximize the system throughput. However, acquiring CSI can be challenging due to channel aging or pilot contamination [4].

For systems with a large number of antennas, energy detection (ED) based receiver is proposed to alleviate the channel estimation burden [5, 6, 7]. It offers a sub-optimal, but low complexity and power efficient solution compared to coherent detection which requires precise CSI [8]. When the number of receiver antennas is sufficiently large, symbol detection can be performed without the knowledge of instantaneous CSI. In fact, ED may even operate without explicit knowledge of the channel statistics as signal energy is collected over an excessive number of receive antennas providing a sample-mean based estimate of the channel energy. Meanwhile, due to noise hardening, additive noise contribution asymptotically approaches to a deterministic term. This increases the reach extension since high signal-to-noise-ratio (SNR) requirement may be circumvented, which is a strong limitation for ED systems with limited number of receiver antennas [9, 10, 11, 12, 13, 14, 8]. Given that the deterministic noise energy may be reliably estimated in a training phase or jointly estimated in the data transmission phase, the noise energy accumulation issue becomes not critical.

For ED systems with a large number of antennas, existing works focus on ISI-free scenarios [5, 15, 7, 16]. In [5, 15, 7], the authors demonstrate that non-coherent detection leads to promising symbol error rate (SER) performance without requiring instantaneous CSI, but the average channel energy. In addition, sub-optimal signal constellation solutions are proposed in [5, 15]. The obtained solutions are asymptotically optimal with an increasing number of antennas and constellation size. Since the constellation design depends on the selected channel distributions, it may be very sensitive to channel model uncertainties [15]. Different from the above-mentioned works, in [16], information-theoretic bounds are derived based on Gaussian approximations of the probability density function of channel energy and observation signal etc. The proposed bounds are shown to be tight at both low and high SNR regimes.

In this work, we investigate the performance of an ED receiver with a large number of antennas under multi-path propagation scenarios. In principle, inter-symbol-interference (ISI) caused by resolvable multi-path components can be alleviated by employing orthogonal frequency-division multiplexing (OFDM) techniques. However, high peak-to-average-ratio impairs the efficiency of the power amplifiers and adding cyclic-prefix decreases the spectral efficiency [3]. We exploit the asymptotic properties brought by a large number of receiver array and propose to use simple equalization methods to combat ISI. Asymptotically, when the number of antennas is sufficiently large, we show that a zero-forcing equalizer, requiring the energy of channel taps instead of all the channel coefficients, can be employed to remove ISI. This method significantly reduces the requirements on CSI, since

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only the energy of channel taps (robust statistics) is required. For millimeter-wave (mmWave) signal processing, it may become an attractive solution since the channel appears sparse. In addition, owning to the asymptotic property, we show that the threshold values for decoding the output of the equalizer take rather simple forms, reducing the computational complexity of the ED receiver. Monte Carlo simulations report that simple equalization techniques lead to promising SER performance and the SER can be significantly reduced as the number of receiver antenna increases.

2. SYSTEM AND SIGNAL MODEL

We consider a system consisting one transmit antenna and a large number of receiver antennas. Contrast to the ISI free cases investigated in [5, 6, 7], we assume that the system bandwidth is sufficiently large such that it is sensible to model the propagation channel as a finite impulse response (FIR) filter with L taps [17]. We assume a block fading channel with coherence time $T_c = N_c T_s$, where T_s denotes the symbol time and N_c is a positive integer. For an arbitrary coherence block, the received signal at time j is then formulated as

$$\mathbf{y}(j) = \sum_{l=0}^{L-1} \mathbf{h}_l x(j-l) + \mathbf{n}(j), \tag{1}$$

where M (no. of receiver antenna) channel coefficients corresponding to the lth path is grouped in $\mathbf{h}_l = [h_{1,l}, \ldots, h_{M,l}]^T$ and $\mathbf{n}(j)$ is a circular complex Gaussian noise vector with component variance σ_n^2. The non-negative transmit symbol $x(j)$ is selected from the constellation set $\{\sqrt{\epsilon_0}, \ldots, \sqrt{\epsilon_{p-1}}\}$, where $\sqrt{\epsilon_p} = p\sqrt{\epsilon}$ with ϵ is a normalization constant. The adoption of a non-negative modulation is due to that signal energy is collected at the output of an ED.

We assume that the channel coefficients are zero mean and

$$E[h_{i,l} h^*_{i',l'}] = \begin{cases} \sigma^2_{h,l}(i), & i = i' \text{ and } l = l' \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where $\sigma^2_{h,l}(i)$ denotes the mean power of the lth path component at antenna element i. Accordingly, the total received power of the lth path at all the received antennas is $\sigma^2_{h,l} = E[\mathbf{h}_l^H \mathbf{h}_l] = \sum_{i=1}^{M} \sigma^2_{h,l}(i)$, where the superscript H denotes the Hermitian operator.

The received signal from each of the M antennas is filtered, squared, and integrated, referred to as an energy collection process, leading to the output of the ED at time j reads [7]

$$z(j) = \frac{||\mathbf{y}(j)||_2^2}{M}, \tag{3}$$

where $||\cdot||_2$ is the L_2 norm. Based on the observation $z(j)$, the task then is to decode the transmit symbol. To proceed, we first analyze the asymptotic behavior of (3) before proposing a finite length zero forcing equalizer to eliminating the ISI caused by the multi-path channel.

3. ENERGY DETECTION IN ISI CHANNELS USING A LARGE NUMBER OF ANTENNAS

In each coherent interval, we rewrite the output of the ED as

$$\begin{aligned} z(j) = {} & \frac{1}{M}||\mathbf{h}_0||_2^2 |x(j)|^2 + \underbrace{\frac{1}{M}\sum_{l=1}^{L-1} ||\mathbf{h}_l||_2^2 |x(j-l)|^2}_{\mathrm{ISI}_1} + \\ & \underbrace{\frac{1}{M}\sum_{l \neq l'} x^*(j-l)x(j-l')\mathbf{h}_l^H \mathbf{h}_{l'}}_{\mathrm{ISI}_2} + \\ & \underbrace{\frac{2}{M}\Re\left(\sum_{l=0}^{L-1} \mathbf{h}_l^H \mathbf{n} x(j-l)\right)}_{\mathrm{ISI}_3} + \frac{1}{M}\mathbf{n}^H(j)\mathbf{n}(j). \end{aligned} \tag{4}$$

Clearly, ISI appears, which jeopardizes the performance of the ED. Therefore, to decode the transmit symbols, the threshold values computed in [7] under an ISI free assumption may not be valid. To remove ISI, guard interval is commonly employed. As the available system bandwidth increases, e.g. mmWave systems, the duration of the transmit pulse can be much shorter compared to the maximum excess delay. Thus, a long guard interval is needed, which compromises the spectral efficiency and data rate. In this work, we exploit the asymptotically behavior of large antenna systems to combat ISI.

For systems with a large number of receiver antennas, invoking asymptotic properties, we obtain 1) channel vectors are asymptotically orthogonal:

$$\frac{1}{M}\mathbf{h}_l^H \mathbf{h}_{l'} \xrightarrow{M \to +\infty} \sigma^2_{h,l}\delta_{l,l'},$$

where $\delta_{l,l'}$ denotes the Dirac delta. Thus, asymptotically, the "ISI_2" term in (4) can be effectively suppressed. 2) Similarly, the noise contribution $\frac{\mathbf{n}^H(j)\mathbf{n}(j)}{M} \xrightarrow{M \to +\infty} \sigma_n^2$ approaches to a deterministic term, which is referred to as noise hardening. 3) As a result of law of large numbers and the channel and noise vectors are independent, the term "ISI_3" approaches to zero. Consequently, we obtain

$$z(j) \xrightarrow{M \to +\infty} \sum_{l=0}^{L-1} \sigma^2_{h,l} |x(j-l)|^2 + \sigma_n^2. \tag{5}$$

Therefore, benefiting from employing a large number of receiver antennas, the symbol detection issue asymptotically converges to a standard equalization problem in an ISI channel. Instead of demanding all the coefficients $\mathbf{h}_l$, for $l = 0, \ldots, L-1$, the required average energy of the channel taps are robust statistics since averaging is performed across all the antennas. Meanwhile, due to noise hardening, the noise contribution can be removed if it is estimated in a training phase or jointly processed with data detection. Thus, the high SNR requirement is not critical.

4. ISI CHANNEL EQUALIZATION: ZERO-FORCING EQUALIZER

The simple expression in (5) motivates us to apply standard equalization techniques. We propose to use a finite length zero-forcing equalizer to combat ISI using the statistics of channel energy. In principle, zero-forcing equalizers require an infinite length filter to "flatten" the frequency selectivity spectrum of propagation channels, which is theoretically sound but practically difficult to implement. Thus, we consider a practical zero-forcing equalizer with finite length $K > L$. For the addressed problem, we define the output of a linear equalizer as

$$\beta(j) = \sum_{k=0}^{K-1} w_k z(j-k) = \mathbf{w}^T \mathbf{z}(j), \tag{6}$$

where $\mathbf{w} = [w_0, \ldots, w_{K-1}]^T$ is the to-be-computed equalizer coefficients and $\mathbf{z}(j) = [z(j), \ldots, z(j-K+1)]^T$. The larger K is, the better equalization performance it may achieve at the cost of requiring a longer memory and vice versa.

To compute the equalizer coefficients $\mathbf{w}$, we write

$$\mathbf{z}(j) = \mathbf{G}\mathbf{s}(j) + \boldsymbol{\xi}, \tag{7}$$

where $\mathbf{s}(j) = [|x(j)|^2, |x(j-1)|^2, \ldots, |x(j-(K+L-2))|^2]^T$. The characteristics of the noise vector $\boldsymbol{\xi} = [\xi(j), \xi(j-1), \ldots, \xi(j-K+1)]^T$ affect the selection of decision regions for decoding, which will be discussed in Section 4.1. The toeplitz matrix $\mathbf{G}$ with dimension $K \times (K+L-1)$, containing the energy of channel taps, reads

$$\mathbf{G} = \begin{pmatrix} \sigma_{h,0}^2 & \sigma_{h,1}^2 & \cdots & \sigma_{h,L-1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{h,0}^2 & \sigma_{h,1}^2 & \cdots & \sigma_{h,L-1}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \sigma_{h,0}^2 & \sigma_{h,1}^2 & \cdots & \sigma_{h,L-1}^2 \end{pmatrix} \tag{8}$$

The matrix $\mathbf{G}$ depends only on the average energy of the channel taps. In fact, if instantaneous channel energy $\frac{\mathbf{h}_l^H \mathbf{h}_l}{M}$, for $l = 0, \ldots, L-1$, is known, it can be used to substitute the corresponding term $\sigma_{h,l}^2$ in $\mathbf{G}$, which may lead to better equalization performance. But they need to be updated every coherence time. When $M \to +\infty$, due to the channel hardening effect, the knowledge on the average channel energy may be sufficient to equalize the ISI channel. These are more robust statistics which can be updated in several coherence time. In any case, L asymptotically static channel energy terms are required for combating ISI in the non-coherent receiver instead of demanding LM channel coefficients for coherent detection, which potentially significantly simplifies the channel estimation task.

Inserting (7) into (6), we obtain

$$\beta(j) = \mathbf{w}^T \mathbf{z}(j) = \mathbf{w}^T \mathbf{G}\mathbf{s}(j) + \mathbf{w}^T \boldsymbol{\xi}. \tag{9}$$

To effectively remove ISI, $\mathbf{w}$ needs to be designed such that $\mathbf{w}^T \mathbf{G} = \mathbf{e}_d$, where $\mathbf{e}_d$ is an all zero vector with the dth entry being unity. Since $\mathbf{G}$ is generally not square, the filter coefficients $\mathbf{w}$ is obtained by selecting the dth row of the pseudo inverse of $\mathbf{G}$, denoted as $\mathbf{G}^\dagger = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T$:

$$\mathbf{w} = \mathbf{e}_d \mathbf{G}^\dagger. \tag{10}$$

Generally speaking, the matrix $\mathbf{G}^\dagger \mathbf{G} \neq \mathbf{I}$ is not an identity matrix. Its diagonal entries are close-to unity while small values appearing on the off-diagonal entries. Therefore, it is important to select d properly. For the problem at hand, we define

$$\mathbf{g}_d = \mathbf{e}_d \mathbf{G}^\dagger \mathbf{G} = [g_1[d], \ldots, g_{K+L-2}[d]]. \tag{11}$$

Then, d is obtained by solving the following optimization problem:

$$d_{opt} = \arg\min_d \frac{|g_d[d]|^2}{\sum_{k=0, k\neq d}^{K+L-2} |g_d[k]|^2}. \tag{12}$$

This approach provides the highest signal to interference level since it takes both the main diagonal and off-diagonal of $\mathbf{G}^\dagger\mathbf{G}$ into account.

Assuming that the ISI is perfectly removed, we obtain the signal model after equalization:

$$\beta(j) = s(j-d) + \mathbf{w}^T \boldsymbol{\xi}, \tag{13}$$

where $s(j-d)$ is the dth entry of $\mathbf{s}(j)$ and a delay d is introduced due to that the dth row of $\mathbf{G}^\dagger$ is selected. We observe that the noise contribution is enhanced by a factor $||\mathbf{w}||_2^2$. We remark that the ISI cannot be fully removed since a finite length filter and asymptotic properties are employed. However, when the filter length K is long enough and M is sufficiently large, the residue interferences are negligible so that (13) can be well justified.

4.1. Decoding Using Asymptotic Properties

Based on (13), $s(j-d)$ can be decoded employing maximum likelihood or other decision rules exploiting the statistics of $\boldsymbol{\xi}$. In the asymptotic case, as $M \to +\infty$, (5) is well justified. The noise contribution $\boldsymbol{\xi}$ in (7) is then simply given by

$$\boldsymbol{\xi} = \sigma_n^2 \mathbf{1}, \tag{14}$$

where $\mathbf{1}$ is an all ones vector. Thus, (13) converges to a deterministic term, which depends only on the filter coefficients

Table 1. Simulation Settings

Modulation order: $P = 4$, SNR = $\frac{E(x^2)}{\sigma_n}$, $N_c = 1000$
No. of channel taps: $L = 4$, No. of filter taps: $K = 7$
$\sigma^2_{h,0} = -0.8\,\mathrm{dB}$, $\sigma^2_{h,1} = -8.6\,\mathrm{dB}$
$\sigma^2_{h,2} = -16\,\mathrm{dB}$, $\sigma^2_{h,3} = -24\,\mathrm{dB}$

and noise variance. Given these two quantities, the decision threshold for deciding $s(j-d) = \epsilon_p$ or ϵ_{p+1} reads

$$\Delta_p = (\epsilon_p + \epsilon_{p+1})/2 + \sigma_n^2 \mathbf{w}^T \mathbf{1}. \tag{15}$$

Thus, computing the threshold values is straightforward, reducing the computational complexity at the receiver.

5. NUMERICAL PERFORMANCE EVALUATION

For an uncoded PAM modulation, we show the performance of the proposed zero-forcing equalizer in a multi-path channel. We compare the zero-forcing equalizer performance in terms of SER using the average and instantaneous energy of each channel tap. Employing instantaneous energy offers a performance benchmark. In ED systems with a large number of receivers, however, the average energy of each channel tap may be sufficient. We conduct Monte Carlo simulations to show the performance gaps between these two approaches. We denote "Inst." and "Avg." as the results obtained from using instantaneous and average channel energy, respectively. We report the simulation settings in Table 1. An exponential decay delay power spectrum is adopted. We employ channel parameters according to the settings in [17].

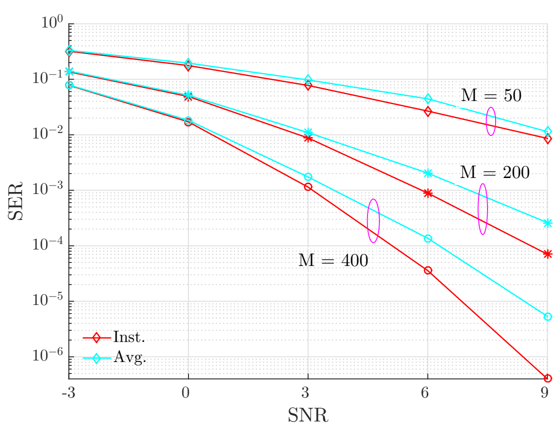

Fig. 1. SER versus SNR for different no. of receiver antennas.

Fig. 1 reports the simulated SER of the proposed equalizer versus SNR for different number of receiver antennas. We observe that using average channel energy achieves promising performance compared to employing instantaneous channel energy. Equipping a large number of antenna in the system significantly reduces the SER and increase the robustness of the system to noise: for example, to achieve SER $= 10^{-3}$, when $M = 200$, 6 dB SNR is required. If M is doubled to 400, we achieve a 3 dB SNR gain to obtain the same SER.

In Fig. 2, we observe that as SNR increases, slight differences in SER performance can be observed. The reason is that the noise factor is weighted by the filter coefficients in (15). Compared to employing average channel energy, using instantaneous channel energy obtains more reliable filter coefficients, which results in slightly better SER performance.

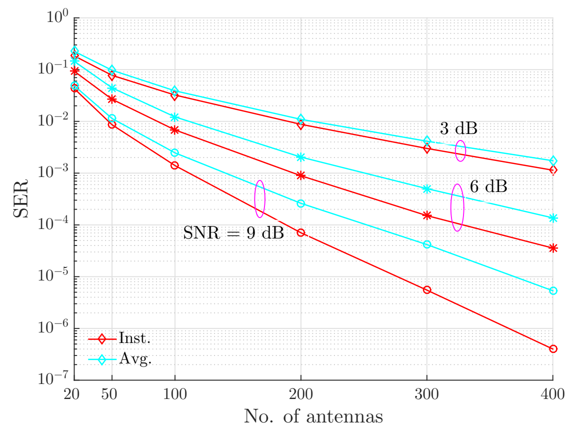

Fig. 2. SER versus no. of receiver antenna for different SNRs.

6. CONCLUSION AND OUTLOOK

In ED systems with a large number of receiver antennas, due to asymptotic properties, the ISI introduced by multi-path propagation can be effectively removed by simple linear equalizers. The filter coefficients can be computed using average channel energies, which are more robust statistics compared to the instantaneous channel energies. Therefore, only L, the number of channel taps, quantities are required for channel equalization, which potentially reduces the effort on acquiring CSI. In addition, we obtain a simple threshold value computation rule, which eases the computation burden at the receiver.

The number of taps L is assumed to be known in this work, which needs to be estimated before performing channel equalization. In mmWave systems, the channel appears sparse, so L may be reliably estimated by using pencil beams. In addition, for systems with medium or small number of antennas, employing the statistics of interference and noise terms may improve the SER performance. But this improvement may be negligible in systems with a large number of receiver antennas.

7. REFERENCES

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