ROBUST RECEIVER DESIGN BASED ON FEC CODE DIVERSITY IN PILOT-CONTAMINATED MULTI-USER MASSIVE MIMO SYSTEMS

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ABSTRACT

This work investigates robust receiver design against pilot-contaminated channel estimation in large multi-antenna systems. Given bounded channel estimate errors, we tackle the multi-user detection problem by presenting a novel idea of exploiting the forward error correction (FEC) code diversity. Unlike the traditional approach based on worst-case or probabilistic channel error, we distribute different channel codes among pilot-interfering users. We then develop a quadratic-programming receiver to utilize the special FEC signature of target user through a set of linear code constraints. Numerical results demonstrate substantial performance gain over existing robust detectors.

***Index Terms*—** Robust receiver, multi-user detection, massive MIMO, pilot contamination, FEC code diversity.

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) ushers in a new period of great excitement for wireless MIMO communications. Asymptotic analysis from random matrix theory establishes that very large antenna array at base station (BS) can deliver potentially orders of magnitude improvement in spectral efficiency and capacity [1]. Additionally, small-scale fading effects are mitigated, while the number of users per cell are independent of cell size. It has been shown that the required energy transmission per bit vanishes as the number of antennas in a cell grows asymptotically large [1]. Moreover, massive MIMO presents the benefits of extensive use of inexpensive low-power components, reduced latency, simplified MAC layer, and robustness against jamming [2].

In spite of the many promising advantages of massive MIMO, such promise can possibly be limited by several physical factors. Among them, one prominent limiting factor is pilot contamination, which results from pilot reuse at different cells. More specifically, in the presence of pilot contamination, the channel estimate of target user is a linear combination of the channel responses of the users in different cells with the same pilot sequence [3]. In cellular uplink, the receiver performance typically depends on the quality of channel state information (CSI). Although there exist some works on pilot decontamination, certain stringent conditions are required [4] or costly mechanisms are needed [5]. Instead of focusing only on improving CSI estimation, we aim to develop robust receivers in a multi-user massive MIMO system against CSI mismatch caused by pilot contamination.

In the literature, minimum output energy (MOE) criterion has been well-established means for rejecting multiple access interference. The MOE criterion minimizes the total output energy while constraining the distortion in the recovered user source signal. This principle ensures that reduction in output energy is a consequence of the interference suppression. The MOE principle was first proposed in [6] as a blind approach for signal recovery and was later used in [7, 8] for CDMA interference rejection. In the face of CDMA code signature mismatch, the authors in [9] devised a robust blind multi-user detector based on second-order cone programming (SOCP). In recent years, MOE was revitalized by a shift to multi-user detection in multi-access MIMO systems. Adopting MOE under the assumption of ideal CSI, a linear minimum variance (MV) receiver for space-time coded signals was presented in [10]. Given imperfect CSI, a plurality of robust receivers were proposed, including diagonal loading minimum variance (DLMV) receiver [10], worst-case robust receiver [11] and chance-constrained SOCP receiver [12].

Although these robust receivers are able to perform reasonably well under mild channel estimate errors, they are not robust enough to ensure reliable data transmissions in the presence of pilot-contaminated CSI. Instead of building robust receiver through worst-case based or chance-constrained approach, we present a new receiver design with the integration of forward error correction (FEC) codes. Owing to their near-capacity performance, low-density parity check (LDPC) codes have been gaining popularity. In this work, in addition to utilizing their FEC capability [13, 14], we propose a novel idea by exploiting FEC code diversity as unique user signature to separate different pilot-interfering users during signal recovery. In particular, we reformulate the MV receiver [10] into an unconstrained quadratic program, while relaxing the strict anchor and interference-mitigation constraint. We then incorporate a set of LDPC code constraints [15] to strengthen the quadratic programming receiver. The proposed receiver is able to utilize the unique code signature of target user signal, thereby achieving much better robustness against channel estimation error compared with existing receivers.

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2. MULTI-USER MIMO MODEL

Consider the uplink transmissions in a multi-cell multi-user MIMO (MC-MU-MIMO) system. As illustrated in Fig. 1, the MC-MU-MIMO network consists of L cells, where the ℓ-th cell served K_ℓ single-antenna mobile users with one N_r-antenna BS. The channels from mobile users to BS's are assumed to be flat-fading. Thus, the channel matrix from all K_ℓ users in the ℓ-th cell to the i-th BS can be represented as

$$\tilde{\mathbf{H}}_{i,\ell} = \begin{bmatrix} \tilde{h}_{i,1,\ell,1} & \dots & \tilde{h}_{i,K_\ell,\ell,1} \\ \vdots & \ddots & \vdots \\ \tilde{h}_{i,1,\ell,N_r} & \dots & \tilde{h}_{i,K_\ell,\ell,N_r} \end{bmatrix}, \quad (1)$$

where $\tilde{h}_{i,k,\ell,n}$ denotes the channel coefficient from the k-th user in the ℓ-th cell to the n-th antenna of the i-th BS. Specifically, $\tilde{h}_{i,k,\ell,n} = \tilde{g}_{i,k,\ell,n}\sqrt{\tilde{d}_{i,k,\ell}}$ consists of complex small-scale fading coefficient $\tilde{g}_{i,k,\ell,n}$ and large-scale fading factor $\tilde{d}_{i,k,\ell}$. Note that large-scale fading factors $\tilde{d}_{i,k,\ell}$'s are the same across different antennas at a BS, but are user-dependent [1].

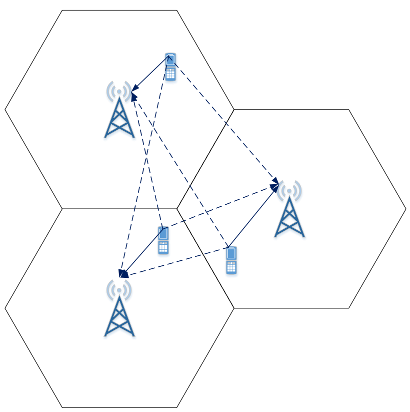

Fig. 1. Uplink transmission in an MC-MU-MIMO system.

Focusing on the signal reception at a specific BS, we drop the subscript i in Eq. (1). To further simplify notations, we ignore the cell identifiers in data transmissions. Instead, we simply let $N_u = \sum_{\ell=1}^{L} K_\ell$ be the total number of users from which this particular BS can receive signals. Therefore, the received signal vector $\tilde{\mathbf{y}} = [\tilde{y}_1 \dots \tilde{y}_{N_r}]^T$ at the BS can be expressed as

$$\tilde{\mathbf{y}} = \left[\tilde{\mathbf{H}}_1 \dots \tilde{\mathbf{H}}_L\right]\tilde{\mathbf{x}} + \tilde{\mathbf{n}} = \left[\tilde{\mathbf{h}}_1 \dots \tilde{\mathbf{h}}_{N_u}\right]\tilde{\mathbf{x}} + \tilde{\mathbf{n}}, \quad (2)$$

where $\tilde{\mathbf{x}} = [\tilde{x}_1 \dots \tilde{x}_{N_u}]^T$ is the signal vector from all users, and $\tilde{\mathbf{n}} = [\tilde{n}_1 \dots \tilde{n}_{N_r}]^T$ is a zero-mean noise vector with complex Gaussian distribution $\mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I})$. For the sake of convenience, we split the real and the imaginary parts of complex symbols as follows $\mathbf{y} = [\text{Re}\{\tilde{\mathbf{y}}\}^T \ \text{Im}\{\tilde{\mathbf{y}}\}^T]^T \in \mathcal{R}^{2N_r\times 1}$, $\mathbf{x} = [\mathbf{x}_1^T \dots \mathbf{x}_{N_u}^T]^T \in \mathcal{R}^{2N_u\times 1}$ where $\mathbf{x}_k = [\text{Re}\{\tilde{x}_k\} \ \text{Im}\{\tilde{x}_k\}]^T$, $\mathbf{n} = [\text{Re}\{\tilde{\mathbf{n}}\}^T \ \text{Im}\{\tilde{\mathbf{n}}\}^T]^T \in \mathcal{R}^{2N_r\times 1}$, and $\mathbf{H} = [\mathbf{H}_1 \dots \mathbf{H}_{N_u}] \in \mathcal{R}^{2N_r\times 2N_u}$, in which

$$\mathbf{H}_\ell = \begin{bmatrix} \text{Re}\{\tilde{\mathbf{h}}_\ell\} & -\text{Im}\{\tilde{\mathbf{h}}_\ell\} \\ \text{Im}\{\tilde{\mathbf{h}}_\ell\} & \text{Re}\{\tilde{\mathbf{h}}_\ell\} \end{bmatrix}.$$

Consequently, we transform Eq. (2) to a real-valued signal equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \quad (3)$$

3. MOE-BASED MULTI-USER DETECTION

Without loss of generality, we designate the first user as the user-of-interest. In this work, our goal is to design a linear receiver $\mathbf{W} = [\mathbf{w}_R \ \mathbf{w}_I]$ such that $\hat{\mathbf{x}}_1 = \mathbf{W}^T\mathbf{y}$. The receiver matrix $\mathbf{W}$ should be able to suppress multi-access interference and while preserving a distortionless response for the target user. As proposed in [10], the MOE-based minimum variance (MV) detector is the solution to the following optimization problem

$$\begin{aligned} \min_{\mathbf{W}}. \quad & \text{tr}\{\mathbf{W}^T\mathbf{R}\mathbf{W}\} \\ \text{s.t.} \quad & \mathbf{W}^T\mathbf{H}_1 = \mathbf{I}_2, \end{aligned} \quad (4)$$

where tr$\{\cdot\}$ denotes the trace of a matrix and $\mathbf{R}$ is the $2N_r \times 2N_r$ covariance matrix $\mathbf{R} = \mathbb{E}\{\mathbf{y}\mathbf{y}^T\}$. In practice, the true covariance matrix $\mathbf{R}$ is replaced by its estimate from Q snapshots of received signals $\hat{\mathbf{R}} = \frac{1}{Q}\sum_{q=1}^{Q}\mathbf{y}_q\mathbf{y}_q^T$.

We note that the MV detector assumes ideal CSI of the target user, i.e., $\mathbf{H}_1$ in this case. In practice, however, training-based channel estimates are typically corrupted by channel noise. When frequency reuse is aggressive among nearby cells, each accommodating a large number of mobile users, CSI estimation errors are farther exacerbated by pilot contamination, for example, in massive MIMO systems. Here, CSI estimation of the target user suffers from both interfering pilots and channel noise [1]. Specifically, when adjacent cells use the same set of (orthogonal) pilot sequences that are transmitted in synchronization, the resulting estimated channel matrix of target user is

$$\hat{\mathbf{H}}_1 = \mathbf{H}_1 + \sum_{\ell\in\mathcal{I}}\mathbf{H}_\ell + \hat{\mathbf{N}}, \quad (5)$$

where $\mathcal{I}$ is the set of interferers during the training period, and $\hat{\mathbf{N}}$ is the noise matrix associated with channel estimates. Confronted with such severe CSI mismatch, however, neither the worst-case based receiver nor the probability-constrained receiver can function robustly. In the rest of this work, we will develop a novel robust receiver by utilizing FEC code information.

4. QUADRATIC PROGRAMMING MULTI-USER DETECTION WITH CODE DIVERSITY

4.1. QP Reformulation of MV Receiver

Though diagonal loading (DL) can improve the receiver robustness to a certain extent [10], the formulation in Eq. (4) is not amendable to further integration of constraints. Moreover, the exact anchor and interference-mitigation constraint $\mathbf{W}^T\hat{\mathbf{H}}_1 = \mathbf{I}_2$ in Eq. (4) may degrade the receiver performance given imperfect CSI $\hat{\mathbf{H}}_1$. In the following, we reformulate the MV receiver through a quadratic programming (QP) approach to overcome these shortcomings.

First, rewrite the MV cost function in quadratic form

$$\mathrm{tr}\{\mathbf{W}^T\hat{\mathbf{R}}\mathbf{W}\} = \begin{bmatrix}\mathbf{w}_R^T & \mathbf{w}_I^T\end{bmatrix}\begin{bmatrix}\hat{\mathbf{R}} & \\ & \hat{\mathbf{R}}\end{bmatrix}\begin{bmatrix}\mathbf{w}_R \\ \mathbf{w}_I\end{bmatrix}. \quad (6)$$

Note that $\mathbf{w} \triangleq [\mathbf{w}_R^T\ \mathbf{w}_I^T]^T$ is the vectorized receiver parameter matrix $\mathbf{W}$. For the constraint, we follow the same vectorization strategy

$$\mathbf{W}^T\hat{\mathbf{H}}_1 = \mathbf{I}_2 \Rightarrow \begin{bmatrix}\hat{\mathbf{H}}_1^T & \\ & \hat{\mathbf{H}}_1^T\end{bmatrix}\begin{bmatrix}\mathbf{w}_R \\ \mathbf{w}_I\end{bmatrix} = \begin{bmatrix}\mathbf{e}_1 \\ \mathbf{e}_2\end{bmatrix}, \quad (7)$$

where the unit vectors $\mathbf{e}_1 = [1\ \ 0]^T$ and $\mathbf{e}_2 = [0\ \ 1]^T$. For convenience, we denote $\mathbf{e} \triangleq [\mathbf{e}_1^T\ \ \mathbf{e}_2^T]^T$.

Since CSI mismatch exists, we would like to enforce less stringent anchor and interference-mitigation constraint. Particularly, we lift the interference residual $\|(\mathbf{I}_2 \otimes \hat{\mathbf{H}}_1^T)\mathbf{w} - \mathbf{e}\|^2$ into the cost function, where $\otimes$ denotes Kronecker product, to arrive at an unconstrained quadratic programming problem

$$\min_{\mathbf{w}}.\ \mathbf{w}^T(\mathbf{I}_2 \otimes \hat{\mathbf{R}})\mathbf{w} + \|(\mathbf{I}_2 \otimes \hat{\mathbf{H}}_1^T)\mathbf{w} - \mathbf{e}\|^2. \quad (8)$$

4.2. Code Diversity Integration

To strengthen the receiver robustness given imperfect channel estimates, we advocate a unified receiver optimization by integration of a set of linear constraints that are generated from LDPC parity checks [15]. Specifically in pilot contamination scenario, we use different FEC codes for pilot interferers.

Consider an (N_c, K_c) LDPC code. Let $\mathcal{M}$ and $\mathcal{N}$ be the set of check nodes and variable nodes of the parity check matrix, respectively, i.e., $\mathcal{M} = \{1, \ldots, N_c - K_c\}$ and $\mathcal{N} = \{1, \ldots, N_c\}$. Denote the neighbor set of the m-th check node as $\mathcal{N}_m$ and let $\mathcal{S} \triangleq \{\mathcal{F}\,|\,\mathcal{F} \subseteq \mathcal{N}_m \text{ with } |\mathcal{F}| \text{ odd}\}$. Then one characterization of fundamental polytope [15] is captured by

$$\sum_{n\in\mathcal{F}} f_n - \sum_{n\in\mathcal{N}_m\setminus\mathcal{F}} f_n \le |\mathcal{F}| - 1,\ \forall m \in \mathcal{M}, \forall \mathcal{F} \in \mathcal{S} \quad (9)$$

plus the box constraints for each bit variable

$$0 \le f_n \le 1, \quad \forall n \in \mathcal{N}. \quad (10)$$

To integrate the constraints (9) and (10) into the quadratic program (8), we need to connect the detected symbols and the bit vector $\mathbf{f} \triangleq [f_1 \ldots f_{N_c}]^T$. For 4-QAM with Gray mapping, the relationship can be described by

$$\begin{aligned}\mathbf{w}_R^T\mathbf{y}_k &= (2f_{2k-1} - 1)/\sqrt{2}, \\ \mathbf{w}_I^T\mathbf{y}_k &= (1 - 2f_{2k})/\sqrt{2}.\end{aligned} \quad (11)$$

4.3. Joint QP Receiver

Before we finalize our joint QP receiver integrated with code constraints, we adopt diagonal loading (DL) by adding a weighted diagonal matrix to the sample covariance matrix, i.e., the new "covariance" matrix $\check{\mathbf{R}} = \hat{\mathbf{R}} + \gamma\mathbf{I}$, where γ is the DL factor. Moreover, we introduce a weighting factor α on the residual term in order to control tightness. In summary, we have the following joint QP formulation

$$\begin{aligned}\min_{\mathbf{w},\mathbf{f}}.\quad & \mathbf{w}^T(\mathbf{I}_2 \otimes \check{\mathbf{R}})\mathbf{w} + \alpha\|(\mathbf{I}_2 \otimes \hat{\mathbf{H}}_1^T)\mathbf{w} - \mathbf{e}\|^2 \\ \text{s.t.}\quad & \mathbf{w}_R^T\mathbf{y}_k = (2f_{2k-1} - 1)/\sqrt{2}, \\ & \mathbf{w}_I^T\mathbf{y}_k = (1 - 2f_{2k})/\sqrt{2}, \\ & \sum_{n\in\mathcal{F}} f_n - \sum_{n\in\mathcal{N}_m\setminus\mathcal{F}} f_n \le |\mathcal{F}| - 1, \\ & 0 \le f_n \le 1, \quad \forall n \in \mathcal{N},\end{aligned} \quad (12)$$

in which the optimization variables are $\mathbf{w} = [\mathbf{w}_R^T\ \mathbf{w}_I^T]^T$ and $\mathbf{f} = [f_1 \ldots f_{N_c}]^T$. Off-the-shelf interior-point method can be applied to solve this quadratic program.

5. NUMERICAL RESULTS

In this section, we provide a number of tests to demonstrate the performance of the proposed QP receiver in terms of bit error rate (BER) under 4-QAM modulation. In particular, we will compare the joint QP receiver with the DLMV receiver in [10] and the SOCP receiver in [12]. Throughout the simulation section, we apply the MOSEK solver [16] to solve QP and SOCP. Further, we assume Rayleigh fading channels. For test purpose, we consider a total of 4 users, whose channel gains are 1, 0.1, 0.9 and 0.3, respectively. The first user is our target user, while other users are interferers. We use tuple (c_1, c_2, c_3, c_4) to denote the code combination for the 4 users in terms of code length. The codes used are (256,192), (128,96) and (512,256). As noted in [10], block length Q can affect the sample covariance matrix $\hat{\mathbf{R}}$ and consequently the BER. Here, we arbitrarily set $Q = 512$. The outage probability for SOCP receiver is set to 0.95, as in [12]. Note that we can further apply soft decoding using the sum-product algorithm (SPA). In all figures, solid lines are generated from hard decision while dashed lines are based on soft decoding.

First, we show the effect of FEC code diversity. The number of receive antenna $N_r = 12$, the DL factor $\gamma = 80$ and the weighting factor $\alpha = 50$. The first user is our target user, while the second user contaminates the target user during CSI estimation, i.e., $\hat{\mathbf{H}}_1 = \mathbf{H}_1 + \mathbf{H}_2 + \hat{\mathbf{N}}$. Given the page constraint, we only show the results of joint QP receiver under various code combinations. DLMV and SOCP receivers are in fact insensitive to FEC code diversity in our tests. As we can see in Fig. 2, the BER performance degrades clearly in high SNR regime when pilot interferer uses the same FEC code as the target user, whereas the FEC codes of other interferers have only slight effect when compared with the most diverse code combination (256,128,512,512).

Next, we demonstrate the performance comparisons between DLMV, SOCP and QP receivers. Set $N_r = 18$, $\gamma = 125$ and $\alpha = 50$. For practicality, we allocate the same code for all users in one cell. In the following, we assume users 1 and 3 are in the same cell, while another cell contains users 2 and 4. We present the first example under noisy channel estimates (no pilot contamination), i.e., $\hat{\mathbf{H}}_1 = \mathbf{H}_1 + \hat{\mathbf{N}}$. Note that

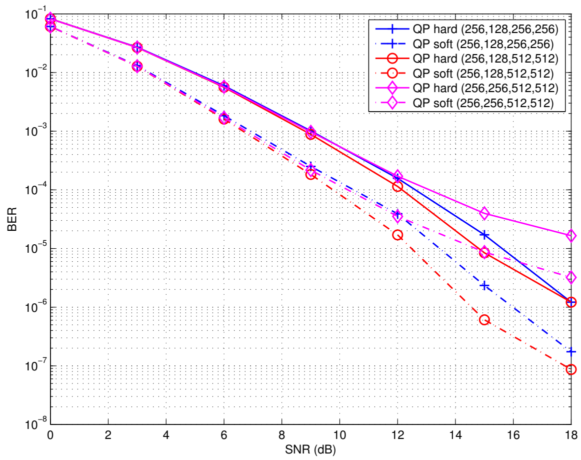

Fig. 2. BER curves showing FEC code diversity of QP receiver under various code combinations in pilot contamination scenario. $N_r = 12$, $\gamma = 80$ and $\alpha = 50$.

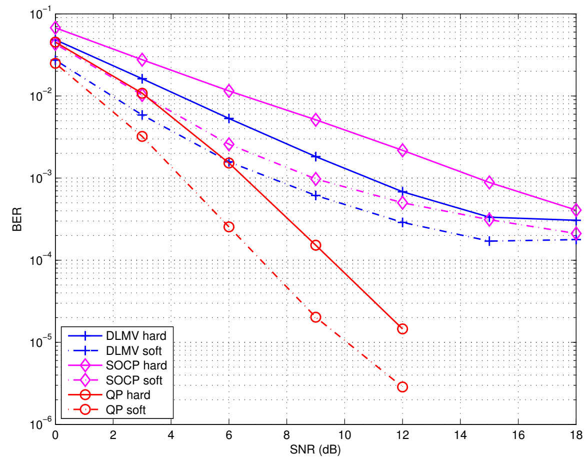

Fig. 4. BER comparisons of DLMV, SOCP and QP receivers using pilot-contaminated channel estimates under code combination (128,256,128,256). $N_r = 18$, $\gamma = 125$ and $\alpha = 50$.

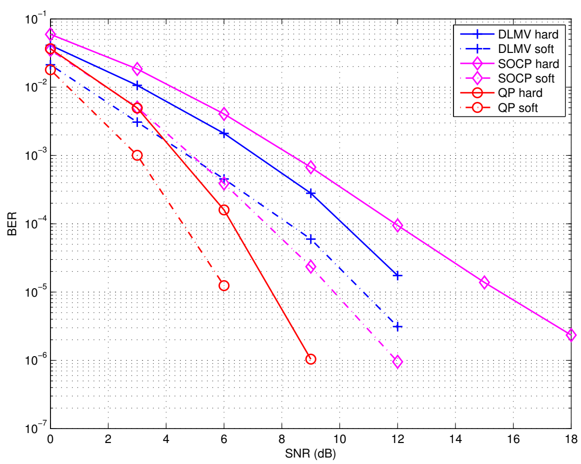

Fig. 3. BER comparisons of DLMV, SOCP and QP receivers using noisy channel estimates under code combination (256,128,256,128). $N_r = 18$, $\gamma = 125$ and $\alpha = 50$.

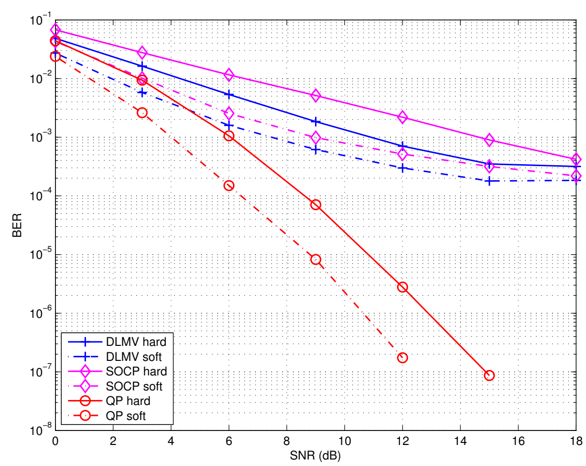

Fig. 5. BER comparisons of DLMV, SOCP and QP receivers using pilot-contaminated channel estimates under code combination (256,128,256,128). $N_r = 18$, $\gamma = 125$ and $\alpha = 50$.

the channel estimation error vanishes as the signal-to-noise ratio (SNR) grows. We illustrate the results in Fig. 3, where all receivers can reach quite low BERs without showing any error floors. Nonetheless, the proposed QP receiver outperforms DLMV and SOCP receivers by several dBs in terms of both hard-decision and soft-decision BER. The second example is shown under pilot-contaminated channel estimates $\hat{\mathbf{H}}_1 = \mathbf{H}_1 + \mathbf{H}_2 + \hat{\mathbf{N}}$. We demonstrate results under 2 code combinations (128,256,128,256) and (256,128,256,128), as shown in Fig. 4 and Fig. 5, respectively. Although the interfering channel gain is mild (the gain of $\mathbf{H}_2$ is merely 0.1), DLMV and SOCP receivers exhibit rather high error floors. In contrast, BER of the proposed QP receiver falls down sharply without apparent error floor.

6. CONCLUSION

We investigate robust receiver for signal recovery in massive MIMO multi-user systems under the constraints of pilot contaminated CSI estimates. Unlike existing optimization formulations, we proposed a joint QP receiver with the direct integration of LDPC code constraints as user signatures to distinguish the target user from other pilot-interfering users. The proposed QP receiver demonstrates improved robustness against pilot contamination. Nevertheless, we do not dismiss the optimization formulations that are based on statistical channel estimation errors. We may actually consider building our joint receiver upon the worst-case or probabilistic approach. Furthermore, future works may provide theoretical analysis of the proposed receiver as well as faster distributed implementations.

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