ARRAY THINNING FOR ANTENNA SELECTION IN MILLIMETER WAVE MIMO SYSTEMS

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ABSTRACT

This paper addresses the problem of designing thinned arrays with minimized side lobe levels for antenna selection in millimeter wave MIMO systems. We propose a new optimization solution based on compressed sensing techniques and convex optimization relaxation which we show to be a heuristic that solves the original binary optimization problem of side lobe level minimization. We compare the proposed method with other approaches from the literature like simulated annealing and genetic algorithms showing the superiority of the method in terms of performance, running time and ease of parameter tuning. The simulation results cover a wide range of dimensions and situations.

***Index Terms*—** array thinning, convex relaxation, millimeter wave MIMO

1. INTRODUCTION

Synthesis of sparse arrays deals with the design of non-uniform antenna arrays with a minimum number of elements whose radiation pattern complies with a given specification. Reducing the number of elements with respect to a uniform array brings several advantages, mainly reduction in power consumption, weight and cost. In the last decades several strategies have been proposed to synthesize sparse or thinned arrays. All of them are based on heuristic methods like genetic algorithms [1, 2], simulated annealing[6, 7, 8, 10], dynamic programming [3, 4], etc. Techniques based on convex optimization, and ℓ_1 norm minimization, have also been proposed in [11, 12, 13, 14].

At milimeter wave (mmWave) frequencies, large MIMO systems with steerable arrays have to be used to achieve the desired directivity pattern. Antenna selection techniques have been proposed at these frequencies as an alternative to phased arrays due to their lower complexity and lower power consumption [15]. Some antenna selection techniques have been designed to effectively estimate the MIMO channel using compressive measurements or to steer the beam in a given direction [15]. Another mmWave application making use of antenna selection is antenna secure modulation (ASM) [16]. It is a physical layer security technique that imposes artificial randomness in the constellations received in undesired directions. Randomness in the constellations is achieved by activating a random number of the available array antennas.

In these mmWave MIMO systems when making use of antenna selection [15, 16] a common problem appears: there are many subsets of antennas of a given size with a similar main lobe but with side lobes that can be quite different. An interesting question is how to choose the M active antennas out of the N antennas in the array such that the side lobe levels are minimized in some sense. The traditional algorithms for synthesis of sparse arrays [1, 2, 3, 4] however do not deal with this question, because their goal is minimizing the number of elements, considering some constraints which account for the the specifications of the desired beampattern. In antenna selection at mmWave frequencies the goal is not to minimize the number of antenna elements, since that is a fixed parameter in the MIMO system, but finding the subset with a minimum side lobe level between all the subsets sharing a similar main lobe. The idea is to create codebooks of antenna subsets with low side lobes, so that the active antenna patterns are selected from these dictionaries.

This paper describes an algorithm for the design of thinned arrays with low peak side lobe levels for antenna selection in mmWave MIMO systems. The proposed method is based on convex optimization techniques involving reweighted ℓ_1 norm regularization. We choose specific weights to deal better with the binary nature of the optimization problem, applied on the convex relaxation of the original discrete optimization problem. We formulate the main problem as an ℓ_∞ optimization problem. We show that the proposed solution solves a norm maximization optimization problem and performs better than simulated annealing or genetic algorithms.

2. ARRAY THINNING FOR ANTENNA SELECTION

We seek to solve the following discrete, non-convex, optimization problem over the "on-off" coefficients of each array element:

$$\text{minimize} \quad \max |\text{SLL}| \tag{1}$$

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where SLL stands for side lobe level, and max $|\text{SLL}| = \max_{\theta\in\Omega} |F(\theta)|$ where $F(\theta)$ is the array factor and Ω is the set of angles where we are interested in the minimization, outside the main lobe range. The results are shown in dB, $|\text{SLL}|^2_{\text{dB}} = 20\log_{10}|F(\theta)|$, but the problem equivalently uses $|F(\theta)|$.

The synthesis of thinned arrays is a combinatorial optimization problem (due to the binary nature – select or not a particular antenna in the array) and has received extensive attention in the past. The effects of array thinning are understood when considering the array gain and the main lobe width but little can be said analytically in general about the side lobe levels. Because exhaustive search is not realistic for large N and M, the search space is $\binom{N}{M} = \frac{N!}{M!(N-M)!}$, in the past solutions like genetic algorithms (GA), dynamic programming (DP), simulated annealing (SA) and convex optimization have been proposed.

The convex optimization approach in [11] also deals with a reweighted ℓ_1 optimization problem but in that case the beamforming vector is the working variable. In our case, the beamforming vector is fixed for example by the ASM technique in [16] and the only working variable is binary and has its support of fixed size, hence a binary optimization problem.

3. THE PROPOSED SOLUTION

In this paper we describe a method for array thinning based on compressed sensing ideas, namely the Iteratively Reweighed ℓ_1 (IRL1) [17] convex optimization formulation. First, let us define the working variables. We denote by $\boldsymbol{H}_\Omega$ the matrix of size $|\Omega| \times N$ that contains all the steering vectors corresponding to a given array geometry and to the directions for which SLL minimization takes place for angles $\theta_i \in \Omega$ on a discrete grid $\boldsymbol{H}_\Omega^H = \begin{bmatrix}\boldsymbol{h}(\theta_1) & \boldsymbol{h}(\theta_2) & \dots & \boldsymbol{h}(\theta_{|\Omega|})\end{bmatrix}$. Regardless of the antenna array structure the design constraints are contained in $\boldsymbol{H}_\Omega$. Thus, the discretized minimization of the SLL with antenna selection becomes:

$$\underset{\boldsymbol{b};\ \sum_{k=1}^N b_k = M,\ b_k\in\{0,1\},}{\text{minimize}} \quad \|\boldsymbol{H}_\Omega \boldsymbol{b}\|_\infty \tag{2}$$

where $\|\boldsymbol{y}\|_\infty = \max_i |y_i|$ is the ℓ_∞ norm and the binary variable $\boldsymbol{b}$ denotes the antenna selection pattern and is constrained to the selection of M out of the N antennas. This objective function deals directly with the radiation pattern of the array and is equivalent to the objective in (1). Although we would like to solve an $\mathcal{L}_\infty$ problem over all possible directions, we instead solve the tractable, finely discretized, ℓ_∞ problem without significant loss in accuracy. Due to the binary variables, this formulation is still non-convex. We relax the binary constraints in (2) to the convex problem [9]:

$$\underset{\boldsymbol{b};\ \sum_{k=1}^N b_k = M,\ b_k\in[0,1]}{\text{minimize}} \quad \|\boldsymbol{H}_\Omega \boldsymbol{b}\|_\infty + M^{-1}\boldsymbol{w}^T\boldsymbol{b} \tag{3}$$

Algorithm 1 – Array thinning by CR+LS.

Input: The design matrix $\boldsymbol{H}_\Omega \in \mathbb{C}^{|\Omega|\times N}$, the number of active antennas M, the number of iterations K and the length of the local search step ℓ.

Output: Selection pattern $\boldsymbol{b}$ such that the SLL is reduced.

Convex Relaxation (CR) step:

1. Set $\boldsymbol{w} = \boldsymbol{1}$.

2. For maximum K iterations (or until convergence of $\boldsymbol{w}$):

- Solve (3) with weights $\boldsymbol{w}$.
- Update $\boldsymbol{w}$: $w_k = 1 - b_k$ for $k = 1, \dots, N$.

3. Establish the support of $\boldsymbol{b}$:

$$\mathcal{K} = \{k \mid b_k > \epsilon \text{ with } k = 1, \dots, N\}.$$

4. If necessary, reduce the support size $|\mathcal{K}|$ to M:

for $|\mathcal{K}|$ down to M set $\mathcal{K} = \mathcal{K} \setminus \{k^*\}$ with

$$k^* = \underset{\mathcal{K}'=\mathcal{K}\setminus\{k\} \text{ for each } k\in\mathcal{K}}{\arg\min} \|\boldsymbol{H}_\Omega \mathbb{1}_{\mathcal{K}'}\|_\infty$$

Local Search (LS) step:

1. Start a local search close to the set $\mathcal{K}$:

$$\{\mathcal{A}, \mathcal{Z}\} = \underset{\mathcal{A}\subset\{\mathcal{K}^{(c)}\cup\mathcal{Z}\}, |\mathcal{A}|=\ell}{\arg\min} \|\boldsymbol{H}_\Omega \mathbb{1}_{\mathcal{K}\setminus\mathcal{Z}\cup\mathcal{A}}\|_\infty$$

for each set $\mathcal{Z} = \{z_1, \dots, z_\ell\} \subset \mathcal{K}, |\mathcal{Z}| = \ell$.

2. Set $\mathcal{K} = \mathcal{K} \setminus \mathcal{Z} \cup \mathcal{A}$ and return $\boldsymbol{b} = \mathbb{1}_\mathcal{K}$.

The regularization term, the second term in the objective, acts as a reweighted ℓ_1 penalty (just sum since $\boldsymbol{b}$ is non-negative) that is simplified due to the N positivity constraints, i.e., $\|\boldsymbol{W}\boldsymbol{b}\|_1 = \sum_{k=1}^N w_{kk}b_k = \boldsymbol{w}^T\boldsymbol{b}$ where $\boldsymbol{w} = \text{diag}(\boldsymbol{W})$ are the weights on the diagonal $\boldsymbol{W}$. Initially, the weight vector is $\boldsymbol{w} = \boldsymbol{1}$, the constant vector, leading to the classical ℓ_1 penalty regularization. Since we solve (3) in an iterative fashion, after the first step, the new weights are $\boldsymbol{w} = \boldsymbol{1} - \boldsymbol{b}$. These weights force entries of the solution $\boldsymbol{b}$ that are close to zero to be driven exactly to zero while the larger entries are allowed to reach magnitudes as high as possible (in this case up to one). We do expect this relaxation to lead naturally to the discrete structure of $\boldsymbol{b}$. The weights $\boldsymbol{w}$ originate from:

$$w_k = 1 - |b_k|\|\boldsymbol{b}\|_\infty^{-1}, \tag{4}$$

adapted to our particular case. If the solution converges after the K steps then notice that the penalty term is a stationary point of $\boldsymbol{1}^T\boldsymbol{b} - \|\boldsymbol{b}\|_2^2\|\boldsymbol{b}\|_\infty^{-1}$ which in our case reduces to $M - \|\boldsymbol{b}\|_2^2$ assuming that at least one entry in $\boldsymbol{b}$ is one. Thus the heuristic with the proposed weights is trying to solve a norm maximization problem for which the binary solution achieves its highest objective function value. To describe the algorithm it is convenient to introduce a set notation. We denote $\boldsymbol{b} = \mathbb{1}_\mathcal{K}$ a binary vector with ones in the positions indexed by the set

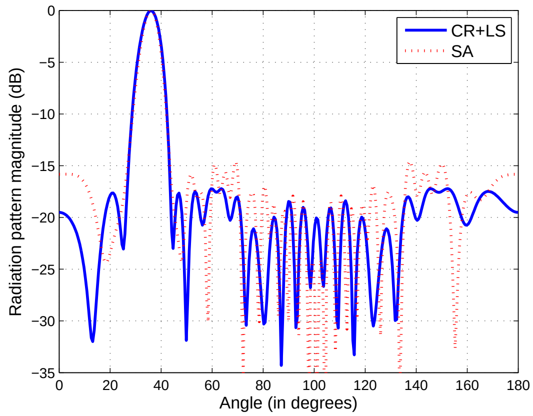

Fig. 1: Array thinning with $N = 35$, $M = 20$ and transmission angle $\theta_T = 36°$ in an ASM environment. SA achieves -14.5663 dB and the proposed CR+LS approach achieves -17.2095 dB maximum side lobe level. The global minimum, check via exhaustive search, is actually the level reached by CR+LS. Only CR reaches -16.9377.

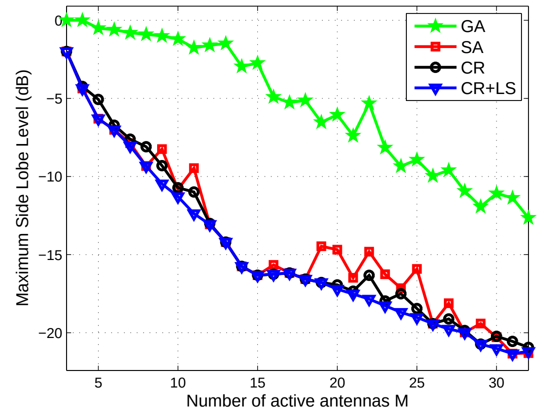

Fig. 2: Maximum SLL (dB) for $N = 35$ and various $M \in \{3, \dots, 32\}$. For these small dimensions the SA performs close to the proposed approach CR+LS and in some situations the local search (LS), with $\ell = 4$, is important to outperform or match SA in all cases.

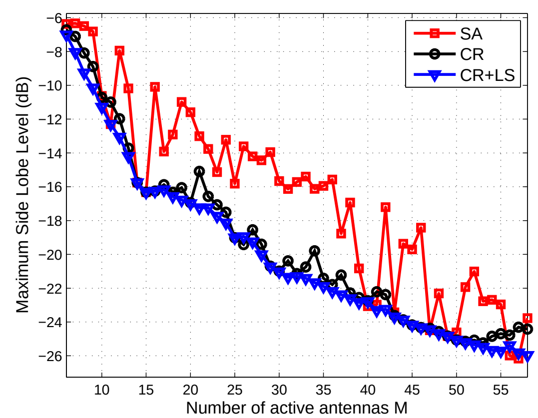

Fig. 3: Maximum SLL (dB) for $N = 64$ and various $M \in \{6, \dots, 58\}$. The local search parameter is $\ell = 3$.

$\mathcal{K}$, $|\mathcal{K}| = M$ denotes the size. The set $\mathcal{K}^{(c)}$ is the complement.

The proposed iterative procedure is presented in Algorithm 1. We solve problem (3) using updated weights $\boldsymbol{w}$ for a fixed number of K iterations. After the iterative process is over we compute the support of the solution $\boldsymbol{b}$ in the set $\mathcal{K}$. The first goal is to bring the size of the support down to M. Due to the relaxed constraints on the entries of $\boldsymbol{b}$ it is always true that the support of the solution is larger than or equal to M, but never smaller. After the size of the support is brought down to M a local search is started that finds the best replacement of size ℓ to the support such that the objective function is maximally minimized. This is indeed a combinatorial search so values of ℓ should be relative small. This extra search step is applied in order to make an additional reduction in the objective function by exploiting the fact that a good starting support (the one previous computed by CR) is available. In general we do not expect this step to produce a large improvement in the solution but as we will see there are situations in which this step helps lower the objective function. Of course, larger ℓ can only lead to better performance but the running time cost might be prohibitively large.

4. RESULTS

Figure 1 shows the radiation patterns of a thinned array designed via Simulated Annealing (SA) and one designed via the proposed CR+LS method. The result is for an ASM based system with $N = 35$ and $M = 20$ with a target angle at $\theta_T = 36°$ and the discretized grid has $|\Omega| = 271$ points. There is an approximate 3 dB performance gap that favors CR+LS. Actually the solution reached by CR+LS is the global minimizer in this case. The local search run with parameter $\ell = 3$ and does help decrease the SLL. The CR step always runs for $K = 7$ iterations.

We further analyze by simulations the behavior of the proposed algorithm as compared with the literature. In Figure 2 we show for fixed $N = 35$ and variable M the performance of the most popular algorithms in the literature: GA and SA. We keep the same experimental setup as for Figure 1 but the proposed CR and CR+LS methods can be easily extended to various other problems or antenna configurations (for example uniform and non-uniform linear or planar arrays). All depends on the design matrix $\boldsymbol{H}_\Omega$.

The GA approach performs the worst. Indeed in this case there are many parameters that can be tuned: the number of generations, the population, the selection mechanism, the cross-over method, the mutation procedure just to name a few. We followed the ideas from [1] to design the GA but performance is still quite poor when compared to the other approaches. The algorithm runs for 100 generations with a

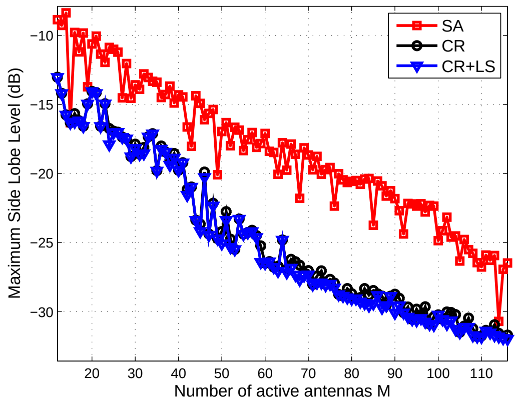

Fig. 4: Maximum SLL (dB) for $N = 128$ and various $M \in \{12, \ldots, 116\}$. The local search parameter is $\ell = 2$ but the LS step is not necessary in any case to outperform SA.

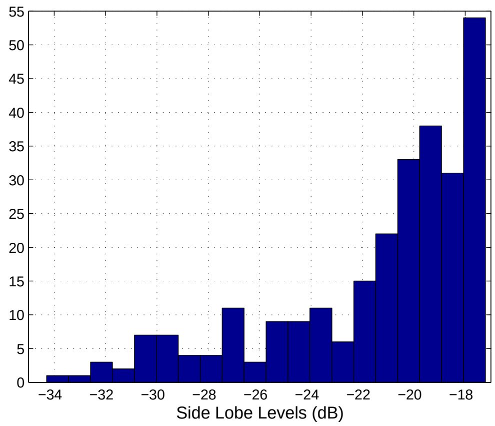

Fig. 5: Empirical side lobe level histogram for the antenna selection designed via CR+LS with $N = 35$ and $M = 20$.

population of size 10^5.

SA performs much better and actually is quite close to the performance of the proposed method. We follow mostly the approach in [10]. The perturbations to the solutions guarantee that the number of active antennas is always M throughout the optimization procedure while an exponential cooling scheduling seems to produce the best performance (this was check by extensive simulations). The parameters that were used are: the number of iterations is 5×10^5, the initial temperature is $T = 10^4$ and the cooling factor is $\beta = 0.98$. We keep the best solution that is reached.

In the case of $N = 35$ the performance gap between SA and CR or CR+LS is not very big. Figures 3 and 4 further provide experimental results for larger $N \in \{64, 128\}$. In these cases the performance gap between CR+LS and SA increases. Notice that SA performs quite well when the search space is relatively small, i.e., small N and/or M close to N. The largest performance gap seems to be in the case of $M \approx N/2$ when the combinatorial search space is the largest. Notice that there are several cases in which the SA actually produces a solution as good as the one provided by CR+LS. Actually, in a few cases like $N = 64$ and $M \in \{40, 56, 57\}$ SA performs slightly better, 0.4 dB better. In all the other cases CR+LS outperforms SA. The results of GA are now shown since they are always the worst. In the last simulation result we show in Figure 5 the histogram of side lobe levels for the antenna selection designed for $N = 35$ and $M = 20$ via CR+LS whose radiation pattern is shown in Figure 1. In Figure 5 we notice the effect of the ℓ_∞ norm minimization: most side lobe levels occur close to the maximum value of -17.2095 dB.

The running times of the proposed methods are also studied. For example, notice that as the size N increases we decrease the length of the local search ℓ such that to keep the running time under control. The optimization problem at the heart of CR is affected only by the choice of N and not by that of M. The running times of the full CR step are on average approximately: 4.5 seconds, 7.1 seconds and 15.2 seconds for $N = 35$, $N = 64$ and $N = 128$ respectively. The implementation uses Matlab 2014a© and the convex optimization library CVX. We take into account the extra formatting time that CVX uses to formulate the optimization problem for the underlying solver. If speed is of any concern then new developments involving fast iteratively reweighted ℓ_1 based on homotopy can be deployed [20]. GA and SA running times depend on the selected parameters. For example, SA with the parameters described in this paper has on average the following approximate running times: 100 seconds for $N = 35$, 110 seconds for $N = 64$ and finally 130 seconds for $N = 128$.

An additional difficulty when using GA and SA is the relatively high number of choices (parameters, selection procedures etc.) that needs to be made. All these choices have an important effect upon performance. In the case of CR there are virtually no parameters. One might argue that a regularization parameter λ might be added to the optimization problem (3). The overall idea when tuning this optimization problem is to adjust it such that the sparsity of the solution is close to M, possibly equal. This avoids the use of the heuristic step to reduce the support to M.

5. CONCLUSIONS

This paper describes an algorithm for the design of thinned arrays for antenna subset modulation that minimizes the side lobe levels. The method is based on ideas from convex optimization and compressed sensing and can be considered a relaxation of the original discrete non-convex optimization problem. We test the proposed approach against other methods in the literature and show its effectiveness both in terms of performance and running time. We also underline the simplicity of the method, which virtually needs no parameter tuning. Here the simulations involve only uniform linear arrays, but the method can be easily applied to different array types.

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