PILOT AIDED DIRECTION OF ARRIVAL ESTIMATION FOR MMWAVE CELLULAR SYSTEMS

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ABSTRACT

Direction of arrival (DoA) estimation of high-resolution beams is critical for cell search and maintaining communication in millimeter wave (mmWave) cellular systems. All-digital solutions for DoA estimation, though desirable for their flexibility and performance, are impractical because of their use of high-speed, power hungry Analog-Digital Converters (ADCs) at each antenna element. In this paper we take a novel approach to formulate a fully digital DoA estimation solution. Our method utilizes mmWave propagation characteristics to properly design a pilot signal that enables the reduction of the ADC speed and the number of antenna by a significant amount while achieving a desired performance goal. Our proposed method applies subspace-based methods like MUSIC on the heavily subsampled received signals, and performs the DoA estimation on a small subset of the dominant multi-paths in the channel one at a time, thus maintaining the integrity of the DoA estimation algorithm while significantly reducing computational complexity.

Index Terms— DoA estimation, mmWave, Pilot

1. INTRODUCTION

The growth of the mobile and multimedia market has increased the need for greater capacity and higher data rates in wireless communications. Millimeter wave communications can fulfill the high bandwidth needs of 5G, and hence are a potential solution for next-generation wireless communications [1], [2]. Transmission in mmWaves propagation is challenging because it is subject to huge propagation and reflection losses, allowing mostly line-of-sight (LOS) paths and a few strong reflections paths to survive. Omnidirectional transmission in such situations is impractical as the antenna gain is insufficient to cover the desired range of transmission. Fortunately, the small wavelengths of mmWaves make it possible to implement large-scale antenna design to mitigate these transmission losses using narrow beamforming [3], [4]. It is advantageous to transmit signals in high-resolution narrow beams along the surviving propagation paths, as this saves power and reduces interferences in the system. Narrow beam transmission poses the challenge of searching for unobstructed directional beams between a base-station and a mobile device in the dynamic mmWave channel environment. A valid up-to-date direction of arrival (DoA) estimation of the narrow beams is critical to effectively establishing the high-resolution beamforming in the mmWave channel. An all-digital solution for DoA estimation is desirable for its flexibility and performance. However, all digital beamforming is impractical due to the high-speed ADC requirements for the GHz-bandwidth transmission received at each of the antennas [5]. Power consumption and cost of the ADC circuit are key bottlenecks hindering the development of an all-digital beamforming solution for DoA estimation. Some alternative analog and hybrid beam forming and beam-search techniques were recently proposed in [6]–[9]. The limitations of these architectures are that they can only detect DoAs one or few at a time, and are not as accurate and flexible as a digital solution. Barati in [10] proposed a digital solution that saves power by using fewer quantization bits for ADC, and argued that the purely digital option is desirable over analog or hybrid due to the additional overhead they add in the beam search process.

In this research, we propose a novel technique for using the pilot signal to assist in the estimation of DoAs of the beams. We show that by using a properly designed pilot, it is possible to significantly reduce both the number of antennas and the ADC speed without compromising the performance of DoA estimation. Specifically we design a fully digital DoA estimation approach that utilizes the sparse channel characteristics of the mmWave channel as well as prior knowledge of the pilot signal. Utilizing the low delay spread characteristics of the mmWave channel; we design a pilot that can be aggressively subsampled at the receiver antennas. Our pilot design is such that the subsampled subsequences enjoy a very distinct circular correlation structure. We exploit this structure to formulate a fully digital subspace based DoA estimation algorithm that can achieve high performance estimates of the DoAs with the least amount of antennas and low-speed ADCs. Our simulation results show that our proposed method maintains the integrity of the DoA estimation algorithm while significantly reducing computational complexity.

2. MMWAVE CHANNEL MODEL

Our understanding of the mmWave propagation channel model is based on measurements and studies done on the 60GHz band [11], [23]. The delay spread in mmWave for a typical indoor LOS scenarios ranges from 4 to 11ns. For a GHz bandwidth system, this is equivalent to a duration of less than 12 symbols. Applying beamforming can further contain the delay spread to 0.5ns for LOS scenarios and 5ns for non-line of sight (NLOS) scenarios, which corresponds to 1 and 4 symbol-durations in a GHz bandwidth system respectively. Delay spread numbers for mmWave are much smaller than conventional cellular communication systems in the lower frequency bands. These smaller delay spreads found in mmWave propagation together with beamforming technology makes a mmWave channel an ideal candidate for being modeled mathematically as a sparse channel. This paper utilizes such a channel model as shown in figure 1 below.

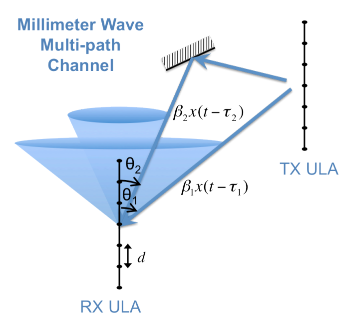

Figure 1. Millimeter Wave Channel Model

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We define a millimeter wave propagation channel consisting of a small number of narrow beam propagation paths between a transmitter and a receiver pair. Our model assumes that the receiver antenna structure is a uniform linear antenna array (ULA) with m antenna elements. Assuming a narrow band signal model, given a direction of arrival (DoA) θ, the array manifold vector also referred to as the direction vector is:

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{m}} e^{i\frac{(m-1)}{2}\psi} \begin{bmatrix} 1 \\ e^{i\psi} \\ \vdots \\ e^{i(m-1)\psi} \end{bmatrix} \quad (1)$$

where $\psi = -\frac{2\pi d}{\lambda}\cos(\theta)$.

Here d is the distance between two antennas in the antenna array, and λ is the wavelength of the mmWave.

There are p propagation paths and each path goes through a distinct delay $\tau_j, j = 1,\cdots,p$, a propagation gain $\beta_j, j = 1,\cdots,p$, and a direction of arrival $\theta_j, j = 1,\cdots,p$.

Let the known pilot signal be called $x(t)$. If we define the following matrices as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}(\theta_1) & \cdots & \mathbf{a}(\theta_p) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \beta_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \beta_p \end{bmatrix}, \mathbf{x}(t) = \begin{bmatrix} x(t-\tau_1) \\ \vdots \\ x(t-\tau_p) \end{bmatrix}.$$

Then, the received signals at the antenna array can be modeled as a vector of m signals $\mathbf{y}(t) \in C^{m\times 1}$ as follows:

$$\mathbf{y}(t) = \mathbf{ABx}(t) + \mathbf{n}(t) \quad (2)$$

Each element of the noise vector $\mathbf{n}(t) \in C^{m\times 1}$ is an additive white Gaussian noise with equal noise variance σ^2 and they are uncorrelated to each other.

In the next few sections, we show how one can formulate a pilot assisted DoA estimation algorithm using the afore-mentioned channel model.

3. SUBSPACE BASED DOA ESTIMATION

Our choice of DoA estimation algorithm uses subspace-based methods like MUSIC or its derivatives. These methods are well-studied approaches for the DoA estimation problem [12]–[16]. Some limited research has been conducted on DoA estimation on known pilot signals [17]–[20]. However, these studies do not specifically design the pilot to leverage the channel characteristics for simplifying the estimation process. In this paper we design a pilot signal with the objective to minimize the antenna array size and reduce ADC speed by subsampling, for mmWave channel conditions. In this problem we focus on detecting a single pilot signal. We assume for multi-user case the pilot signal of each source to be poorly correlated to the other sources' pilot signals and therefore contribute as additive white Gaussian noise.

Subspace-based methods like MUSIC can find all p DoAs with m number of receive antennas as long as m is greater than p. Let the covariance matrix of the received vector $\mathbf{y}(t)$ be defined as follows:

$$\mathbf{R}_{\mathbf{yy}} = E\left\{\mathbf{y}(t)\mathbf{y}(t)^H\right\} = \mathbf{ABPB}^H\mathbf{A}^H + \sigma^2\mathrm{I} \quad (3)$$

where, $\mathbf{P}$ is the covariance matrix of the delayed pilot signals defined as: $\mathbf{P} = E\{\mathbf{x}(t)\mathbf{x}(t)^H\}$ and I is identity matrix. Here the notations $(\cdot)^H$ and $E\{\cdot\}$ represent conjugate transpose, and expectation of a random variable respectively.

For a MUSIC like method to estimate all p DoAs, the covariance matrix $\mathbf{P}$ has to be non-singular. This is possible if all the propagation delays are distinct and the pilot signal has good autocorrelation properties. Further, since our received signal is a subsampled version of the pilot signals, the decimated subsequences of the pilot signal must preserve the non-singularity property of $\mathbf{P}$. That is, each decimated subsequence of the pilot must have good auto correlation property over the delay spread of the channel and the cross-correlation among the decimated sequences with different phase shifts must be low. In our problem formulation we design a pilot signal that has an additional cyclic prefix whose length is greater than the delay spread of the channel. This allows us to focus our attention on circular auto-correlation and cross-correlation properties of the subsequences instead of linear correlation properties.

In the next section, we propose a novel DOA estimation method using a Zadoff-Chu sequence as a pilot.

4. PROPOSED PILOT DESIGN FOR SUBSAMPLE BASED MUSIC ALGORITHM

Zadoff-Chu (ZC) sequence is a CAZAC (Constant Amplitude Zero Auto-Correlation) sequence. In addition to having perfect circular auto-correlation properties it also has a constant amplitude (polyphase sequence). The constant amplitude property allows one to design a more efficient power amplifier making it an attractive choice for wireless communication systems. In 4G LTE, Zadoff-Chu sequences are used extensively for synchronization and channel estimation purposes [24]. More details on the Zadoff-Chu sequences can be found in [21], [22].

The Zadoff-Chu sequence $s[n]$ of length L is defined as:

$$s[n] = \begin{cases} e^{-\frac{i\pi u n^2}{L}}, & \text{for } L \text{ even} \\ e^{-\frac{i\pi u n(n+1)}{L}}, & \text{for } L \text{ odd} \end{cases}$$

Here, u is called the root of the sequence and is relative prime to L. These sequences are periodic with a periodicity of L. When these sequences are conjugated or scaled by a complex constant they preserve the constant amplitude, zero circular correlation (CAZAC) properties. Further it is shown in [21] that adding a frequency offset by multiplying the sequences with the sequence $e^{i\frac{2\pi q}{L}n}, n = 0,\cdots,L-1$, where q is any integer, maintains the CAZAC sequence properties. The last property is also evident from the L-point Discrete Fourier Transforms (DFTs) of the sequences as they are constant amplitude for all frequencies due to the zero circular correlation property. Therefore, circular shifting the DFTs by any integer frequency q preserves the constant amplitude property and hence the zero circular correlation property.

In this paper, we prove that Zadoff Chu sequences of length, $L = ND^2$, where N and D are any positive integer pair, when decimated by D, demonstrate the following desirable properties:

1) All D subsequences have zero circular cross-correlations among themselves.
2) The circular auto-correlation of each of the sub-sequences is non-zero at every N lag and zero elsewhere. Further, the magnitudes of the non-zero entries of the circular auto-correlation have a constant amplitude and a linear phase drift.

Proof: Our proof will encompass three cases of N and D. First, we consider when N is any even positive integer and D is either any even or any odd positive integer. The Second case includes all odd positive integers for N and all even positive integers for D, and the third case covers all odd positive integers for N and all odd positive integers for D. We start off by defining the decimated subsequence with the j^{th} phase offset, where $j = 0,1,\ldots,D-1$, as:

$$s_j[k] = s[j + Dk], k = 0,\cdots,ND-1$$

This expression can be simplified to separate out the constant phase term, the linear phase term and the linear frequency term respectively, for k=0,…,ND-1, as follows:

$$s_j[k] = \begin{cases} e^{\frac{-i\pi uj^2}{ND^2}} e^{\frac{-i2\pi(uj)}{ND}k} e^{\frac{-i\pi uk^2}{N}}, & N \text{ even}, D \text{ even/odd} \\ e^{\frac{-i\pi uj(j+1)}{ND^2}} e^{\frac{-i2\pi(uj+u\frac{D}{2})}{ND}k} e^{\frac{-i\pi uk(k+1)}{N}}, & N \text{ odd}, D \text{ even} \\ e^{\frac{-i\pi j(j+1)}{ND^2}} e^{\frac{-i2\pi(uj+u\frac{(D+1)}{2})}{ND}k} e^{\frac{-i\pi uk(k+1)}{N}}, & N \text{ odd}, D \text{ odd} \end{cases}$$

The constant phase term alters neither the circular auto-correlation properties of each subsequence nor the magnitude of the cross-correlation properties among the sub-sequences.

The root parameter u is relatively prime to both N and D. This is because by definition u is relatively prime to $L=ND^2$. Therefore the third linear frequency term is a ZC sequence with length N and root u repeated D times. The ND-point DFT of this term will therefore be non-zero every D frequencies, and magnitudes at these frequencies will be constant according to the property of ZC sequence. As a result, the circular auto-correlation of the third term is non-zero every N lag and the magnitude at these lags are constant.

The second term adds a frequency offset to each subsequence of length ND. The frequency offset circularly rotates the DFT of the third term by $(uj + z)$, for $j = 0,\cdots,D-1$, where z is an integer defined as follows:

$$z = \begin{cases} 0, & N \text{ even}, D \text{ even/odd} \\ \left(\frac{D}{2}\right), & N \text{ odd}, D \text{ even} \\ \left(\frac{D+1}{2}\right), & N \text{ odd}, D \text{ odd} \end{cases}$$

Therefore, like the third term, each subsequence has non-zero circular auto-correlation at every N lag and the phases of each of the circular auto-correlation functions have a linear phase drift of $(uj + z)$.

Since the length of each subsequence is a multiple of D, and the non-zero DFT magnitudes are a constant and repeat every D frequencies, we are only concerned with the set of frequency offsets defined in the following set:

$$\Omega = \{((uj+z))_D;\ \text{for } j = 0,\cdots,D-1\}$$

Since u is relatively prime to D, and a constant integer, z is added to (uj) in all three cases, the above set is nothing but $\{0,\cdots,D-1\}$. Therefore each of the ND-point DFT of the circular cross correlations between any two subsequences will be zero. This concludes our proof.

The proof is illustrated with an example. Consider the following figure showing the magnitude of the DFT of the D decimated subsequences of a ZC sequence, where $N = 55, D = 9, u = 14$. It is clearly seen that the constant non-zero magnitude of each sequence occurs every D frequencies and the circular frequency shifts of the D subsequences are distinct and from the set $\{0,\cdots,D-1\}$.

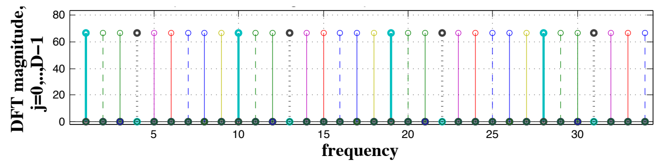

Figure 2. DFT magnitudes of the subsequences

4.1. Applying Zadoff-Chu sequence for decimated MUSIC method for DoA estimation

We can efficiently apply the properties of the decimated sequences of a Zadoff-Chu sequence of length $L = ND^2$, where N and D are any positive integers, and root u, to formulate a DoA estimation method that minimizes the number of antennas needed and maximizes the amount of subsampling using subspace based MUSIC method.

Let us define the maximum delay spread in the channel as, $\tau_{\max}$ symbol-duration. We assume that the ZC sequence is transmitted with an additional $\tau_{\max}$ symbol-duration cyclic prefix. We intend to subsample this sequence at the receiver by D. According to the properties of the decimated subsequences proven earlier, as long as $\tau_{\max}/D < N$, the zero cross-correlation property among all possible decimated multi-path components will be preserved.

The covariance matrix, used in MUSIC algorithm, of a signal vector, say $\tilde{\mathbf{y}}[k] \in C^{m\times 1}, k = 0,\cdots,ND-1$, can be estimated in the time domain or in the frequency domain as follows:

$$\hat{\mathbf{R}}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}} = \frac{1}{ND}\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^H = \frac{1}{ND}(\tilde{\mathbf{Y}}\mathbf{F})(\mathbf{F}^H\tilde{\mathbf{Y}}^H) \tag{4}$$

Here, the matrix $\tilde{\mathbf{Y}} \in C^{m\times ND}$ is defined as:

$$\tilde{\mathbf{Y}} = \begin{bmatrix} \tilde{\mathbf{y}}[0] & \tilde{\mathbf{y}}[1] & \cdots & \tilde{\mathbf{y}}[ND-1] \end{bmatrix}$$

The matrix $\mathbf{F} \in C^{ND\times ND}$ is the ND-point DFT matrix and the $(j,l)^{th}$ element of the matrix is $\frac{1}{\sqrt{ND}} e^{\frac{-i2\pi jl}{ND}}$. Since the DFT matrix is unitary, i.e., $\mathbf{F}\mathbf{F}^H = \mathbf{I}$, the time domain and frequency domain estimates of the covariance matrix are equivalent. Later in the section, we explain why applying the MUSIC estimator on the frequency domain signal vectors can reduce computational complexity.

In our proposed method we first perform circular correlation on the decimated received signal by any subsequence of the ZC pilot sequence. The circular correlation output is particularly easy to compute in the frequency domain due to the sparse non-zero elements of the DFTs of the subsequences (non-zero every D frequencies). Since all D subsequences have zero circular cross-correlations among themselves, the correlation output has no contribution from all the subsequences other than the chosen one. This way we can decouple the contribution of each subsequence of the ZC pilot, one at a time. Since the maximum delay in the channel is $\tau_{\max}$ symbols, the subsampled received signal can have at most $\lceil \tau_{\max}/D \rceil$ number of multi-paths from each of the subsequences of the ZC sequence. The decoupling of the contribution from each of the subsequences makes it possible to apply the MUSIC algorithm on the correlation output from each of the subsequences with an antenna array size of only $\lceil \tau_{\max}/D \rceil + 1$. This is a significant reduction in the number of antennas. Regardless of the number of multipath present in the received signal, we only need an antenna array size of $\lceil \tau_{\max}/D \rceil + 1$.

Note that there will be only a few dominant multi-paths present in the received signal due to the sparse nature of the mmWaves channel. The subsequences that contribute to the dominant multi-paths can be identified from the subsampled received signals' DFTs. These subsequences produce stronger DFT magnitudes at their corresponding frequency bins. By only utilizing the correlation outputs of the dominant multi-path subsequences, we can further reduce the computation necessary in our proposed method.

A useful by-product of this method is the unique identification of the propagation delays of the dominant multi-paths from the correlation outputs of the selected subsamples. After beamforming these propagation delays can be used as critical parameters for solving the channel equalization problem. In the next section, we simulate a channel model where we apply our proposed DoA estimation algorithm and demonstrate its benefits.

5. SIMULATION RESULTS

We use a conventional MUSIC algorithm as our baseline method for all-digital DoA estimation. Both the baseline and our proposed approaches perform DoA estimation on the same number of data samples. The baseline method uses 128 samples of complex Gaussian random numbers as the pilot symbols. The pilot symbols in our proposed method is a Zadoff-Chu sequence with the length $L = ND^2$, where N=4, D=32, and the root u=11. The received signals at the antenna array for the proposed method is subsampled by a factor D=32, hence, the number of data samples used in both methods are the same.

The propagation channel is known to have a maximum delay spread of, $\tau_{\max} = 32$ and has 5 multi-paths with the following parameters:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 10 \\ 28 \\ 17 \end{bmatrix}, \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.75 \\ 0.7 \\ 0.15 \\ 0.1 \end{bmatrix}, \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} 18^o \\ 30^o \\ 123^o \\ 213^o \\ 220^o \end{bmatrix}.$$

The signal-to-noise ratio of the received signal at each antenna is 10dB. Since the 4th and 5th multi-paths are very close together, we use an antenna array size of m=5 to estimate the four DoAs.

About twenty simulation runs of the root-MUSIC spectrums, the baseline method, are shown in figure 3. The solid black lines correspond to the ideal DoA angles of the signal, and their amplitudes represent the relative gains at the corresponding multi-paths. The deviations of the DoA angles corresponding to the root-MUSIC peaks, from the ideal DoAs, give us a measure of the performance of the estimator.

The plots in figure 4 demonstrate the method proposed in this paper. We use an antenna array size of only 2, and use subsampling by 32 to resolve the DoAs of the multi-path components. The reduction in antenna array size to 2 is possible since $\tau_{\max}/D$ is only 1 in our example. DFTs of the subsampled signals at the two antenna elements are used to select the subsequences of the ZC sequences that can resolve each of the multi-path components. Correlation output with the selected subsequences is then computed in frequency domain. Finally twenty simulation runs of the root MUSIC spectrums are plotted on each of the selected frequency domain correlation outputs from the antennas.

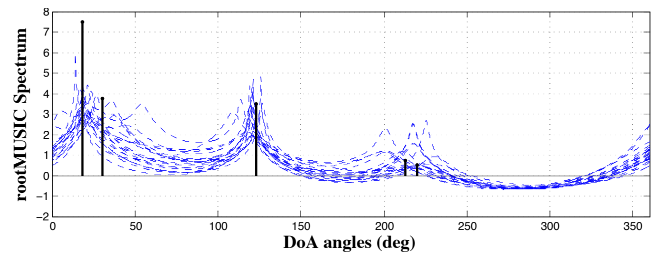

Figure 3. Root MUSIC Spectrum (Baseline)

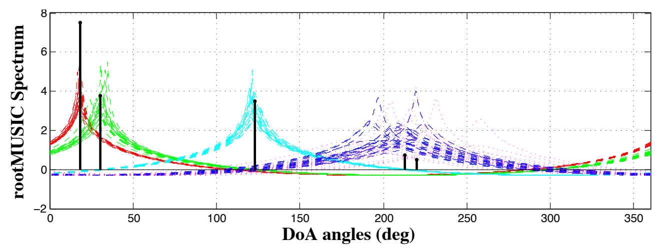

Figure 4. Root MUSIC Spectrum (Proposed Method)

The DoA estimations of the strongest three multipath components corresponding to the first three solid black lines (in figure 4), perform significantly better than the estimations from the conventional MUSIC estimator depicted in figure 3. Thus we can conclude that our proposed DoA estimation method, using aggressive sub-sampling and far fewer antenna elements, can successfully estimate all the DoA's better than the baseline MUSIC methods. These simulation results support the merit of using a well-designed pilot signal specific to the channel characteristics for DoA estimation.

In our simulation results we have only shown integer symbol delays on the pilot signal. As future work we can investigate designing a suitable pulse-shaping filter for the pilot symbols, which will preserve good correlation properties for fractional delays of the Zadoff-Chu sequences and maintain specified spectral mask.

5. CONCLUSION

In this paper we propose a subspace based DoA estimation method performed on the subsampled received signals at each antenna. Our method takes advantage of the low delay spread, $\tau_{\max}$ of the mmWave propagation channel and judiciously selects a subsample factor D that reduces the number of antennas and corresponding ADC pairs to $\lceil \tau_{\max}/D \rceil + 1$. This is done by choosing a Zadoff-Chu sequence as the pilot signal, whose length is any multiple of D^2. We have proven that subsequences of such pilot sequences that are decimated by a factor D have very special circular correlation properties. We have also demonstrated how these properties are utilized to reduce the ADC speed and the antenna array size significantly without compromising DoA estimation performance and complexity.

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