Low-Complexity Recursive Convolutional Precoding for OFDM-based Large-Scale Antenna Systems

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***Abstract*—*Large-scale antenna* (LSA) has gained a lot of attention recently since it can significantly improve the performance of wireless systems. Similar to *multiple-input multiple-output* (MIMO) *orthogonal frequency division multiplexing* (OFDM) or MIMO-OFDM, LSA can be also combined with OFDM to deal with frequency selectivity in wireless channels. However, such combination suffers from substantially increased complexity proportional to the number of antennas in LSA systems. In this paper, we propose a low-complexity recursive convolutional precoding to address the issues above. The traditional ZF precoding is implemented through the recursive convolutional precoding in the time domain so that only one IFFT is required for each user and the matrix inversion can be also avoided. Simulation results show that the proposed approach can achieve the same performance as that of ZF but with much lower complexity.**

***Index Terms*—Large-scale antenna, massive MIMO, precoding, OFDM.**

I. INTRODUCTION

By installing hundreds of antennas at the *base station* (BS), *large-scale antenna* (LSA) systems can significantly improve performance of cellular networks [1], [2]. Even if LSA can be regarded as an extension of the traditional *multiple-input multiple-output* (MIMO) systems, which has been widely studied during the last couple of decades [3], many special properties of LSA due to extremely large number of antennas make it a potential technique for future wireless systems and thus has gained lots of attention recently. Similar to the philosophy of MIMO-*orthogonal frequency division multiplexing* (OFDM) [4] or MIMO-OFDM [5], LSA can be also combined with OFDM to deal with frequency selectivity in wireless channels. Although straightforward, such combination suffers from substantially increased complexity.

First, the precoding is conducted in the frequency domain for traditional MIMO-OFDM [6]. In this case, each antenna at the BS requires an *inverse fast Fourier transform* (IFFT) for OFDM modulation and the number of IFFTs is equal to the antenna number. Therefore, the number of IFFTs will increase substantially as the rising of the antenna number in LSA systems, leading to a huge computational burden.

Second, *zero-forcing* (ZF) precoding is required to support more users in LSA systems. As indicated in [1], [2], the MF precoding can perform as well as the ZF precoding in LSA systems because the *inter-user-interference* (IUI) can be suppressed asymptotically through the MF precoding if the antenna number is large enough and the channels at different antennas and different users are independent. In practical systems, however, the antenna number is always finite. Moreover, the channels at different antennas will be correlated when placing so many antennas in a small area. In this sense, there will be residual IUI for the MF precoding, and the ZF precoding is thus still required [7]. As a result, the matrix inversion of the ZF precoding will substantially increase the complexity, especially when the user number is large.

To address the issues above, we propose a low-complexity recursive convolutional precoding for LSA-OFDM in this paper.

First, a convolutional precoding filter in the time domain is used to replace the traditional precoding in the frequency domain. In this way, only one IFFT is required for each user no matter how many antennas there are. Meanwhile, by exploiting the frequency-domain correlation of the traditional precoding coefficients, the length of the precoding filter can be much smaller than the FFT size. As a result, the complexity can be greatly reduced, especially when the antenna number is large. Even though the convolutional precoding has been reported in [8] for traditional MIMO-OFDM systems, the advantage of convolutional precoding can be hardly recognized in traditional systems because the antenna number there is small. In this paper, we highlight that such advantage becomes remarkable when the antenna number is large and thus it is more suitable to adopt the convolutional precoding rather than the traditional frequency-domain precoding for the transceiver design in LSA-OFDM systems.

Second, based on the order recursion of Taylor expansion, the convolutional precoding filter works recursively in this paper such that we can not only avoid direct matrix inverse of traditional ZF precoding but also provide a way to implement the traditional ZF precoding through the convolutional precoding filter with low complexity. Taylor expansion has already been used for *Truncated polynomial expansion* (TPE) in [9]–[12]. In [9], it is used to approximate the matrix inverse in ZF precoding. The precoding can be conducted iteratively so that the matrix inverse can be avoided. A similar approach is adopted in [10] where the TPE is based on Cayley-Hamilton theorem and Taylor expansion is used for optimization of polynomial coefficients. Different from the existing works that are based on a matrix form Taylor expansion in the frequency domain, the recursive ZF precoding in this paper is implemented through the recursive filter in the time domain such that it can be naturally combined with the convolutional precoding.

The rest of this paper is organized as follows. The system model is introduced in Section II. The proposed approach is derived in Section III. Simulation results are presented Section IV. Finally, conclusions are drawn in Section V.

II. System Model

Consider downlink transmission in an LSA-OFDM system where a BS employs M antennas to serve P users, each with one antenna, simultaneously at the same frequency band. As in [1], we assume $M \gg P$.

Denote $x_p[n,k]$ with $\mathrm{E}(|x_p[n,k]|^2) = E_s$ to be the transmit symbol for the p-th user at the k-th subcarrier of the n-th OFDM block. In an LSA-OFDM based on traditional OFDM implementation, the precoding is carried out in the frequency domain, and therefore the transmit signal at the l-th sample of the n-th OFDM block at the m-th antenna for the p-th user is

$$s_{m,p}[n,l] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} u_{m,p}[n,k] x_p[n,k] e^{j\frac{2\pi kl}{K}}, \tag{1}$$

where K denotes the subcarrier number for the OFDM modulation and $u_{m,p}[n,k]$ denotes the precoding coefficient for the k-th subcarrier of the n-th OFDM block at the m-th antenna for the p-th user. A *cyclic prefix* (CP) will be added in front of the transmit signal to deal with the delay spread of wireless channels.

After removing the CP and OFDM demodulation, the received signal at the p-th user can be expressed as

$$y_p[n,k] = \sum_{m=1}^{M} h_{p,m}[n,k] \left(\sum_{p=1}^{P} u_{m,p}[n,k] x_p[n,k] \right) + z_p[n,k], \tag{2}$$

where $z_p[n,k]$ is the additive white noise with $\mathrm{E}(|z_p[n,k]|^2) = N_0$, and $h_{p,m}[n,k]$ is the *channel frequency response* (CFR) corresponding to the k-th subcarrier of the n-th block at the m-th antenna for the p-th user, which can be expressed as

$$h_{p,m}[n,k] = \sum_{l=0}^{L-1} c_{p,m}[n,l] e^{-j\frac{2\pi lk}{K}}, \tag{3}$$

where $c_{p,m}[n,l]$ is the *channel impulse response* (CIR) and L denotes the channel length which is usually much smaller than the FFT size. The CFR is assumed to be complex Gaussian distributed with zero mean and $\mathrm{E}\{h_{p,m}[n,k] h^*_{p_1,m_1}[n,k]\} = g_p \delta[m-m_1]\delta[p-p_1]$, where g_p denotes the square of the large-scale fading coefficient for the p-th user and $\delta[\cdot]$ denotes the Kronecker delta function. It means the CFRs have been assumed to be independent for different users an different antennas.

The received signal in (2) can be rewritten in a vector form as

$$\begin{aligned} \mathbf{y}[n,k] &\triangleq (y_1[n,k], \cdots, y_P[n,k])^{\mathrm{T}} \\ &= \mathbf{H}[n,k]\mathbf{U}[n,k]\mathbf{x}[n,k] + \mathbf{z}[n,k], \end{aligned} \tag{4}$$

where

$$\begin{aligned} &\mathbf{x}[n,k] = (x_1[n,k], \cdots, x_P[n,k])^{\mathrm{T}}, \\ &\mathbf{z}[n,k] = (z_1[n,k], \cdots, z_P[n,k])^{\mathrm{T}}, \\ &\mathbf{U}[n,k] = \{u_{m,p}[n,k]\}_{m,p=1}^{M,P} = (\mathbf{u}_1[n,k], \cdots, \mathbf{u}_P[n,k]), \\ &\mathbf{H}[n,k] = \{h_{p,m}[n,k]\}_{p,m=1}^{P,M} = (\mathbf{h}_1[n,k], \cdots, \mathbf{h}_P[n,k])^{\mathrm{T}}, \end{aligned}$$

with $\mathbf{u}_p[n,k] = (u_{1,p}[n,k], \cdots, u_{M,p}[n,k])^{\mathrm{T}}$ being the corresponding precoding vector of the p-th user and $\mathbf{h}_p[n,k] = (h_{p,1}[n,k], \cdots, h_{p,M}[n,k])^{\mathrm{T}}$ being the CFR vector for the p-th user.

III. Low-Complexity Recursive Convolutional Precoding

A. Recursive Updating

The ZF precoding is considered in this paper, then the desired precoding matrix can be expressed as

$$\mathbf{U}_o[n,k] = \mathbf{H}^{\mathrm{H}}[n,k] \left(\mathbf{H}[n,k]\mathbf{H}^{\mathrm{H}}[n,k] \right)^{-1}. \tag{5}$$

With the help of Taylor expansion, the matrix inverse in (5) can be substituted by an order-recursive relation as

$$\begin{aligned} \mathbf{U}^{(Q+1)}[n,k] = \mathbf{U}^{(Q)}[n,k] + \\ \frac{\mu}{M}\mathbf{H}^{\mathrm{H}}[n,k]\mathbf{G}^{-1}(\mathbf{I} - \mathbf{H}[n,k]\mathbf{U}^{(Q)}[n,k]), \end{aligned} \tag{6}$$

where $\mathbf{G} = \mathrm{diag}\{g_p\}_{p=1}^{P}$ and $\mathbf{U}^{(Q)}[n,k]$ denotes the corresponding precoding matrix with the Q-th order expansion and μ is a step size that affects the convergence, as we will discuss in Section IV. The order-recursive relation in (6) can be also rewritten in a vector form as

$$\begin{aligned} \mathbf{u}_p^{(Q+1)}[n,k] = \mathbf{u}_p^{(Q)}[n,k] + \\ \frac{\mu}{M} \sum_{i=1}^{P} g_i^{-1} \mathbf{h}_i^*[n,k] (\delta[i-p] - \mathbf{h}_i^{\mathrm{T}}[n,k]\mathbf{u}_p^{(Q)}[n,k]). \end{aligned} \tag{7}$$

In (7), the order-recursive updating is driven by the expansion order, Q. Mathematically, the expansion order in (7) can be viewed as a *recursion counter*, which increases as the recursion proceeds. In this sense, the OFDM block index can be also used as that *recursion counter*. In other words, (7) can be also driven by the OFDM block index if replacing expansion order, Q, with OFDM block index, n, that is

$$\begin{aligned} \mathbf{u}_p[n+1,k] = \mathbf{u}_p[n,k] + \\ \frac{\mu}{M} \sum_{i=1}^{P} g_i^{-1} \mathbf{h}_i^*[n,k] (\delta[i-p] - \mathbf{h}_i^{\mathrm{T}}[n,k]\mathbf{u}_p[n,k]). \end{aligned} \tag{8}$$

As a result, the order recursion in (7) is converted to the time recursion in (8). Essentially, the order recursion in (7) can be converted to the time recursion in (8) is just because they have a similar expression except that one is driven by Q and

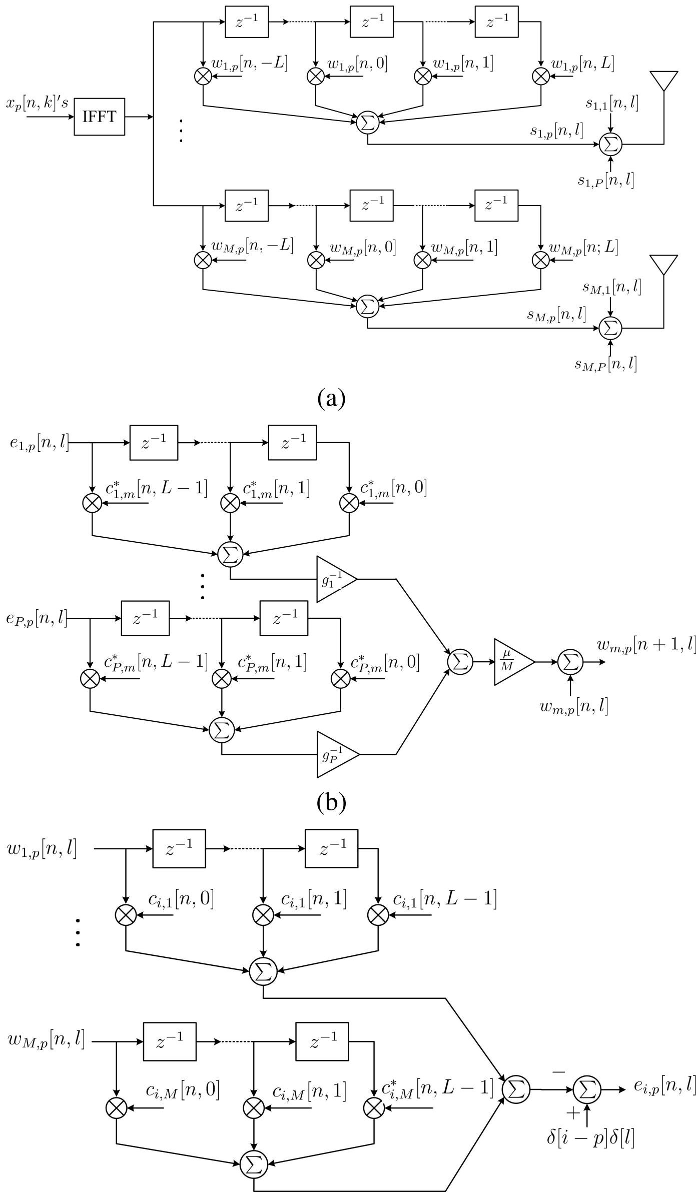

Fig. 1. (a) Convolutional precoding with (b) recursive coefficient updating and (c) estimation error calculation.

the other is driven by n. Using the time recursion in (8), the actual calculation can be conducted in the time domain even though the principle for avoiding the matrix inverse is based on the order recursion in (7). In this way, we can not only reduce the complexity since there is not need to repeat the order recursions from the zeroth order for each OFDM block, but also track the time-varying channels as long as the channel changes slowly.

B. Convolutional Precoding

Although the matrix inverse is avoided through (8), the precoding is still conducted in the frequency domain. In this subsection, we will convert it into the time-domain convolutional precoding by exploiting the frequency-domain correlation of the precoding matrices. Denote $\mathbf{u}_{m,p}[n] = (u_{m,p}[n,0],\cdots,u_{m,p}[n,K-1])^{\mathrm{T}}$, which contains the precoding coefficients from all subcarriers of the n-th OFDM block at the m-th antenna for the p-th user. Then, (8) can be rewritten as

$$\mathbf{u}_{m,p}[n+1] = \mathbf{u}_{m,p}[n] + \frac{\mu}{M}\sum_{i=1}^{P} g_i^{-1}\left(\delta[i-p]\mathbf{I} - \mathbf{D}_{i,p}[n]\right)\mathbf{h}^*_{i,m}[n], \quad (9)$$

where $\mathbf{h}_{p,m}[n] = (h_{p,m}[n,0],\cdots,h_{p,m}[n,K-1])^{\mathrm{T}}$ is the corresponding CFR vector from the m-th antenna to the p-th user, and $\mathbf{D}_{i,p}[n]$ is a $K\times K$ diagonal matrix with the (k,k)-th element given by

$$\{\mathbf{D}_{i,p}[n]\}_{(k,k)} = \sum_{m=1}^{M} h_{i,m}[n,k]u_{m,p}[n,k]. \quad (10)$$

Denote $w_{m,p}[n,l]$ to be the coefficient for the l-th tap of the precoding filter at the m-th antenna for the p-th user corresponding to the n-th OFDM block. Then, we have

$$\begin{aligned}\mathbf{w}_{m,p}[n] &\triangleq (w_{m,p}[n,0],\cdots,w_{m,p}[n,K-1])^{\mathrm{T}}\\ &= \frac{1}{K}\mathbf{F}^{\mathrm{H}}\mathbf{u}_{m,p}[n], \end{aligned}\quad (11)$$

where $\mathbf{w}_{m,p}[n]$ is the corresponding precoding vector, and $\mathbf{F}$ is the *discrete Fourier transform* (DFT) matrix with the (m,n)-th element given by

$$\{\mathbf{F}\}_{(m,n)} = e^{-j\frac{2\pi mn}{K}}, \qquad m,n\in[0,K-1]. \quad (12)$$

By taking the inverse DFT of (9), we can obtain the coefficients for the time-domain convolutional precoding filter as

$$w_{m,p}[n+1,l] = w_{m,p}[n,l] + \frac{\mu}{M}\sum_{i=1}^{P} g_i^{-1}c^*_{i,m}[n,-l]*e_{i,p}[n,l], \quad (13)$$

where $e_{i,p}[n,l]$ is the estimation error given by

$$e_{i,p}[n,l] = \delta[i-p]\delta[l] - \sum_{m=1}^{M} c_{i,m}[n,l]*w_{m,p}[n,l]. \quad (14)$$

The resulted recursive convolutional precoding is shown in Fig. 1. The precoding is carried out in the time domain via the precoding filter. In this case, only one IFFT is required for each user no matter how many antennas there are at the BS. Therefore, the number of IFFTs is equal to the number of users, which is much smaller than the antenna number in LSA systems. By exploiting the correlation of frequency-domain precoding coefficients, the coefficients of the precoding filter is sparse and thus can be truncated. For the single user case, the precoding filter is exactly the conjugate of the CIR and thus the length of the precoding filter will be $0\le l\le L-1$. In the case of multiple users, we use one more tap, as a rule of thumb, for the positive taps and another L taps to include the significant coefficients on the negative taps. As a result, $w_{m,p}[n,l]$ can be truncated within the range $-L\le l\le L$

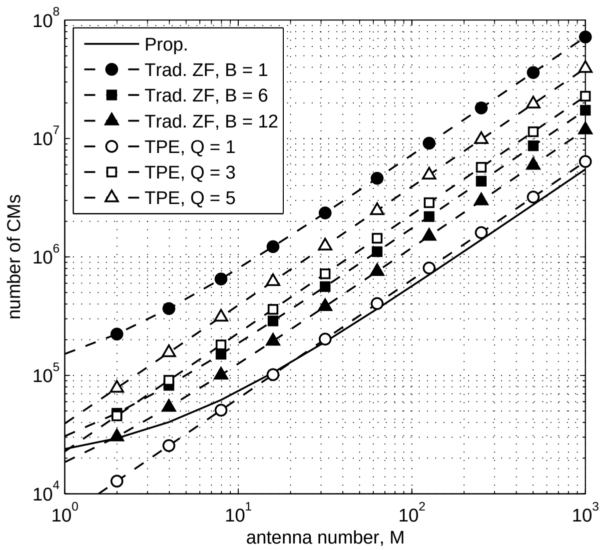

Fig. 2. An example for the complexity comparison.

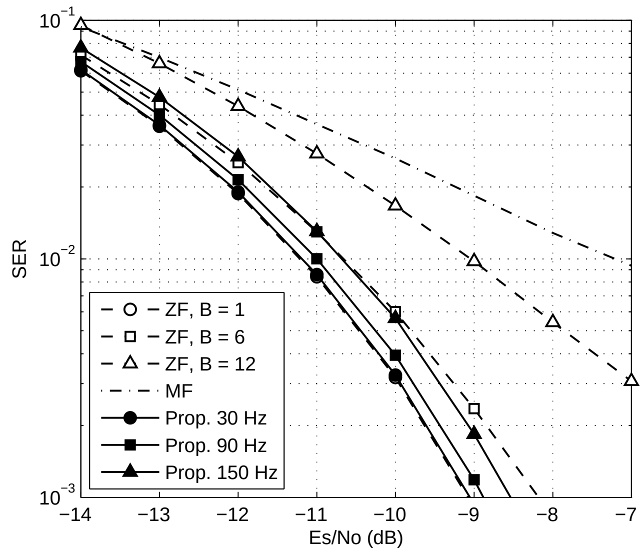

Fig. 3. SER versus Es/N_0.

(modulo K).

C. Complexity

As a comparison of complexity, fig. 2 presents the *complex multiplications* (CMs) required by the proposed approach, the traditional ZF precoding, and the TPE precoding for the typical 5 MHz bandwidth in LTE where the size of FFT is $K = 512$ [14]. For the traditional ZF precoding, B consecutive subcarriers ($B = 12$ in *long-term evolution* (LTE)) can share the same precoding coefficients by exploiting the frequency-domain correlation of the precoding coefficients. For the TPE precoding, it requires $Q - 1$ iterations for each OFDM block because the iterations are repeated from the zeroth order for each OFDM block. As expected, the complexity of the convolutional precoding is substantially reduced compared with existing approaches when the antenna number is large. When antenna number is small, however, the complexity reduction is not so significant as that for the case of large antenna number. The traditional ZF or TPE may even require fewer CMs than the proposed approach with larger B or smaller Q, at the cost of performance degradation, as will be shown in Section V. In fact, the advantage of the convolutional precoding can be hardly observed in traditional systems since the antenna number there is small, and it only becomes remarkable when the antenna number is very large. Therefore, it is more suitable to adopt the convolutional precoding rather than the traditional frequency-domain precoding for the transceiver design in LSA-OFDM systems.

IV. Simulation Results

In this section, we evaluate the proposed approach using computer simulation. We consider a BS equipped with $M = 100$ antennas and $P = 10$ users in the system. A *quadrature-phase-shift-keying* (QPSK) modulated OFDM signal is used, where the subcarrier spacing is 15 KHz corresponding to an OFDM symbol duration about 66.7μs. For a typical 5 MHz channel, the size of FFT is 512 with 300 subcarriers used for data transmission and the others used as guard band as in LTE [14]. Each frame consists of 14 OFDM symbols. A normalized ETU channel model is used, which has 9 taps and the maximum delay $\tau_{\max} = 5\mu$s. Without loss of generality, we assume $g_p = 1$ for all users.

Fig. 3 shows the SER versus Es/N_0 with different Doppler frequencies. For the proposed approach, we also assume the expansion order is large enough for initialization such that $\mathbf{U}[0, k] = \mathbf{U}_o[0, k]$. As the increasing of the Doppler frequencies, the SER performances degrade because the channels cannot be efficiently tracked when the Doppler frequency is large. As comparisons, the MF precoding and the traditional ZF precodings with $B = 1, 6, 12$ are also included. Since the ZF and MF precodings are conducted for each OFDM block individually, the SER performances will be the same for different Doppler frequencies. When the Doppler frequency is small, the proposed approach can achieve the same SER as the traditional ZF precoding with $B = 1$. As the increasing of B, the performance of ZF precoding will degrade although the complexity can be reduced. Meanwhile, the proposed approach can significantly outperform the MF precoding since the latter cannot completely remove the IUI.

V. Conclusions

In this paper, low-complexity convolutional precoding has been proposed for the precoder design in an LSA-OFDM system. Our results have shown that it is more suitable to adopt the convolutional precoding rather than the traditional frequency-domain precoding for the transceiver design in LSA-OFDM systems.

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