A JOINT DESIGN APPROACH FOR SPECTRUM SHARING BETWEEN RADAR AND COMMUNICATION SYSTEMS

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ABSTRACT

A joint design approach is proposed for the coexistence of MIMO radars and a communication system, for a scenario in which the targets fall in different range bins. Radar transmit precoding and adaptive communication transmission are adopted, and are jointly designed to maximize signal-to-interference-plus-noise ratio (SINR) at the radar receiver subject to the communication system meeting certain rate and power constraints. We start with the design of a system in which knowledge of the target information is used. Such design can be used to benchmark the performance of schemes that do not use target information. Then, we propose a design which does not require target information. In both cases, the optimization problems are nonconvex with respect to the design variables and have high computational complexity. Alternating optimization and sequential convex programming techniques are used to find a local maximum. Based on the analysis of the obtained solution, we propose a reduced dimensionality design, which has reduced complexity without degrading the radar SINR. Simulation results validate the effectiveness of the proposed spectrum sharing framework.

Index Terms— Collocated MIMO radar, spectrum sharing, alternating optimization, sequential convex programming

1. INTRODUCTION

The operating frequency bands of communication and radar systems often overlap, causing one system to exert interference to the other. For example, the high UHF radar systems overlap with GSM communication systems, and the S-band radar systems partially overlap with Long Term Evolution (LTE), and WiMax systems [1–4]. Spectrum sharing is a new line of work whose goal is to enable radar and communication systems to share the spectrum efficiently by minimizing interference effects [3–10].

Spectrum sharing between MIMO radar and communication systems has been considered in [4–7], where the radar interference to the communication system is eliminated by projecting the radar waveforms onto the null space of the interference channel from radar to communication systems. However, projection-type techniques might miss targets lying in the row space of the interference channel. Spatial filtering at the radar receiver is proposed in [8] to reject interference from the communication systems. This approach, however, works only if the target is not in the direction of the interference coming from the communication system.

Most of the existing radar-communication spectrum sharing literature addresses interference mitigation either for the communication systems [4–7], or for the radar [8]. To the best of our knowledge, co-design of radar and communication systems for spectrum sharing

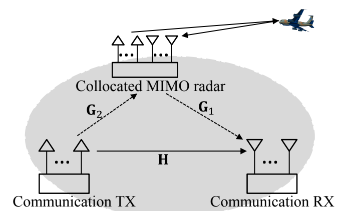

Fig. 1: A MIMO communication system sharing spectrum with a colocated MIMO radar system.

was proposed in [11–13] for the first time. Compared to the radar design approaches of [4–8], the joint design has the potential to improve the spectrum utilization due to increased number of design degrees of freedom. However, the results of [11–13] were developed for a scenario in which all targets fall in the same range bin, and the propagation delay is properly compensated.

In this paper, we propose a spectrum sharing framework for the coexistence of MIMO radars and a communication system, for a scenario in which the targets fall in different range bins. The coexistence model considers the radar operation pattern, *i.e.*, transmitting a short pulsed waveform and listening target echoes for a much longer period. Radar transmit (TX) precoding and adaptive communication transmission are adopted and are jointly designed. Unlike the radar waveform projection based methods, the joint design approach could potentially align the target returns and the communication interference separately in different subspaces, and thus suppress the interference without degrading the target returns. We formulate the design problem as maximization of SINR at the radar receiver subject to the communication system meeting certain rate and power constraints. We start with the design of a system in which knowledge of the target information (e.g., delays, reflectivities) is used. Such design can be used to benchmark the performance of schemes that do not use target information. Then, we propose a design which does not require target information. In both cases, the optimization problems are nonconvex with respect to (w.r.t.) the design variables and have high computational complexity. Alternating optimization and sequential convex programming techniques are used to find a local maximum. Analysis on the obtained solution indicates that a two-level constant communication rate over the radar TX period and the radar listening-only period could achieve the same radar SINR as the adaptive transmission. Based on this fact, we propose a new design with a much lower dimension which has reduced complexity without degrading the radar SINR. Simulation results validate the effectiveness of the proposed spectrum sharing methods over methods based on noncooperative spectrum access.

The paper is organized as follows. Section 2 introduces the coexistence model of a MIMO radar system and a communication system. The proposed spectrum sharing method is given in Section 3. Numerical results and conclusions are provided respectively in Sections 4 and 5. *Notation:* $\mathcal{CN}(\mu, \mathbf{\Sigma})$ denotes the circularly symmetric

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complex Gaussian distribution with mean μ and covariance matrix $\mathbf{\Sigma}$. $|\cdot|$ and $\mathrm{Tr}(\cdot)$ denote the matrix determinant and trace respectively. The set $\mathbb{N}_L^+$ is defined as $\{1, \ldots, L\}$. δ_{ij} denotes the Kronecker delta. $\lfloor x \rfloor$ denotes the largest integer not larger than x. $\mathbf{A}^T$ and $\mathbf{A}^H$ respectively denote the transpose and Hermitian transpose of $\mathbf{A}$.

2. SYSTEM MODELS

Consider a MIMO communication system which coexists with a MIMO radar system as shown in Fig. 1, sharing the same carrier frequency. The MIMO radar system uses $M_{t,R}$ TX and $M_{r,R}$ RX collocated antennas for target detection/estimation. The communication transmitter and receiver are equipped with $M_{t,C}$ and $M_{r,C}$ antennas, respectively. The communication channel is denoted as $\mathbf{H} \in \mathbb{C}^{M_{r,C} \times M_{t,C}}$. The interference channel from the radar TX antennas to the communication receiver is denoted as $\mathbf{G}_1 \in \mathbb{C}^{M_{r,C} \times M_{t,R}}$ [4, 5, 7]; the interference channel from the communication transmitter to the radar RX antennas is denoted as $\mathbf{G}_2 \in \mathbb{C}^{M_{r,R} \times M_{t,C}}$. It is assumed that the channels $\mathbf{H}$, $\mathbf{G}_1$ and $\mathbf{G}_2$ are block fading [14] and perfectly known at the communication transmitter. In practice, the channel state information can be obtained through the transmission of pilot signals [4, 15]. The detailed signal models for the MIMO radar and communication systems are described in the sequel. We do not assume perfect carrier phase synchronization between the two systems. A graphical illustration of the received signal at the radar and communication receivers is provided in Fig. 2.

The MIMO radar employs narrowband orthogonal waveforms, each of which contains L coded sub-pulses, each of duration T_b. Let $\mathbf{s}_m \triangleq [s_{m1}, \ldots, s_{mL}]^T$ denote the orthogonal code vector for the m-th TX antenna. It holds that $\langle \mathbf{s}_m, \mathbf{s}_n \rangle = \delta_{mn}$. The waveforms are first precoded by matrix $\mathbf{P} \in \mathbb{C}^{M_{t,R} \times M_{t,R}}$, and then transmitted over carrier f_c periodically, with pulse repetition interval T_{PRI}. Suppose that there are K targets on the same plane with the antennas, each at directions of arrival $\{\theta_k\}$ and range $\{d_k\}$ w.r.t. the radar phase center. During each pulse, the target echoes and communication interference received at the radar RX antennas are demodulated to baseband and sampled every T_b seconds. The discrete time signal model for sampling time index $l \in \mathbb{N}_{\tilde{L}}^+$ is expressed as

$$\mathbf{y}_R(l) = \sum_{k=1}^{K} \beta_k \mathbf{v}_r(\theta_k) \mathbf{v}_t^T(\theta_k) \mathbf{P} \mathbf{s}(l - l_k) + \mathbf{G}_2 \mathbf{x}(l) e^{j\alpha_2(l)} + \mathbf{w}_R(l), \tag{1}$$

where $\tilde{L} = \lfloor T_{PRI}/T_b \rfloor$ denotes the total number of samples in one PRI; $\mathbf{y}_R(l)$ and $\mathbf{x}(l)$ respectively denote the radar received signal and communication waveform symbol at time lT_b; $\mathbf{s}(l) = [s_{1l}, \ldots, s_{M_{t,R}l}]^T$; $\mathbf{w}_R(l)$ is noise distributed as $\mathcal{CN}(\mathbf{0}, \sigma_R^2 \mathbf{I})$; $l_k = \lfloor \tau_k / T_b \rfloor$ with $\tau_k \triangleq 2d_k/v_c$; β_k denotes the complex radar cross section for the k-th target; the Swerling II target model is assumed, *i.e.*, the β_k's vary from pulse to pulse and have distribution $\mathcal{CN}(0, \sigma_{\beta k}^2)$; and $\mathbf{v}_r(\theta) \in \mathbb{C}^{M_{r,R}}$ is the receive steering vector defined as

$$\mathbf{v}_r(\theta) \triangleq \left[e^{j2\pi \langle \mathbf{d}_1^r, \mathbf{u}(\theta) \rangle / \lambda_c}, \ldots, e^{j2\pi \langle \mathbf{d}_{M_{r,R}}^r, \mathbf{u}(\theta) \rangle / \lambda_c} \right]^T,$$

with $\mathbf{d}_1^r \triangleq [x_m^r \, y_m^r]^T$ denoting the two-dimensional coordinates of the m-th RX antenna, $\mathbf{u}(\theta) \triangleq [\cos(\theta) \, \sin(\theta)]^T$, and λ_c denoting the carrier wavelength. $\mathbf{v}_t(\theta) \in \mathbb{C}^{M_{t,R}}$ is the transmit steering vector and is respectively defined. The second term on the right hand side of (1) denotes the interference due to the communication transmission $\mathbf{x}(l) \in \mathbb{C}^{M_{t,C}}$. $e^{j\alpha_2(l)}$ is introduced to denote the random phase offset resulting from the random phase jitter of the oscillators at the communication transmitter and the MIMO radar receiver Phase-Locked Loops [12]. In the literature [16–18], phase jitter is

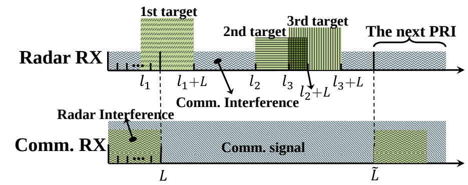

Fig. 2: An illustration of the received signal at radar and communication receivers. At the radar receiver, echoes returned from three targets are present during periods $\mathcal{L}_k, k = 1, 2, 3$ with $\mathcal{L}_k \triangleq \{l_k, ..., l_k + L - 1\}$, while the interference from the communication system is present during the whole PRI. Echoes from the second and third targets overlap with each other. At the communication receiver, the radar interference is only present during the first L symbols.

modeled as zero-mean Gaussian process. Note that $\mathbf{s}(l)$ is nonzero only for $l \in \mathbb{N}_L^+$. The echo from the k-th target appears starting from l_k and lasts for L samples.

The MIMO communication system uses the same carrier frequency f_c. The baseband signal at the communication receiver is sampled according to the symbol rate T_s, which could be different than the radar waveform symbol duration T_b. In this paper, we only consider the matched case, *i.e.*, $T_s = T_b$; the extension of the proposed methods to the mismatched case is straightforward [12]. The discrete time communication signal has the following form

$$\mathbf{y}_C(l) = \mathbf{H}\mathbf{x}(l) + \mathbf{G}_1 \mathbf{P} \mathbf{s}(l) e^{j\alpha_1(l)} + \mathbf{w}_C(l), \; l \in \mathbb{N}_{\tilde{L}}^+, \tag{2}$$

where $\mathbf{x}(l) \in \mathbb{C}^{M_{t,C}}$ denotes the transmit vector at the communication transmitter at time index l; $e^{j\alpha_1(l)}$ denotes the random phase offset between the radar TX carrier and the communication RX reference carrier [12]; the additive noise $\mathbf{w}_C(l)$ has distribution $\mathcal{CN}(\mathbf{0}, \sigma_C^2 \mathbf{I})$. Note that the radar waveform $\mathbf{s}(l)$ equals zero when $l > L$, which means that the communication system is interference free during this period. The above model assumes that the radar transmission is the only interference, while the target returns do not reach the communication system.

3. PROPOSED SPECTRUM SHARING FRAMEWORK

The figure of merit for the communication system is the achievable channel capacity. For the communication receiver, there are two distinct periods: one containing $l \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+$, during which only additive noise is present, and another one containing $l \in \mathbb{N}_L^+$, during which both interference and noise are present. Let the interference covariance during the latter period be $\mathbf{R}_{\mathrm{C}il} = \mathbf{G}_1 \mathbf{P} \mathbb{E}\{\mathbf{s}(l)\mathbf{s}^H(l)\} \mathbf{P}^H \mathbf{G}_1^H$. In this paper, as in [13] we choose $\mathbf{S}$ as a random orthonormal matrix. Note that the entries of $\mathbf{S}$ are not independent anymore. However, based on [19, Theorem 3], if $M_{t,R} = \mathcal{O}(L/\ln L)$, the entries of $\mathbf{S}$ can be approximated by i.i.d. Gaussian random variables with distribution $\mathcal{N}(0, 1/L)$. The communication system is aware that $\mathbf{S}$ is orthonormal but has no access to the specific realization of $\mathbf{S}$. Based on the above, the radar interference covariance matrix equals $\mathbf{R}_{\mathrm{C}il} \equiv \mathbf{R}_{\mathrm{Ci}} \triangleq \mathbf{G}_1 \mathbf{\Phi} \mathbf{G}_1^H$ for any $l \in \mathbb{N}_L^+$, where $\mathbf{\Phi} \triangleq \mathbf{P}\mathbf{P}^H / L$ is positive semidefinite.

The overall communication system capacity can be maximized using adaptive rate transmission [20, 21]. For $l \in \mathbb{N}_L^+$, the *instaneous* capacity is unknown because the interference plus noise is not necessarily Gaussian due to the random phase offset $\alpha_1(l)$. In this paper, we are interested in a lower bound of the capacity. However, Gaussian noise with covariance matrix equal to the actual noise covariance is the worst-case noise for additive noise channels [22]. The lower bound of the capacity is given by $\underline{C}(\mathbf{R}_{xl}, \mathbf{\Phi}) \triangleq \log_2 \left| \mathbf{I} + \mathbf{R}_{\mathrm{Cin}l}^{-1} \mathbf{H} \mathbf{R}_{xl} \mathbf{H}^H \right|$, which is achieved when the codeword $\mathbf{x}(l)$, $l \in \mathbb{N}_L^+$ is distributed as $\mathcal{CN}(0, \mathbf{R}_{xl})$. Similar to the definition of ergodic capacity [20], the overall communication capacity should be the average over $\tilde{L}$ symbols, *i.e.*,

$$\mathrm{C}_{\mathrm{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}) \triangleq 1/\tilde{L}\sum_{l=1}^{\tilde{L}} \log_2 \left|\mathbf{I} + \mathbf{R}_{\mathrm{Cin}l}^{-1}\mathbf{H}\mathbf{R}_{xl}\mathbf{H}^H\right|, \quad (3)$$

where $\{\mathbf{R}_{xl}\}$ denotes the set of all $\mathbf{R}_{xl}$'s, and $\mathbf{R}_{\mathrm{Cin}l}$ equals $\mathbf{R}_{\mathrm{Ci}} + \sigma_C^2\mathbf{I}$ if $l \in \mathbb{N}_L^+$, otherwise $\sigma_C^2\mathbf{I}$.

For the radar system, the SINR has been commonly used as figure of merit in the waveform design literature with the prior knowledge of targets and the surrounding environment [23–26]. The covariance of the interference exerted at the radar RX antennas during the l-th symbol equals $\mathbb{E}\{\mathbf{G}_2\mathbf{x}(l)e^{j\alpha_2(l)}e^{-j\alpha_2(l)}\mathbf{x}^H(l)\mathbf{G}_2^H\} = \mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H$. The echoes returned from the k-th target are present during $\mathcal{L}_k \triangleq \{l_k, \cdots, l_k + L - 1\}$, and have covariance $\mathbf{D}_k\mathbf{\Phi}\mathbf{D}_k^H$ for any $l \in \mathcal{L}_k$, where $\mathbf{D}_k \triangleq \sigma_{\beta k}\mathbf{v}_r(\theta_k)\mathbf{v}_t^T(\theta_k)$. The local SINR associated with the k-th target is averaged over $\mathcal{L}_k$ [25]

$$\mathrm{SINR}_k = 1/L\sum_{l\in\mathcal{L}_k} \mathrm{Tr}\big(\mathbf{R}_{\mathrm{Rin}l}^{-1}\mathbf{D}_k\mathbf{\Phi}\mathbf{D}_k^H\big), \quad (4)$$

where $\mathbf{R}_{\mathrm{Rin}l} \triangleq \mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H + \sigma_R^2\mathbf{I}$. The overall SINR is defined as $\mathrm{SINR} \triangleq 1/K\sum_{k=1}^K \mathrm{SINR}_k(\{\mathbf{R}_{xl}\}_{\mathcal{L}_k}, \mathbf{\Phi})$.

In the following, we first present the formulation based on target prior information, *i.e.*, knowledge based spectrum sharing, and then the formulation for the worst case design strategy, which does not rely on target information, *i.e.*, robust spectrum sharing.

1) Knowledge-based spectrum sharing. In some cases, targets information can be maintained from the detection and tracking history [24, 27]. Since in most cases such information does not exist, the design for this case will be used to benchmark other methods that are more practical. Assuming that $\{\sigma_{\beta k}^2\}$, $\{l_k\}$, and $\{\theta_k\}$ are known, the design problem is to maximize the radar SINR, subject to satisfying the communication rate and TX power constraints:

$$(\mathbf{P}_1) \max_{\{\mathbf{R}_{xl}\}\succeq 0, \mathbf{\Phi}\succeq 0} \mathrm{SINR}, \ \text{s.t.}\ \mathrm{C}_{\mathrm{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}) \geq C, \quad (5a)$$

$$\sum_{l=1}^{\tilde{L}} \mathrm{Tr}\,(\mathbf{R}_{xl}) \leq P_C, L\mathrm{Tr}\,(\mathbf{\Phi}) \leq P_R, \quad (5b)$$

The constraint of (5a) restricts the communication rate to be at least C, in order to avoid service outage. The constraints of (5b) restrict the total communication and radar TX power to be no larger than P_C and P_R, respectively.

2) Robust spectrum sharing with unknown $\{\sigma_{\beta k}^2\}$ and $\{l_k\}$. Here we consider the scenario where the radar searches in particular directions of interest given by set $\{\theta_k\}$ for targets with unknown RCS variances and delays [26, 28]. The worst possible target RCS variances are given by $\{\sigma_{\beta k}^2\} \equiv \sigma_\beta^2$, where σ_β^2 is the smallest target RCS variance that could be detected by the radar. Since $\mathcal{L}_k$ is unknown, the local SINR_k associated with the k-th target is relaxed to the whole PRI

$$\mathrm{SINR}'_k = 1/\tilde{L}\sum_{l\in\mathbb{N}_{\tilde{L}}^+} \mathrm{Tr}\left(\mathbf{R}_{\mathrm{Rin}l}^{-1}\mathbf{D}_k\mathbf{\Phi}\mathbf{D}_k^H\right), \quad (6)$$

where $\mathbf{D}_k \triangleq \sigma_\beta\mathbf{v}_r(\theta_k)\mathbf{v}_t^T(\theta_k)$. The overall SINR is given by $\mathrm{SINR}' \triangleq 1/K\sum_{k=1}^K \mathrm{SINR}'_k$. Now, the spectrum sharing problem can be formulated as

$$(\mathbf{P}_2) \max_{\{\mathbf{R}_{xl}\}\succeq 0, \mathbf{\Phi}\succeq 0} \mathrm{SINR}', \ \text{s.t. same constraints as in}\ (\mathbf{P}_1).$$

Both $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$ are nonconvex w.r.t. variable pair $(\{\mathbf{R}_{xl}\}, \mathbf{\Phi})$. In the following, we will focus on the algorithm that solves $(\mathbf{P}_2)$, which could also be adapted to solve $(\mathbf{P}_1)$.

3.1. Iterative algorithm for solving $(\mathbf{P}_2)$

A solution can be obtained via alternating optimization. Let $(\{\mathbf{R}_{xl}^n\}, \mathbf{\Phi}^n)$ be the variable at the n-th iteration. *First*, we solve $\{\mathbf{R}_{xl}^n\}$ while fixing $\mathbf{\Phi}$ to be $\mathbf{\Phi}^{n-1}$:

$$\begin{aligned}(\mathbf{P}_\mathbf{R}) &\max_{\{\mathbf{R}_{xl}\}\succeq 0} 1/K\sum_{k=1}^K \mathrm{SINR}'_k(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}^{n-1}) \\ &\text{s.t.}\ \mathrm{C}_{\mathrm{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}^{n-1}) \geq C, \sum_{l=1}^{\tilde{L}} \mathrm{Tr}\,(\mathbf{R}_{xl}) \leq P_C.\end{aligned} \quad (7)$$

Let us rewrite the objective as $\sum_{l=1}^{\tilde{L}} f(\mathbf{R}_{xl})$, with

$$f(\mathbf{R}_{xl}) \triangleq \mathrm{Tr}\left(\left(\mathbf{G}_2\mathbf{R}_{xl}\mathbf{G}_2^H + \sigma_R^2\mathbf{I}\right)^{-1}\mathcal{D}^{n-1}\right), \quad (8)$$

where $\mathcal{D}^{n-1} = \sum_{k=1}^K \mathbf{D}_k\mathbf{\Phi}^{n-1}\mathbf{D}_k^H$, and constant scale factors are omitted. It can be shown that $f(\mathbf{R}_{xl})$ is convex w.r.t $\mathbf{R}_{xl}$. Problem $(\mathbf{P}_\mathbf{R})$ is nonconvex w.r.t. $\mathbf{R}_{xl}$, because it maximizes a convex function. The sequential convex programming technique is used to find a local optimal solution [29]. $f(\mathbf{R}_{xl})$ can be approximated by the first order Taylor series expansion at $\bar{\mathbf{R}}_{xl}$ as

$$\begin{aligned}f(\mathbf{R}_{xl}) \approx& \tilde{f}(\mathbf{R}_{xl}) \triangleq f(\bar{\mathbf{R}}_{xl}) \\ &+ \mathrm{Tr}\left[\left(\frac{\partial f(\mathbf{R}_{xl})}{\partial\Re(\mathbf{R}_{xl})}\right)^T_{\mathbf{R}_{xl}=\bar{\mathbf{R}}_{xl}}(\mathbf{R}_{xl} - \bar{\mathbf{R}}_{xl})\right],\end{aligned}$$

where $\frac{\partial f(\mathbf{R}_{xl})}{\partial\Re(\mathbf{R}_{xl})} = -[\mathbf{G}_2^H\mathbf{R}_{\mathrm{Rin}l}^{-1}\mathcal{D}^{n-1}\mathbf{R}_{\mathrm{Rin}l}^{-1}\mathbf{G}_2]^T$.

We can see that $\tilde{f}(\mathbf{R}_{xl})$ is now an affine function of $\mathbf{R}_{xl}$. Problem $(\mathbf{P}_\mathbf{R})$ can be approximated by the following convex problem:

$$\begin{aligned}(\tilde{\mathbf{P}}_\mathbf{R}) &\max_{\{\mathbf{R}_{xl}\}\succeq 0} \sum_{l=1}^{\tilde{L}} \tilde{f}(\mathbf{R}_{xl}) \\ &\text{s.t.}\ \mathrm{C}_{\mathrm{avg}}(\{\mathbf{R}_{xl}\}, \mathbf{\Phi}^{n-1}) \geq C, \sum_{l=1}^{\tilde{L}} \mathrm{Tr}\,(\mathbf{R}_{xl}) \leq P_C.\end{aligned} \quad (9)$$

which can be solved with available convex programming packages. The original problem $(\mathbf{P}_\mathbf{R})$ could be solved via several iterations of solving $(\tilde{\mathbf{P}}_\mathbf{R})$. At each iteration, $\{\bar{\mathbf{R}}_{xl}\}$ is updated with the optimal solution of the previous iteration. The iteration stops when the increase of SINR is small. In addition, we observe that both the objective and constraints are separable functions of $\{\mathbf{R}_{xl}\}$. Dual decomposition technique could be used to solve (9) with lower computation complexity.

Second, the obtained $\{\mathbf{R}_{xl}^n\}$ are used to solve the following problem for $\mathbf{\Phi}^n$:

$$(\mathbf{P}_\Phi) \max_{\mathbf{\Phi}\succeq 0} \mathrm{Tr}\,(\mathbf{Q}^n\mathbf{\Phi}) \ \text{s.t.}\ \mathrm{C}_{\mathrm{avg}}(\{\mathbf{R}_{xl}^n\}, \mathbf{\Phi}) \geq C, L\mathrm{Tr}\,(\mathbf{\Phi}) \leq P_R,$$

where $\mathbf{Q}^n \triangleq \sum_{k=1}^K \mathbf{D}_k^H\left[\sum_{l=1}^{\tilde{L}}\left(\mathbf{G}_2\mathbf{R}_{xl}^n\mathbf{G}_2^H + \sigma_R^2\mathbf{I}\right)^{-1}\right]\mathbf{D}_k$.

Let $C_l(\mathbf{R}_{xl}^n, \mathbf{\Phi}) \triangleq \log_2\left|\mathbf{I} + \mathbf{R}_{\mathrm{Cin}l}^{-1}\mathbf{H}\mathbf{R}_{xl}^n\mathbf{H}^H\right|$; this is function of $\mathbf{\Phi}$ only if $l \in \mathbb{N}_L^+$. The first constraint in $(\mathbf{P}_\Phi)$ can be rewritten as $\sum_{l=1}^L C_l(\mathbf{R}_{xl}^n, \mathbf{\Phi}) \geq \tilde{L}C - \sum_{l=L+1}^{\tilde{L}} C_l(\mathbf{R}_{xl}^n) \triangleq \tilde{C}^n$. We could express $C_l(\mathbf{R}_{xl}^n, \mathbf{\Phi}), \forall l \in \mathbb{N}_L^+$, as follows

$$\begin{aligned}C_l(\mathbf{R}_{xl}^n, \mathbf{\Phi}) &= \log_2\left|\mathbf{R}_{\mathrm{Cin}l} + \mathbf{H}\mathbf{R}_{xl}^n\mathbf{H}^H\right| - \log_2|\mathbf{R}_{\mathrm{Cin}l}| \\ &= \log_2|\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \tilde{\mathbf{R}}_{xl}^n| - \log_2|\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I}|,\end{aligned} \quad (10)$$

where $\tilde{\mathbf{R}}_{xl}^n \triangleq \sigma_C^2\mathbf{I} + \mathbf{H}\mathbf{R}_{xl}^n\mathbf{H}^H$. We can see that $C_l(\mathbf{R}_{xl}^n, \mathbf{\Phi})$ is in the form of a concave function plus a convex function. It can be shown that $C_l(\mathbf{R}_{xl}^n, \mathbf{\Phi})$ is actually a convex function of $\mathbf{\Phi}$. Thus, $(\mathbf{P}_\Phi)$ is nonconvex because the above constraint imposes a nonconvex feasible set on $\mathbf{\Phi}$. A similar problem is considered in [13, Eq. (5)]. As in [13], we introduce a slack variable $\mathbf{\Psi}$ to overcome the non-convexity and apply alternating optimization again as an inner iteration. Let $(\mathbf{\Phi}^{ni}, \mathbf{\Psi}^{ni})$ be the variables at the i-th inner iteration corresponding to the n-th outer alternating iteration. $\mathbf{\Phi}^{ni}$ is initialized as $\mathbf{\Phi}^{n-1}$ for $i = 0$. Given $\mathbf{\Phi}^{n(i-1)}$, $\mathbf{\Psi}^{ni}$ is obtained as follows

$$\mathbf{\Psi}^{ni} = \left(\mathbf{G}_1\mathbf{\Phi}^{n(i-1)}\mathbf{G}_1^H + \sigma_C^2\mathbf{I}\right)^{-1}. \quad (11)$$

Based on $\mathbf{\Psi}^{ni}$, $\mathbf{\Phi}^{ni}$ is obtained by solving the following problem

$$(\mathbf{P}'_\Phi) \quad \max_{\mathbf{\Phi}\succeq 0} \mathrm{Tr}\left(\mathbf{Q}^n\mathbf{\Phi}\right), \quad \text{s.t. } L\mathrm{Tr}\left(\mathbf{\Phi}\right) \le P_R,$$

$$\sum_{l\in\mathbb{N}_L^+} \log_2 \left|\mathbf{I} + \mathbf{G}_1^H(\tilde{\mathbf{R}}_{xl}^n)^{-1}\mathbf{G}_1\mathbf{\Phi}\right| - L\mathrm{Tr}\left(\mathbf{G}_1^H\mathbf{\Psi}^{ni}\mathbf{G}_1\mathbf{\Phi}\right) \ge C',$$

where $C' \triangleq \tilde{C}^n + L\left\{\sigma_C^2\mathrm{Tr}(\mathbf{\Psi}^{ni}) - \log_2|\mathbf{\Psi}^{ni}| - M_{r,C}\right\} - \sum_{l\in\mathbb{N}_L^+}\log_2|\tilde{\mathbf{R}}_{xl}^n|$. $(\mathbf{P}'_\Phi)$ is convex w.r.t. $\mathbf{\Phi}$ and thus can be solved using available software packages [29].

The complete proposed spectrum sharing algorithm alternately solves $(\mathbf{P}_R)$ and $(\mathbf{P}_\Phi)$ as stated above. It is easy to show that the value of SINR is nondecreasing during the alternating iterations. Also, the SINR has a upper bound. Therefore, the algorithm converges. The iteration stops if the improvement of SINR is smaller than a certain threshold.

3.2. Discussion

The adaptive transmission technique adopted by the communication system greatly increases the complexity of the spectrum sharing problem $(\mathbf{P}_2)$. The following property can be used to reduce the complexity of solving $(\mathbf{P}_2)$ without any performance degradation.

Proposition 1. *Suppose that* $\{\mathbf{R}_{xl}\}$ *is initialized by* $\{\mathbf{R}_{xl}\} \equiv \mathbf{R}_x^0$. *Then, the optimal value of* $(\mathbf{P}_R)$ *in every iteration of the proposed algorithm could be achieved by* $\{\mathbf{R}_{xl}^n\}$ *such that for any* $l, l' \in \mathbb{N}_L^+$ *(or* $l, l' \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+$*), it holds that* $\mathbf{R}_{xl}^n = \mathbf{R}_{xl'}^n$.

The proof can be found in the extended version [30]. The above proposition indicates that it suffices to use only two matrix variables, $\mathbf{R}_{x1}$ and $\mathbf{R}_{x2}$, as the communication transmission covariance matrices respectively for two periods, the one during which radar transmits and the one during which radar only receives. The spectrum sharing problem can be reformulated as following

$$(\mathbf{P}'_2) \quad \max_{\{\mathbf{R}_{xl}\}\succeq 0, \mathbf{\Phi}\succeq 0} \frac{1}{K}\sum_{l=1}^{2} \mathrm{Tr}\left(\eta_l \mathbf{R}_{\mathrm{Rin}l}^{-1} \sum_{k=1}^{K} \mathbf{D}_k\mathbf{\Phi}\mathbf{D}_k^H\right)$$

$$\text{s.t. } \sum_{l=1}^{2}\eta_l C_l(\mathbf{R}_{xl}, \mathbf{\Phi}) \ge C, \sum_{l=1}^{2}\eta_l\tilde{L}\mathrm{Tr}\left(\mathbf{R}_{xl}\right) \le P_C, L\mathrm{Tr}\left(\mathbf{\Phi}\right) \le P_R,$$

where $\eta_1 \triangleq L/\tilde{L}$ is called the duty cycle and $\eta_2 = 1 - \eta_1$. Again, alternating optimization and sequential convex programming techniques used in Section 3.1 could be applied to solve $(\mathbf{P}'_2)$, which could achieve the same radar SINR objective as $(\mathbf{P}_2)$. We can observe that the robust communication transmission scheme for unknown target ranges is constant rate transmission over two periods. This is reasonable in the sense that the achieved radar SINR would be constant across different target ranges, and thus abrupt SINR degradation for certain target ranges would be avoided.

4. NUMERICAL RESULTS

We next conduct some simulation results to quantify the comparative performance of the designs based on solving $(\mathbf{P}_1)$, $(\mathbf{P}_2)$, $(\mathbf{P}'_2)$, and also include results based the projection method of [7].

We set the number of samples per PRI to $\tilde{L} = 32$, the number of radar waveform symbols to $L = 8$, the noise variance to $\sigma_C^2 = \sigma_R^2 = 0.01$, and the number of antennas to $M_{t,R} = M_{r,R} = M_{t,C} = M_{r,C} = 4$. The MIMO radar system consists of collocated TX and RX antennas forming half-wavelength uniform linear arrays. The radar waveforms are chosen from the rows of a random orthonormal matrix [11]. There are three stationary targets at angles $-60°$, $0°$ and $60°$ w.r.t. to the arrays, and the corresponding target propagation delays are $6, 18$ and 22. This corresponds to the scenario depicted in Fig. 2. For the communication capacity and power constraints, we take $C = 24$ bits/symbol and $P_C = \tilde{L}M_{t,C}$ (the power is normalized by the power of the radar waveform). The interference channels $\mathbf{G}_1$ and $\mathbf{G}_2$ are generated with entries which are independent and distributed as $\mathcal{CN}(0, 0.01)$. The channel $\mathbf{H}$ has independent entries, distributed as $\mathcal{CN}(0, 1)$. The communication covariance matrix and the radar precoding matrix are jointly optimized according to $(\mathbf{P}_1)$, $(\mathbf{P}_2)$ and $(\mathbf{P}'_2)$ in Section 3. For comparison, we implement methods based on uniform precoding, *i.e.*, $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{I}$, and null space projection (NSP) precoding, *i.e.*, $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{V}\mathbf{V}^H$, where $\mathbf{V}$ contains the basis of the null space of $\mathbf{G}_1$ [7]. In both of the aforementioned methods, selfish communication is considered, *i.e.*, the communication system minimizes the transmit power to achieve capacity C without any concern about the interferences it exerts to the radar system.

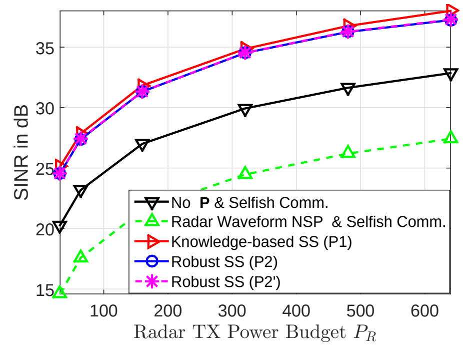

Fig. 3: SINR performance vs different values of radar TX power.

Fig. 3 shows the SINR results for different values of the radar transmit power budget (P_R). The radar power budget per antenna ranges from 1 to 20 times of the communication power budget per antenna. The highest SINR, as expected, corresponds to the case in which pretty much everything is known about the targets, *i.e.*, via the joint design of $\mathbf{P}$ and $\{\mathbf{R}_{xl}\}$ resulting from $(\mathbf{P}_1)$. Interestingly, the design of $(\mathbf{P}_2)$, which uses no knowledge about the targets incurs an SINR loss of 1 dB only. Also interestingly, the low complexity spectrum sharing method of $(\mathbf{P}'_2)$, which does not use any knowledge about the targets, achieves the same SINR performance as $(\mathbf{P}_2)$. For this particular example, as compared to $(\mathbf{P}_2)$, in $(\mathbf{P}'_2)$ the number of matrix variables is reduced from 33 to 3.

As expected, the selfish communication schemes with no precoding involves no cooperation between the radar and communication systems, and thus achieves the worst performance. The projection-type method of [7] performs even worse, because targets may fall in the row space of $\mathbf{G}_1$.

5. CONCLUSION

We have considered a general spectrum sharing framework between a MIMO radar and a MIMO communication system. Depending on the availability of target range information, a knowledge-based and a robust spectrum sharing approach were proposed to maximize the radar SINR while satisfying the communication requirements. The resulting nonconvex problems were solved by using alternating optimization and sequential convex programming. Simulation results have validated the effectiveness of the proposed spectrum sharing methods. We would like to point out that radar and communication coexistence is a new line of work with limitations and challenges. It calls for not only more research efforts on system modeling and design, but also cooperation across public and private sectors on regulation and policy revision.

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