CONSTRUCTIVE INTERFERENCE EXPLOITATION FOR DOWNLINK BEAMFORMING BASED ON NOISE ROBUSTNESS AND OUTAGE PROBABILITY

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ABSTRACT

Quality of service (QoS) is commonly measured in terms of signal to interference plus noise ratio (SINR), where multiuser interference is mitigated in order to improve the performance. As opposed to conventional suppression, interference can be exploited constructively to enhance the desired signal. With the aid of channel state information (CSI) at the transmitter and data information, we study symbol-level downlink beamforming problems based on noise robustness and outage probability, respectively, subject to power constraints. We further show that an equivalence relationship between the noise robustness and outage probability symbol-level downlink beamforming problems can be obtained. Finally, we provide an analytic symbol error rate (SER) upper bound of the worst user by solving the outage probability-based problem. Our simulations demonstrate that the proposed techniques provide substantial performance improvements over conventional downlink beamforming techniques.

***Index Terms*—** Downlink beamforming, robust design, error probability, convex optimization, constructive interference.

1. INTRODUCTION

In wireless networks, downlink beamforming is an attractive approach as an effective way of simultaneously transmitting an individual data for each user to achieve demand in high data rate. [1, 2]. In addition to the urge for high throughputs and limited power expenses, quality of service (QoS) is also a main criterion in modern communications systems. With the knowledge of channel state information (CSI) at the transmitter, designing downlink beamformers to improve the QoS for downlink scenario has been studied extensively [3–9].

Zero-forcing (ZF) precoding is commonly employed to downlink problem. The multiuser interference signal is nulled in wireless communications [10, 11]. The advantage of ZF precoding is that the algorithm is simple to apply. However, it is not fully optimized. To obtain the optimal solutions, the optimization-based downlink beamforming problems were developed [4, 5, 12–15]. One form of downlink problems is to maximize the minimum SINR subject to a total power constraint [4]. The problem is efficiently solved using an iterative algorithm. Taking the CSI mismatch into account, channel robust worst-case downlink beamforming optimization was considered [5, 12–14]. To provide more flexibility than the worst-case scenario, channel outage probability-based downlink beamforming optimization has been introduced [14, 15]. It has been proved that both the worst channel robustness and outage probability-based problems are equivalent.

In the SINR-based downlink problem, beamformers are designed to guarantee that the SINR constraints are satisfied. However, the drawback of SINR criteria is that power is wasted by suppressing the interference. Rather than mitigating, one can exploit constructive interference to enhance the useful signal by making use of both the CSI and data information. By exploiting the constructive part of interference to achieve higher performance, the closed-form linear and non-linear precoders were discussed [16–22]. Nonetheless, these precoders are not the optimal design. Optimization-based downlink beamforming precoders by exploiting constructive interference was considered [23, 24].

In line with the above, this paper is based on the symbol-level downlink beamforming optimization by exploiting constructive interference to amplify the signal [23, 24]. In the following analysis, phase-shift keying (PSK) modulation is selected. We assume that a time division duplexing (TDD) transmission, e.g., downlink channels can be determined by using the knowledge of uplink CSI and uplink-downlink channel reciprocity [25], the availability of perfect CSI at the transmitter and instantaneous data information, as in [23, 24]. We propose a symbol-level downlink beamforming problem based on noise robust design in Section 4 by introducing a geometrical analysis to the optimization problem studied in [23]. We reformulate the optimization to address the symbol-level downlink beamforming problem based on outage probability design in Section 5 by use of duality with the noise robust case. All proposed approaches can be formulated into convex optimizations and can be solved efficiently. We provide an analytic symbol error rate (SER) upper bound of the worst user by solving the error probability-based optimization.

Notation: $\mathrm{E}(\cdot)$, $\mathrm{Pr}(\cdot)$, $|\cdot|$, $\|\cdot\|$, $(\cdot)^*$ $(\cdot)^T$, denote statistical expectation, the probability, the absolute value, the Euclidean norm, the complex conjugate, the transpose, respectively. $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ are the real part, and the imaginary part, respectively.

2. SYSTEM MODEL AND CONVENTIONAL DOWNLINK BEAMFORMING

Let us consider a downlink scenario with a single N-antenna at the base station (BS). We assume that there are K single-antenna users. Let b_i be the transmitted data with the unit amplitude of the M-order PSK modulation and the given maximum angular shift $\theta = \pi/M$. The transmitted signal at the BS is the $N \times 1$ vector

$$\mathbf{x} = \sum_{i=1}^{K} \mathbf{t}_i b_i, \qquad (1)$$

where $\mathbf{t}_i$ is the $N \times 1$ beamforming vector for the ith user. The received signal for the ith user is given by

$$y_i \quad = \quad \mathbf{h}_i^T \mathbf{x} + n_i, \qquad (2)$$

where n_i is a complex white Gaussian noise and $\mathbf{h}_i$ is the $N \times 1$ channel vector for the ith user. We present a common downlink

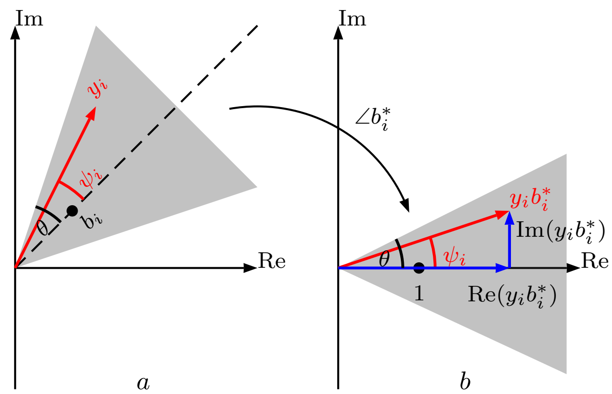

Fig. 1: In M-PSK, (a) constructive interference y_i within correct detection region; (b) vector decomposition of $y_i b_i^*$ after rotation by $\angle b_i^*$.

beamforming optimization problem in the literature [4–6], which maximizes the minimum SINR subject to a total transmitted power constraint. The problem can be formulated as [4]

$$\begin{aligned}
&\max_{\mathbf{t}_i,\gamma} \ \gamma \\
&\text{s.t.} \ \frac{|\mathbf{h}_i^T \mathbf{t}_i|^2}{\sum_{\substack{j=1\\ j\neq i}}^{K} |\mathbf{h}_j^T \mathbf{t}_j|^2 + \sigma^2} \geq \gamma, \ \forall i=1,\ldots,K, \\
&\quad \sum_{i=1}^{K} \|\mathbf{t}_i\|^2 \leq P_0, \qquad (3)
\end{aligned}$$

where γ is the minimum SINR and P_0 is the given total transmitted power threshold and σ^2 is the noise variance.

3. CONSTRUCTIVE INTERFERENCE OPTIMIZATION-BASED PRECODING

By jointly exploiting the knowledge of the CSI and user data information at the transmitter, the constructive interference-based optimization precoder in [23] improves upon the above conventional optimization. The precoder maximizes the shifted distance $\tau\sigma$ of correct detection region away from origin along with the direction of the corresponding transmitted symbol b_i by designing the beamformers. The optimal beamformers can guarantee that the resultant received symbol $\mathbf{h}_i^T\mathbf{x}$ still falls within the corresponding region. Under the design criterion, the resultant received symbol moves away from the original decision thresholds of the constellation. This leads to an improvement of QoS. The reader interested in additional details of the underlying concept is referred to [23]. The optimization problem can be written in mathematical form as [23]

$$\begin{aligned}
&\max_{\mathbf{x},\tau} \ \tau \\
&\text{s.t.} \ |\mathrm{Im}(b_i^* \mathbf{h}_i^T \mathbf{x})| \leq (\mathrm{Re}(b_i^* \mathbf{h}_i^T \mathbf{x}) - \tau\sigma)\tan\theta, \\
&\quad \|\mathbf{x}\|^2 \leq P_0, \ \forall i=1,\ldots,K, \qquad (4)
\end{aligned}$$

where P_0 is the predefined total transmitted power threshold. The constraints of (4) stem from the fact that the resultant received symbol for the ith user lays on correct detection region, if and only if $-\theta \leq \phi_i \leq \theta$, where ϕ_i is an angle such that

$$\phi_i(\mathbf{x},\tau) = \begin{cases} \tan^{-1}\left(\frac{\mathrm{Im}(b_i^* \mathbf{h}_i^T \mathbf{x})}{\mathrm{Re}(b_i^* \mathbf{h}_i^T \mathbf{x}) - \tau\sigma}\right) & \mathrm{Re}(b_i^* \mathbf{h}_i^T \mathbf{x}) > \tau\sigma, \\ 0 & b_i^* \mathbf{h}_i^T \mathbf{x} = \tau\sigma. \end{cases} \qquad (5)$$

The disadvantage of (4) are that it is hard to quantify the QoS in terms of τ. In particular, [23] did not provide the relationship between τ and the worst user's SER performance. We address this issue in Section 5. In the next section we present a noise robustness-based optimization by exploiting the constructive interference.

4. NOISE ROBUST BEAMFORMING OPTIMIZATION

In this section, we introduce a noise robust adaptation together with exploiting the constructive interference. First of all, we present an improved systematic treatment of constructive interference for the received signal. For PSK modulation, interference is constructive[1] if the received signal y_i lays on the correct detection region, which is the shaded area shown in Fig. 1(a). Under the definition of constructive interference, we obtain the following lemma.

Lemma 1. *The received signal y_i is said to receive constructive interference, if and only if*

$$-\theta \leq \psi_i \leq \theta \qquad (6)$$

where ψ_i in Fig. 1(a) is the angle between the received signal y_i and the transmitted symbol b_i such that

$$\psi_i(\mathbf{x}, n_i) = \begin{cases} \tan^{-1}\left(\frac{\mathrm{Im}(y_i b_i^*)}{\mathrm{Re}(y_i b_i^*)}\right) & \mathrm{Re}(y_i b_i^*) > 0, \\ 0 & y_i b_i^* = 0. \end{cases} \qquad (7)$$

The criterion in (6) can be directly reformulated as the following constraints

$$|\mathrm{Im}(y_i b_i^*)| - \mathrm{Re}(y_i b_i^*) \tan\theta \leq 0. \qquad (8)$$

Proof. Suppose that the received signal y_i is within the correct detection region. To obtain the angle ψ_i, we first rotate Fig. 1(a) to Fig. 1(b) by shifting the constellation by a phase equal to $\angle b_i^*$, i.e., by multiplying b_i^*. As b_i is a unit power, $y_i b_i^*$ does not change the magnitude. Then we obtain the inequities in (7) where $\mathrm{Im}(y_i b_i^*)$ and $\mathrm{Re}(y_i b_i^*)$ are the projection of $y_i b_i^*$ onto the real and imaginary axis, respectively. ☐

4.1. Noise Uncertainty Radius Maximization

The idea of the symbol-level downlink beamforming problem based on noise robustness is to design the beamformers such that the received signal is constructive interference if the noise is within the noise uncertainty set. To improve the noise robustness of the design given the noise variance σ^2, we maximize the radius $\Gamma\sigma$ of the noise uncertainty set such that it can still satisfy the constraints (8) under the power constraint. The noise robustness-based optimization problem by exploiting constructive interference can be written as

$$\begin{aligned}
&\max_{\mathbf{x},\Gamma} \ \Gamma \ \text{s.t.} \ \max_{\|n_i\| \leq \Gamma\sigma} |\psi_i(\mathbf{x}, n_i)| \leq \theta, \ \forall i=1,\ldots,K, \\
&\qquad \|\mathbf{x}\|^2 \leq P, \qquad (9)
\end{aligned}$$

[1] Note that we consider the resultant received symbol plus noise in our case, while [23] discussed the resultant received symbol $\mathbf{h}_i^T\mathbf{x}$ in the formulation.

where P is the given total transmit power. By Lemma 1, we rewrite (9) as

$$\max_{\mathbf{x},\Gamma}\ \Gamma \text{ s.t. } \max_{\|n_i\|\leq\Gamma\sigma} |\operatorname{Im}(y_i b_i^*)| - \operatorname{Re}(y_i b_i^*)\tan\theta \leq 0,$$
$$\|\mathbf{x}\|^2 \leq P,\ \forall i=1,\ldots,K. \tag{10}$$

To simplify above problem, we can first solve the inner maximization in (10).

Corollary 1. *For a fixed $\tilde{\mathbf{x}}$, the inner maximization in (10) has the following optimal solution as*

$$|\operatorname{Im}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}})| + \Gamma\sigma/\cos\theta - \operatorname{Re}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}})\tan\theta. \tag{11}$$

Proof. Let $\tilde{y}_i = b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}} + n_i$. The dual Lagrange function is given by

$$\mathcal{L}(\kappa_i, n_i) = -|\operatorname{Im}(\tilde{y}_i)| + \operatorname{Re}(\tilde{y}_i)\tan\theta + \kappa_i(\|n_i\|^2 - \Gamma^2\sigma^2), \tag{12}$$

where $\kappa_i \geq 0$. Note that

$$\operatorname{Im}(b_i^* n_i) = n_{Ii}b_{Ri} - n_{Ri}b_{Ii}, \tag{13}$$
$$\operatorname{Re}(b_i^* n_i) = n_{Ii}b_{Ii} + n_{Ri}b_{Ri}. \tag{14}$$

where $n_i \triangleq n_{Ri} + i n_{Ii}$, and $b_i \triangleq b_{Ri} + i b_{Ii}$. Setting $\frac{\partial\mathcal{L}}{\partial n_{Ri}} = 0$ and $\frac{\partial\mathcal{L}}{\partial n_{Ii}} = 0$, we obtain

$$b_{Ri}\tan\theta + b_{Ii}\alpha_i + 2\kappa_i^\star n_{Ri}^\star = 0, \tag{15a}$$
$$-b_{Ri}\alpha_i + b_{Ii}\tan\theta + 2\kappa_i^\star n_{Ii}^\star = 0, \tag{15b}$$

where $\alpha_i = \operatorname{Im}(\tilde{y}_i)/|\operatorname{Im}(\tilde{y}_i)|$ and $a^\star$ is the optimal value of a. If we suppose that $\kappa_i^\star = 0$, then (15) implies that $b_{Ri} = b_{Ii} = 0$, which leads to the contradiction. Therefore, we conclude that $\kappa_i^\star > 0$ and

$$\|n_i^\star\|^2 = \Gamma^2\sigma^2, \tag{16}$$

by the complementary slackness. Putting (15) into (16) and noticing the fact that b_i is a unit power symbol, we obtain

$$\kappa_i^\star = (2\Gamma\sigma\cos\theta)^{-1}. \tag{17}$$

We substitute (17) back into (15), then we get

$$n_{Ri}^\star = -(b_{Ri}\tan\theta + b_{Ii}\alpha_i)\Gamma\sigma\cos\theta, \tag{18a}$$
$$n_{Ii}^\star = (b_{Ri}\alpha_i - b_{Ii}\tan\theta)\Gamma\sigma\cos\theta. \tag{18b}$$

Taking (18) into problem (10), we rewrite the inner maximization in (10) as

$$\alpha_i\operatorname{Im}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}}) - n_{Ri}^\star(b_{Ri}\tan\theta + b_{Ii}\alpha_i)$$
$$+ n_{Ii}^\star(b_{Ri}\alpha_i - b_{Ii}\tan\theta) - \operatorname{Re}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}})\tan\theta$$
$$= |\operatorname{Im}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}})| + \Gamma\sigma/\cos\theta - \operatorname{Re}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}})\tan\theta, \tag{19}$$

where $\operatorname{Im}(\tilde{y}_i)$ and $\operatorname{Re}(b_i^*\mathbf{h}_i^T\tilde{\mathbf{x}})$ have the same sign because we can assume that the received noise cannot dominate the received signal. □

According to Corollary 1, we reformulate (10) as a function $\Gamma^\star(\cdot)$ for any given $P \geq 0$ such that

$$\Gamma^\star(P):\ \max_{\mathbf{x},\Gamma}\ \Gamma$$
$$\text{s.t. } |\operatorname{Im}(b_i^*\mathbf{h}_i^T\mathbf{x})| + \Gamma\sigma/\cos\theta \leq \operatorname{Re}(b_i^*\mathbf{h}_i^T\mathbf{x})\tan\theta,$$
$$\|\mathbf{x}\|^2 \leq P,\ \forall i=1,\ldots,K. \tag{20}$$

Problem (20) can be solved using available convex optimization tools [26]. Finally, we obtain the optimal beamformer $\mathbf{t}_i^\star$ in (1) as

$$\mathbf{t}_i^\star = \mathbf{x}^\star b_i^*/K, \tag{21}$$

where $\mathbf{x}^\star$ is the optimal solution in (20).

Remark 1*:* Suppose $\mathbf{x}_{SD}^\star$ and $\mathbf{x}_{NR}^\star$ are optimal solutions of (4) and (20), respectively. Then $\sin\theta\mathbf{x}_{NR}^\star = \mathbf{x}_{SD}^\star$. Hence we can treat them as equivalence problems.

5. OUTAGE PROBABILITY APPROACH

We assume a noise at the receiver is complex Gaussian with zero mean. In this section, we present a new approach to constructive interference-based downlink beamforming by the noise outage probability. In the concept of noise outage probability, we replace the noise robust downlink beamforming constraints by more flexible probabilistic constraints. We define the noise outage probability for the ith constraint as the probability that received signal lays outside the correct detection region bounded by either the angle θ or $-\theta$. The problem can be written as

$$\min_{\mathbf{x},p}\ p \text{ s.t. } \Pr\Big(\pi \geq \psi_i(\mathbf{x}, n_i) \geq \theta\Big) \leq p,\ \forall i=1,\ldots,K, \tag{22a}$$
$$\Pr\Big(-\pi \geq \psi_i(\mathbf{x}, n_i) \geq -\theta\Big) \leq p,\ \forall i=1,\ldots,K, \tag{22b}$$
$$\|\mathbf{x}\|^2 \leq P.$$

Remark 2*:* Problem (22) and the channel outage probability based downlink beamforming problem in [14, 15] are different. The constraints in [14, 15] are outage probabilistic SINR-based with channel random variables, while the constraints are outage probabilistic constructive interference-based with noise random variables. The SER upper bound of the worst user is equal to $2p$, which is originated from that the worst case possibility of the received signal laying outside the correct detection region bounded by the angle $\pm\theta$ is p respectively. It will be shown in the simulation result that the worst user's SER performance calculations close to the upper bound.

According to Lemma 1, problem (22) can be expressed as

$$\min_{\mathbf{x},p}\ p \text{ s.t. } \Pr\Big(\operatorname{Im}(y_i b_i^*) \geq \operatorname{Re}(y_i b_i^*)\tan\theta\Big) \leq p, \tag{23a}$$
$$\Pr\Big(\operatorname{Im}(y_i b_i^*) \leq -\operatorname{Re}(y_i b_i^*)\tan\theta\Big) \leq p, \tag{23b}$$
$$\|\mathbf{x}\|^2 \leq P, \forall i=1,\ldots,K.$$

The constraints in (23a) and (23b) can be rewritten as

$$\Pr(z_i + \tilde{n}_i \geq 0) \leq p, \tag{24}$$

where

$$z_i = \pm\operatorname{Im}(b_i^*\mathbf{h}_i^T\mathbf{x}) - \operatorname{Re}(b_i^*\mathbf{h}_i^T\mathbf{x})\tan\theta, \tag{25}$$
$$\tilde{n}_i = \pm\operatorname{Im}(b_i^* n_i) - \operatorname{Re}(b_i^* n_i)\tan\theta. \tag{26}$$

As n_i is complex Gaussian, we obtain

$$\mathrm{E}\{\operatorname{Re}(b_i^* n_i)^2\} = \mathrm{E}\{\operatorname{Im}(b_i^* n_i)^2\} = b_{Ri}^2\frac{\sigma^2}{2} + b_{Ii}^2\frac{\sigma^2}{2} = \frac{\sigma^2}{2}, \tag{27}$$
$$\mathrm{E}\{\operatorname{Re}(b_i^* n_i)\operatorname{Im}(b_i^* n_i)\} = b_{Ri}b_{Ii} - b_{Ri}b_{Ii} = 0. \tag{28}$$

The variance of n_i is given by

$$\mathrm{E}\{\tilde{n}_i^2\} = (1 + \tan^2\theta)\sigma^2/2 = \sigma^2/(2\cos^2\theta). \tag{29}$$

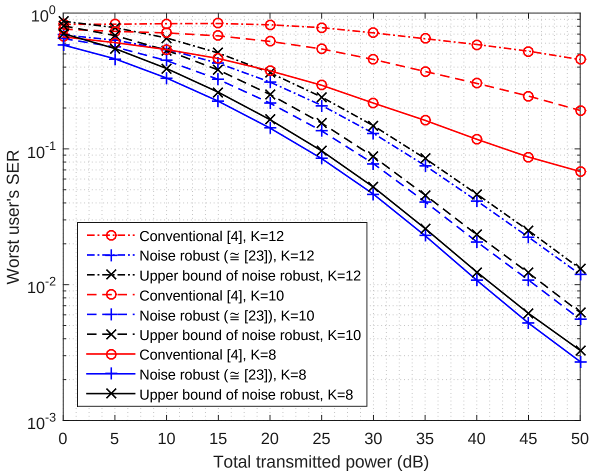

Fig. 2: The worst user's SER performance versus transmit power with $N = 10$.

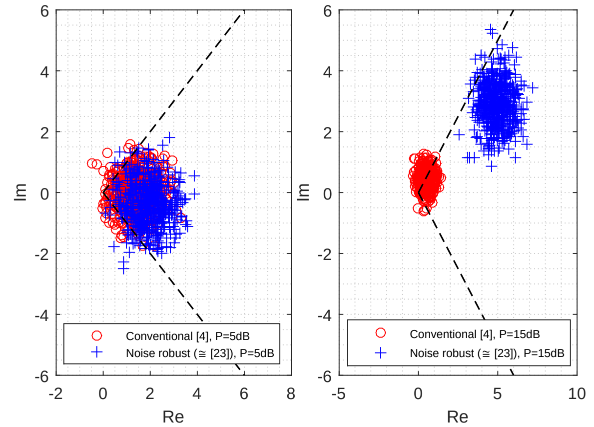

Fig. 3: Distribution of received signals on complex plane with $N = 10$, and $K = 10$.

Therefore, $\tilde{n}_i \sim \mathcal{N}(0, \frac{\sigma}{\sqrt{2}\cos\theta})$. By ensuring reliable communication link, the noise outage probability must be close to 0. According to [15], we assume that $p \leq 0.5$. The outage probability constraints in (24) can be expressed in terms of the Gaussian error function $\mathrm{erf}(\cdot)$ as

$$\frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{-z_i\cos\theta}{\sigma}\right) \leq p, \tag{30}$$

or equivalently,

$$|\,\mathrm{Im}(b_i^*\mathbf{h}_i^T\mathbf{x})| + \frac{\mathrm{erf}^{-1}(1-2p)\sigma}{\cos\theta} \leq \mathrm{Re}(b_i^*\mathbf{h}_i^T\mathbf{x})\tan\theta,\ \forall i. \tag{31}$$

Hence, the outage probability problem (23) can be written as a function $p^\star(\cdot)$ for any given $P \geq 0$ such that

$$\begin{aligned} p^\star(P):\ &\min_{\mathbf{x},p}\ p \\ &\text{s.t. } \mathrm{Im}(b_i^*\mathbf{h}_i^T\mathbf{x})| + \frac{\mathrm{erf}^{-1}(1-2p)\sigma}{\cos\theta} \leq \mathrm{Re}(b_i^*\mathbf{h}_i^T\mathbf{x})\tan\theta, \\ &\|\mathbf{x}\|^2 \leq P,\ \forall i = 1,\ldots,K. \end{aligned} \tag{32}$$

and the optimal values of (20) and (32) have the following relations:

$$\Gamma^\star(P) = \mathrm{erf}^{-1}(1-2p^\star(P)), \tag{33}$$

$$p^\star(P) = \frac{1}{2} - \frac{1}{2}\mathrm{erf}(\Gamma^\star(P)), \tag{34}$$

$$\mathbf{x}_p^\star(P) = \mathbf{x}_\Gamma^\star(P), \tag{35}$$

where $\mathbf{x}_p^\star(\tilde{P})$ is an optimal solution of (32) for a given power $\tilde{P}$.

6. SIMULATIONS

In our simulations, the system with 4-PSK modulation is considered, i.e., $\theta = \pi/4$, while it is intuitive that the benefits of the proposed approaches extend to other modulation schemes. The white complex zero-mean Gaussian noise n_i is with the variance $\sigma^2 = 1$. We consider a constructive interference-based downlink beamforming network with $N = 10$ antennas, while it is obvious that the benefits shown extend to different numbers of antennas. Let ω_i be a uniformly distributed random number between $-\pi/2$ and $\pi/2$. We model the downlink channel between the BS and ith user as [27]

$$\mathbf{h}_i = \left[1, e^{j\pi\sin\omega_i}, \ldots, e^{j\pi(N-1)\sin\omega_i}\right]^T. \tag{36}$$

We compare two different techniques: 'Conventional [4]' refers to the SINR balancing problem in [4]; 'Noise robust (≅ [23])' stands for the problem (20). Note that (20) is equivalent to (4), which is proposed in [23]. 'Upper bound of noise robust' stands for the SER upper bound of the worst user by solving noise robust approach and it is equal to $2p$ according to Remark 2, where p is the outage probability of (32). Since we have shown in Section 5 that the noise robust approach of (20) and the outage probability approach of (32) are equivalent, we only consider the noise robust approach in the following simulations.

Fig. 2 compares the worst user's SER performance for the different techniques. In Fig. 2, we fix the number of users and compare the worst user's SER performance of our proposed approaches and the conventional approach of [4] versus the total transmitted power P with different numbers of user K. It can be seen from the figure that the noise robust approach outperforms the conventional method of (3). Furthermore, the worst user's SER performance calculations of the proposed noise robust approach match close to the SER upper bound.

Fig. 3 displays the distribution of the received signals using the two techniques on complex plane with $P = 5$dB and $P = 15$dB. Here, we set the transmitted symbol to be 1. The right side of dotted line is the constructive area of the constellation. Therefore, the received signals are valid if they lay on the right side behind the dotted line. We observe from Fig. 3 that the received signals of our proposed method can better lay on the correct detection region compared to the conventional method. Moreover, We notice that when the power increases, our technique can shift the received signals further away from the decision threshold than the conventional technique.

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