Uniform Expected Likelihood Solution for Interference Rejection Combining Regularization

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Abstract— **A well known problem of regularization (diagonal loading) of the interference rejection combining (IRC) and IRC / maximum ratio combining (MRC) switching is addressed. Different empirical loading factor selection rules adjusted to specific scenarios have been introduced in the literature. It is expected that future network will be characterized by variety of scenarios, transmission modes, and receiver configurations. Empirical IRC regularization may not be suitable for such networks. In this study we consider the Expected Likelihood (EL) criterion for estimation/selection of the interference plus noise covariance matrix and demonstrate that it gives the diagonal loading selection rules that are effective in wide range of scenarios and receiver configurations.**

Keywords— **Interference rejection combining, maximum ratio combining, regularization, diagonal loading, expected likelihood.**

I. Introduction

Interference aware receivers play a pivotal role in future communications networks, which will be increasingly dense and interference limited [1]. Interference rejection combining is a simple example, which is currently standardized, e.g., in LTE [2]. A very important part of such receivers is estimation of the interference plus noise covariance matrix, which is normally performed over some sets of pilot symbols. Direct application of the conventional sample (maximum likelihood) matrix estimation for a limited number of samples may lead to significant performance degradation depending on the interference scenario. A well known solution for this problem is a regularization (diagonal loading) of the sample covariance matrix. In scenarios, typically considered in radar applications, e.g., [3], with the number of interfering point sources P significantly smaller than the number of receive antenna elements K and very high interference-to-noise ratio (INR) , selection of the diagonal loading factor is proven to be non-critical. Indeed, for INR$\gg 1$ and $P \ll K$, the loading factor $\delta \approx (2 \div 3)\sigma_n^2$, where σ_n^2 is the additive white noise power, yields near-optimal performance [3]. Yet, in wireless communications, the number of interference components may approach or exceed the number of antenna elements and INR may vary significantly. Under these conditions, the robust (scenario invariant) selection of the diagonal loading factor may have a large impact on system performance.

Different empirical loading factor selection rules adjusted to specific scenarios have been introduced. Particularly, generalized likelihood ratio (sphericity) test is studied for scenario classification in [4] and applied to the LTE uplink in [5]. Selection of the regularization parameter proportional to the variance of the diagonal elements of the sample covariance matrix is proposed in [6] and used in [2] for IRC on the LTE downlink. Scene detection depending on the relative weight of the off-diagonal elements of the sample covariance matrix is proposed in [7].

The major drawback for most of these scene dependent and empirical regularization techniques is that the diagonal loading factor selection rules are adjusted to particular scenarios, and usually are not appropriate for other scenarios. Yet, it is envisaged that future network will be characterized by variety of scenarios [8], transmission modes, and receiver configurations. Empirical, simulation based estimates of the interference plus noise covariance matrix may not be suitable for such networks.

Ideally, we would like to have a regularization technique that provides the best performance in any possible scenario. In fact, receiver performance criteria, such as bit error rate (BER), throughput, etc., are indirectly associated with the covariance matrix estimation criteria. Particularly, the maximum likelihood (ML) covariance matrix estimation criterion does not guarantee the best receiver performance. Since direct diagonal loading factor selection based on optimization of receiver performance, for example, minimum BER, is not feasible, we may have to consider interim estimation criteria that could potentially be better suited to the problem in question. Specifically, in this study we consider the Expected Likelihood criterion [9] for estimation/selection of the interference plus noise covariance matrix. The main idea of this approach is based on the fact that the likelihood ratio (LR) of the actual (a priori unknown) covariance matrix is described by distribution that does not depend on this covariance matrix, i.e., it is scenario independent. Therefore, the regularized covariance matrix estimate is threated as appropriate if its likelihood ration is within the support of this distribution, i.e., the EL estimate is "as likely as the actual covariance matrix." Despite having been proven very efficient in radar applications [10], [11] and in wireless communications for establishing the performance bounds in asynchronous semi-blind interference suppression [12], this approach needs thorough investigation in the problem in study. Specifically, two main questions have to be addressed. The first question is to what extend (in terms of scenario parameters) the EL criterion contributes to the optimization of the receiver performance. In other words, how efficient is the regularized estimate if its likelihood is exactly equal to the LR of the actual covariance matrix for the given set of the training data? It is clear that performance of this impractical diagonal loading selection rule, would shed light on the usefulness of this criterion. The second questions is how to select the diagonal loading factor in practical situations. Specifically, for relatively small sample support, the probability density function (p.d.f.) of the "expected", i.e., for the actual covariance matrix, LR is rather broad, and any specific selection within the support of this distribution, for example, median value [9], may have its limits that need to be explored.

The reminder of the pater is organized as follows. Section II describes a problem formulation. Section III formulates the EL-based regularization approach and presents the investigation of its applicability under typical estimation conditions. Section IV contains the simulation results in the LTE downlink scenario including comparison with one empirical solution based on the scene analysis. Section V concludes the paper.

II. Problem formulation and empirical solution example

A conventional IRC formulation for a single stream transmission and K received antenna is as follows [13]:

$$\mathbf{x} = \mathbf{h}s + \mathbf{z}, \tag{1}$$

$$\hat{s} = \hat{\mathbf{w}}^* \mathbf{x}, \tag{2}$$

$$\hat{\mathbf{w}} = \frac{1}{\hat{\mathbf{h}}^* \hat{\mathbf{R}}^{-1} \hat{\mathbf{h}}} \hat{\mathbf{R}}^{-1} \hat{\mathbf{h}}, \tag{3}$$

$$\hat{\mathbf{R}} = L^{-1} \sum_{l=1}^{L} \hat{\mathbf{z}}_l \hat{\mathbf{z}}_l^*, \tag{4}$$

where $\mathbf{x}$, $\mathbf{h}$, and $\mathbf{z}$ are the $(K \times 1)$ vectors of the received signal, propagation channel and interference plus noise signal, s is the desired signal, $\mathbf{w}$ is the $(K \times 1)$ weight vector, $\mathbf{R}$ is the $(K \times K)$ interference plus noise covariance matrix., $(\cdot)^*$ and $\hat{(\cdot)}$ denote complex conjugate transpose and estimation operations, L is the number of samples available for $\hat{\mathbf{R}}$ estimation[1].

Particularly, the IRC adopted for the LTE downlink [2] is normally estimated over one time-frequency resource block (RB), which is the smallest LTE scheduling element, using the cell specific reference signals (CRS) leading to $\hat{\mathbf{z}}_l = \mathbf{x}_l - \hat{\mathbf{h}}_l p_l$, $l = 1, \dots L$, where p_l is the CRS symbol, $\hat{\mathbf{h}}_l$ is the channel estimate corresponding to the lth CRS position, and L is the number of CRSs per RB.

Direct application of the sample covariance matrix (4) in IRC may lead to performance degradation especially for a small number of samples L and in noise limited scenarios, where MRC becomes the optimal receiver. The well known solutions for this problem is a regularization (diagonal loading) and, possibly, IRC/MRC switching. This can be achieved by replacing the sample covariance matrix $\hat{\mathbf{R}}$ in (3) with the regularized matrix

$$\tilde{\mathbf{R}} = (1-\delta)\hat{\mathbf{R}} + \delta\hat{\mathbf{D}}, \tag{5}$$

where $\hat{\mathbf{D}}$ is the diagonal regularization matrix, e.g., $\hat{\mathbf{D}} = \text{diag}(\hat{\mathbf{R}})$, $0 < \delta \leq 1$ is the regularization parameter with $\delta < 1$ corresponding to the IRC diagonal loading and $\delta = 1$ corresponding to the IRC/MRC switching.

As it was mentioned in Section I, different metrics have been proposed for selection of the diagonal loading factor and the switching point. One example of such an empirical scene analysis metric from [7] is

$$\gamma = \frac{\sum_{n \neq m} |\hat{r}_{nm}|}{\text{tr}(\hat{\mathbf{R}})}, \tag{6}$$

where $\hat{r}_{nm}$ is the nmth element of $\hat{\mathbf{R}}$ and $\text{tr}(\mathbf{A})$ is the trace of $\mathbf{A}$. This metric can be used for diagonal loading selection and switching, for example as follows:

$$\delta = \begin{cases} 1 & \gamma \leq 0.9 \\ 0.4 & 0.9 < \gamma \leq 1.25 \\ 0.2 & 1.25 < \gamma \leq 1.5 \\ 0.1 & 1.5 < \gamma \leq 2 \\ 0 & \gamma > 2 \end{cases}, \tag{7}$$

where the corresponding thresholds are found by means of a number of simulation trials to get a reasonable performance illustrated in Section IV in the particular scenario with $K = 4$, $L = 12$, and single interference source.

The problem with such a solution is that it is scenario dependent. Generally, one could repeat simulations in different scenarios and, probably, obtain reasonable thresholds. Future wireless networks will be characterized by variety of scenarios, transmission modes, and receiver configurations. Thus, empirical scenario dependent IRC regularization and IRC/MRC switching solutions may not be suitable for such networks.

III. Uniform EL-based solution

Let us assume that L independent identical distributed Gaussian vectors $\mathbf{z}_l$ are available. Then the likelihood ratio for some estimate $\tilde{\mathbf{R}}$ of the actual covariance matrix $\mathbf{R} = \text{E}(\hat{\mathbf{R}})$ is as follows [14]:

$$\text{LR}(\tilde{\mathbf{R}}) = \frac{\det(\tilde{\mathbf{R}}^{-1}\hat{\mathbf{R}})\exp(K)}{\exp\left[\text{tr}(\tilde{\mathbf{R}}^{-1}\hat{\mathbf{R}})\right]} \leq 1, \tag{8}$$

where $\hat{\mathbf{R}} = L^{-1} \sum_{l=1}^{L} \mathbf{z}_l \mathbf{z}_l^*$ is the sufficient statistics.

Selection $\tilde{\mathbf{R}} = \hat{\mathbf{R}}$ gives LR=1 and $\hat{\mathbf{R}}$ is called the ML estimate of the actual covariance matrix $\mathbf{R}$. The EL approach is based on two main observations [9]:

- All other then $\hat{\mathbf{R}}$ matrices give LR< 1 including the actual matrix: $\text{LR}(\mathbf{R}) < 1$.
- The $\text{LR}(\mathbf{R})$ statistics depend on the dimension of the problem and number of samples, but it does not depend on the actual covariance matrix: $\text{p.d.f}[\text{LR}(\mathbf{R})] = f(L, K) \neq f(\mathbf{R})$.

[1]This IRC formulation is applicable for different communication systems for both uplink and downlink. Our goal in this paper is investigation of the scenario and estimation parameters impact on selection of the diagonal loading factor rather than performance analysis of some particular communication standards.

The second observation means that the corresponding distributions are scenario independent and can be precalculated for any sets of estimation parameters K and L, e.g., using equation (179) in [9] or simple simulations for any $\mathbf{R}$, e.g., $\mathbf{R} = \mathbf{I}$, where $\mathbf{I}$ is the unit matrix. This is illustrated in Fig. 1 for $K = 2, 4, 6$ in 10^6 trials with $L = 12$ $(K \times 1)$ independent Gaussian vectors with unit variance. The corresponding parameters of these distributions are summarized in Table 1, where μ denotes median value, β_1, β_2 define the distribution concentration interval with $\text{Prob}\,[\text{LR}(\mathbf{R}) < \beta_1] = \text{Prob}\,[\text{LR}(\mathbf{R}) > \beta_2] = 5\%$.

Then, the EL estimate of the covariance matrix can be found assuming that on average it should have the same statistical quality as the unknown actual covariance matrix. For the regularized model (5), the EL-based diagonal loading factor can be found from the following equation:

$$\text{LR}\left[(1-\delta)\hat{\mathbf{R}} + \delta\hat{\mathbf{D}}\right] = \nu \in [\beta_1, \beta_2]\,. \tag{9}$$

To illustrate the applicability area of the EL-based diagonal loading selection, let us compare the following two options of ν for the receiver configurations specified in Fig. 1 and Table 1:

- $\nu = \text{LR}(\mathbf{R})$ assuming the known actual covariance matrix;
- $\nu = \mu(K, L)$ assuming that the corresponding median value is selected according to Table 1.

Figs. 2 and 3 present the raw BER simulation results in the idealized scenario with the known channels, QPSK signals, fixed number of pilots $L = 12$, different number of received antennas $K = 2, 4, 6$ and variable SNR, SIR (INR) and number of interference sources assuming the same SIR for all of them. In addition to the IRC results with the EL-based regularization, the conventional MRC ($\delta = 1$) and IRC without regularization ($\delta = 0$) performance is also plotted. Fig. 2 illustrates the IRC/MRC switching scenario and Fig. 3 corresponds to the strong interference limited environment. One can see that:

- In all scenarios, the EL-based solution with $\nu = \mu(K, L)$ demonstrates the desirable behavior. Some performance degradation is observed in the interference limited scenario in Fig. 3, especially for $K = 4, 6$ with $K - 1$ interference sources, where the regularization actually should be switched off ($\delta \approx 0$).
- In all scenarios, the practical $\nu = \mu(K, L)$ and idealistic $\nu = \text{LR}(\mathbf{R})$ targets in (9) give approximately the same results, including some BER performance degradation in Fig. 3.

The second observation means that this degradation represents mismatch between the BER and likelihood criteria as discussed in Section I, which establishes limits (breakdown) of the EL efficiency in the considered problem.

Further adjustments to the regularization selection could be made by means of taking into account some additional a priori information, for example, regarding importance of some special scenarios. Particularly, if the maximum performance needs to be demonstrated in the (test) pure noise limited scenario, then hard IRC/MRC switching may be beneficial. On the contrary, if the highest interference rejection capability is required in the strong interference limited scenarios, then the LR target in (9) could be biased to the upper bound of the corresponding distribution. Then, the modified regularization with hard switching rule could be as follows:

$$\delta_{\text{bias}} = \begin{cases} 1 & \text{LR}\left(\hat{\mathbf{D}}\right) > \beta_1 \\ \delta_2 & \text{LR}\left(\hat{\mathbf{D}}\right) \le \beta_1 \end{cases}, \tag{10}$$

where δ_2 is a root of equation (9) for $\nu = \beta_2(K, L)$.

IV. Simulation results

We simulate the simplified synchronous LTE downlink scenario: 10MHz bandwidth, TM6 transmission mode with QPSK signal and full band allocation for the serving and 1÷3 interfering cells with the same SIR, VA5 propagation channels, conventional two-dimensional CRS-based channel estimation, 1 RB based IRC with $L = 12$ CRSs per RB (only the CRSs overlapping with the data symbols are used for interference plus noise covariance matrix estimation), and $K = 2, 4, 6$ low correlation received antennas. Matlab routine “fminbnd” is used to solve equation (9).

The raw BER performance estimated over 100 subframes for different scenarios and receiver configurations is presented in Figs. 4 and 5 for the MRC, IRC with no regularization, and EL-regularized IRC: δ_{median} according to (9) for $\nu = \mu(K, L)$ and δ_{bias} according to (10). The results for the regularized IRC based on the scene analysis metric (6), (7) designed by means of simulations for $K = 4$ in the scenario in Fig. 4b, are also shown in all simulations. One can see that the empirical regularization demonstrates the appropriate results in Figs. 4b and 5a scenarios, but its performance in other scenarios may be significantly degraded. The thresholds in (7) might have to be adjusted in other scenarios to obtain the desired performance, but it needs more scenario dependent simulations. The EL-based solution demonstrates the desirable behavior in different scenarios without any empirical metrics and thresholds. Some performance degradation in Fig. 5 for 2 and 3 co-channel interference (CCI) sources relates to the EL breakdown as discussed in Section III, which deserves a more detailed study.

Diagonal loading factor distributions are shown in Fig. 6 in the same scenario as in Fig. 4 for $K = 4$. One can see that the biased solution gives higher probability of the hard IRC/MRC switching ($\delta = 1$) in the noise limited scenarios and higher probability of lower regularization factor in the strong interference limited scenario compared to the EL median case. In our simulations, we practically do not see any improvement from hard IRC/MRC switching in Fig. 4, but we keep the EL-based hard switching option in (10) assuming that it may still be useful when the maximum performance needs to be demonstrated in pure noise limited situations.

It is worth emphasizing that even if an additional complexity incurred by the EL based regularization may be a problem for current implementation, especially on the downlink, it still could be useful as a benchmark for some simplified empirical estimators, e.g., those in (6), (7).

V. Conclusions

The Expected Likelihood based rules for selection of the diagonal loading factor have been introduced and simulated in different scenarios. It has been shown that they demonstrate a desirable behavior in wide range of scenarios and receiver configurations, which makes them promising solutions in interference aware receivers design for future wireless networks.

VI. Acknowledgment

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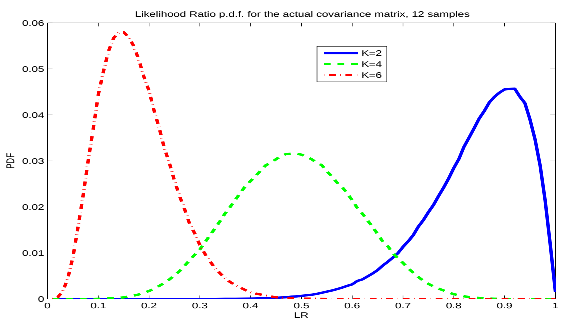

Fig. 1. LR p.d.f. for the actual covariance matrix for $L = 12$ and $K = 2, 4, 6$.

Tabl. 1. Parameters of the LR($\mathbf{R}$) distributions for $L = 12$ and $K = 2, 4, 6$

K	2	4	6
μ	0.86	0.49	0.17
β_1	0.65	0.28	0.07
β_2	0.97	0.68	0.30

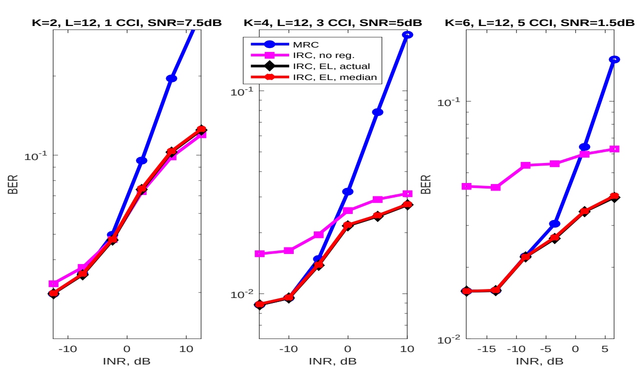

Fig. 2. Raw BER results for the fixed SNR and variable INR in test environment.

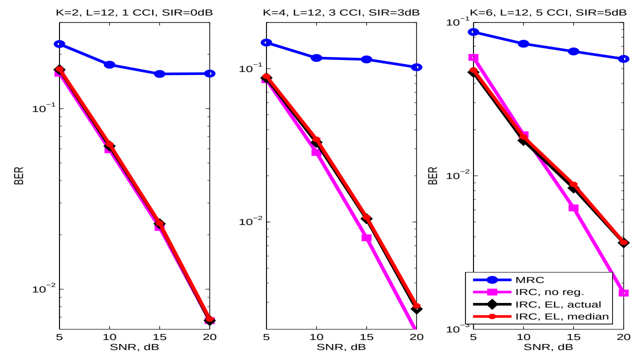

Fig. 3. Raw BER results for the fixed SIR and variable SNR in test environment.

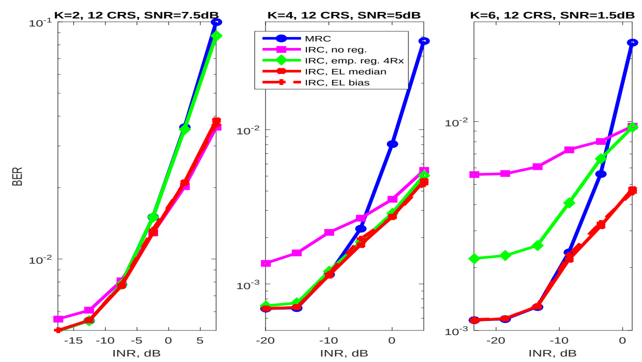

Fig. 4. Raw BER results for the fixed SNR and variable INR in LTE downlink environment.

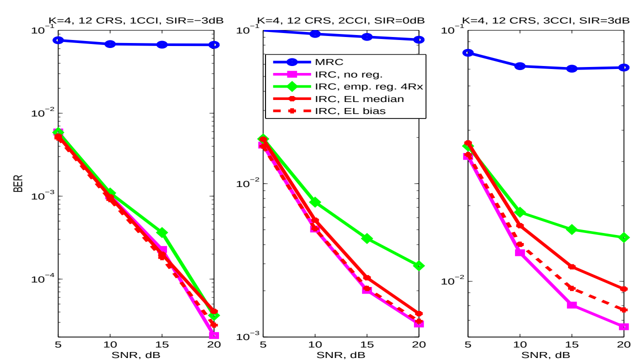

Fig. 5. Raw BER results for the fixed SIR and variable SNR in LTE downlink environment.

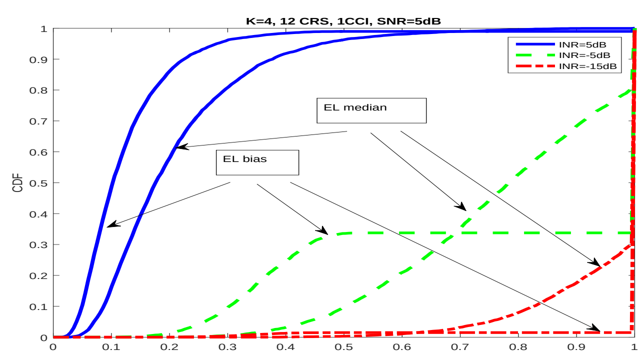

Fig. 6. Diagonal loading factor c.d.f. in Fig. 4 scenario for $K = 4$.

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