EXTENSION OF SEDJOCO AND ITS USE IN A COMBINATION OF MULTICAST AND COORDINATED MULTI-POINT SYSTEMS

Yao Cheng[1], *Arie Yeredor*[2], *and Martin Haardt*[1]

[1] Communications Research Laboratory
Ilmenau University of Technology
P.O. Box 100565, D-98684 Ilmenau, Germany
{y.cheng, martin.haardt}@tu-ilmenau.de

[2] School of Electrical Engineering
Tel-Aviv University
P.O. Box 39040, Tel-Aviv 69978, Israel
arie@eng.tau.ac.il

ABSTRACT

This paper presents a new perspective of beamforming designs in Coordinated Multi-Point (CoMP) downlink systems that are combined with multicast schemes. The beamformer computation is expressed as a joint matrix transformation that can be regarded as an extension of the "Sequentially Drilled" Joint Congruence (SeDJoCo) decomposition. A solution of the proposed joint matrix transformation is devised that takes into account the elimination of the multi-user interference as well as the maximization of the desired signal components. Therefore, it leads to a very effective semi-algebraic solution of beamforming designs for the multicast CoMP downlink, which is evident in the numerical simulations.

Index Terms— joint matrix transformations, beamforming, Coordinated Multi-Point, multicast

1. INTRODUCTION

Joint matrix transformations are a crucial linear algebraic tool in various signal processing fields such as Blind Source Separation (BSS), Independent Vector Analysis (IVA), and beamforming for multi-user Multiple-Input Multiple-Output (MIMO) systems [1], [2], [3], [4]. Recently, a joint matrix transformation termed "Sequentially Drilled" Joint Congruence (SeDJoCo) decomposition has been proposed [5]. It finds applications in the context of both BSS and Coordinated Beamforming (CBF) in the multi-user MIMO downlink. The SeDJoCo transformation is defined as follows: Given a set of K (conjugate-)symmetric "target matrices" of size $K \times K$ denoted by $\boldsymbol{Q}_k$ for $k = 1, \ldots, K$, find a transformation matrix $\boldsymbol{B} \in \mathbb{C}^{K \times K}$ such that the k-th column (and the k-th row) of the k-th transformed matrix

$$\boldsymbol{D}_k = \boldsymbol{B} \cdot \boldsymbol{Q}_k \cdot \boldsymbol{B}^{\mathrm{H}} \in \mathbb{C}^{K \times K},\ k = 1, \ldots, K \tag{1}$$

is "drilled", i.e., all elements in the k-th column and row are zeros, except for the (k, k)-th element. This "drilled" structure of SeDJoCo is depicted in Fig. 1. In the multi-user MIMO downlink with space division multiple access, one base station equipped with K antennas transmits to K users each having K receive antennas. Assuming maximum ratio combining is employed at each user terminal, the precoding vectors for the K users denoted by $\boldsymbol{f}_k \in \mathbb{C}^K$ $(k = 1, \ldots, K)$ are computed such that the multi-user interference is mitigated. In this context, the target matrices and the transformation matrix are constructed as $\boldsymbol{Q}_k = \boldsymbol{H}_k^{\mathrm{H}} \cdot \boldsymbol{H}_k$ and $\boldsymbol{B}^{\mathrm{H}} = \begin{bmatrix} \boldsymbol{f}_1 & \cdots & \boldsymbol{f}_K \end{bmatrix}$, respectively. Here $\boldsymbol{H}_k \in \mathbb{C}^{K \times K}$ represents the channel matrix between the base station and the k-th user. Two solutions of SeDJoCo have been proposed in [5].

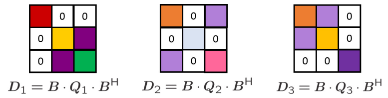

Fig. 1. Illustration of the "drilled" structure in SeDJoCo for $K = 3$

In this contribution, we focus on a more sophisticated Coordinated Multi-Point (CoMP) downlink system combined with multicast schemes (cf. Fig. 2 for a three-cell example). A CBF scheme called Extended FlexCoBF has been developed for a similar scenario in [6], where the transmit and the receive beamformers are computed jointly and iteratively. In each iteration, Extended FlexCoBF requires the exchange of the precoded channels of the cell edge users (e.g., User 3 in Fig. 2) among adjacent cells, resulting in transmission delays and a signaling overhead. Motivated by the intriguing link between SeDJoCo and CBF in the multi-user MIMO downlink, we propose a new joint matrix transformation that helps accomplish the challenging CBF task in multicast CoMP downlink settings. The solution of the proposed joint matrix transformation is inspired by the joint multi-user MIMO system (JMMS) [7] but is largely extended from it. Via numerical simulations, we show that the proposed joint matrix transformation, as an extension of SeDJoCo, is a very effective and efficient semi-algebraic tool for beamforming in multicast CoMP downlink systems. It leads to a comparable sum rate performance as Extended FlexCoBF but requires a significantly smaller signaling overhead.

2. PROBLEM FORMULATION

Consider a multicast CoMP downlink setting where the users are distinguished into cell interior users and cell edge users. An example of a three-cell scenario is illustrated in Fig. 2. The base stations are each equipped with M_{T} transmit antennas. After user scheduling, each base station serves three users. All user nodes have the same number of receive antennas denoted as M_{R}, and a single data stream is sent to each user. Using a cell-specific multicast scheme, the same signal is transmitted to the user with the same index in all the three cells, i.e., denoting the signal for the k-th user as x_k $(k = 1, 2, 3)$, User k in Cell 1, Cell 2, and Cell 3 all receive x_k from their corresponding base stations, respectively. User k in all three cells belong to one multicast group [8], [9], [10], and, for instance, they could be subscribers of the same TV programs. Here User 1 and User 2 in each cell are treated as cell interior users. Both the inter-cell interference and the desired signals from the adjacent cells are assumed to be negligible due to the heavy path loss. Therefore, the cell interior

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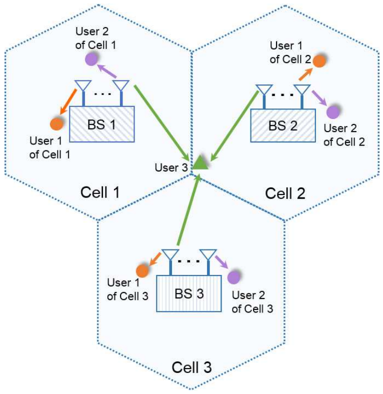

Fig. 2. A three-cell multicast CoMP downlink scenario where two cell interior users and one cell edge user are served in each cell

users only receive signals from their own base station and suffer only from the intra-cell interference, i.e., the multi-user interference as in the single-cell multi-user MIMO downlink scenarios. On the other hand, User 3 is a cell edge user and is affected by both the intra-cell interference and the inter-cell interference. The fact that each cell edge user receives the desired signals from the base stations in the adjacent cells is exploited. Joint transmissions of the neighboring cells to the cell edge user are considered to combat the interference and to deal with the greater path loss compared to the cell interior users.

To avoid cumbersome notations, we take this example to present the data model and the solution in Section 3. The extension to a more general case is addressed at the end of Section 3. The received signal of User m as a cell interior user ($m = 1, 2$) in Cell n ($n = 1, 2, 3$) is expressed as

$$\boldsymbol{r}_{m,n} = \boldsymbol{H}_{m,n} \cdot \boldsymbol{f}_{m,n} \cdot x_m + \boldsymbol{H}_{m,n} \cdot \sum_{\ell \neq m} \boldsymbol{f}_{\ell,n} \cdot x_\ell + \boldsymbol{n}_{m,n}, \quad (2)$$

where $\boldsymbol{H}_{m,n} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{T}}$ denotes the channel matrix from the base station in Cell n to User m, x_m represents the signal for User m, and $\boldsymbol{f}_{\ell,n} \in \mathbb{C}^{M_\mathrm{T}}$ is the precoding vector computed at the base station of Cell n for User ℓ ($\ell = 1, 2, 3$). In this example, $M_\mathrm{T} = M_\mathrm{R} = 3$ and is equal to the number of users served by each cell. It can be seen that for a cell interior user, assuming that the inter-cell interference is negligible, the transmission from the base station in its own cell is the same as in a single-cell multi-user MIMO downlink system. On the other hand, the received signal of User 3 is written as

$$\boldsymbol{r}_3 = \sum_{n=1,2,3} \boldsymbol{H}_{3,n} \cdot \boldsymbol{f}_{3,n} \cdot x_3 + \sum_{n=1,2,3} (\boldsymbol{H}_{3,n} \cdot \sum_{\ell=1,2} \boldsymbol{f}_{\ell,n} \cdot x_\ell) + \boldsymbol{n}_3.$$

Based on the maximum ratio combining criterion, we define the receive combining vectors as follows

$$\boldsymbol{w}_{m,n} = \boldsymbol{H}_{m,n} \cdot \boldsymbol{f}_{m,n}, \ m = 1, 2, \ n = 1, 2, 3 \quad (3)$$

$$\boldsymbol{w}_3 = \sum_{n=1,2,3} \boldsymbol{H}_{3,n} \cdot \boldsymbol{f}_{3,n}. \quad (4)$$

For a cell interior user, User m ($m = 1, 2$) in Cell n ($n = 1, 2, 3$), achieving zero interference via the design of the beamforming vectors requires $\boldsymbol{f}_{m,n}^\mathrm{H} \cdot \boldsymbol{H}_{m,n}^\mathrm{H} \cdot \boldsymbol{H}_{m,n} \cdot \boldsymbol{f}_{\ell,n} = 0$, where $\ell \neq m$. For User 3, as a cell edge user, the desired component can be expressed as

$$\sum_{n=1,2,3} \boldsymbol{f}_{3,n}^\mathrm{H} \cdot \boldsymbol{H}_{3,n}^\mathrm{H} \cdot (\boldsymbol{H}_{3,1} \cdot \boldsymbol{f}_{3,1} + \boldsymbol{H}_{3,2} \cdot \boldsymbol{f}_{3,2} + \boldsymbol{H}_{3,3} \cdot \boldsymbol{f}_{3,3}) \cdot x_3.$$

To guarantee zero interference for User 3, the following has to be fulfilled for $\ell = 1, 2$

$$\sum_{n=1,2,3} \boldsymbol{f}_{3,n}^\mathrm{H} \cdot \boldsymbol{H}_{3,n}^\mathrm{H} \cdot (\boldsymbol{H}_{3,1} \cdot \boldsymbol{f}_{\ell,1} + \boldsymbol{H}_{3,2} \cdot \boldsymbol{f}_{\ell,2} + \boldsymbol{H}_{3,3} \cdot \boldsymbol{f}_{\ell,3}) = 0.$$

To formulate the coordinated beamforming task described above into a joint matrix transformation problem, we define transformation matrices $\boldsymbol{B}_n$ ($n = 1, 2, 3$) with respect to Cell n

$$\boldsymbol{B}_n^\mathrm{H} = \begin{bmatrix} \boldsymbol{f}_{1,n} & \boldsymbol{f}_{2,n} & \boldsymbol{f}_{3,n} \end{bmatrix} \in \mathbb{C}^{3 \times 3}. \quad (5)$$

The target matrices can be distinguished into two sets, i.e.,

$$\boldsymbol{Q}_m^{(n,n)} = \boldsymbol{H}_{m,n}^\mathrm{H} \cdot \boldsymbol{H}_{m,n}, \ m = 1, 2, 3, \ n = 1, 2, 3$$

that are all Hermitian matrices and

$$\boldsymbol{Q}_3^{(1,2)} = \boldsymbol{H}_{3,1}^\mathrm{H} \cdot \boldsymbol{H}_{3,2}, \ \boldsymbol{Q}_3^{(1,3)} = \boldsymbol{H}_{3,1}^\mathrm{H} \cdot \boldsymbol{H}_{3,3}, \ \boldsymbol{Q}_3^{(2,3)} = \boldsymbol{H}_{3,2}^\mathrm{H} \cdot \boldsymbol{H}_{3,3}.$$

The resulting joint matrix transformation is illustrated in Fig. 3.

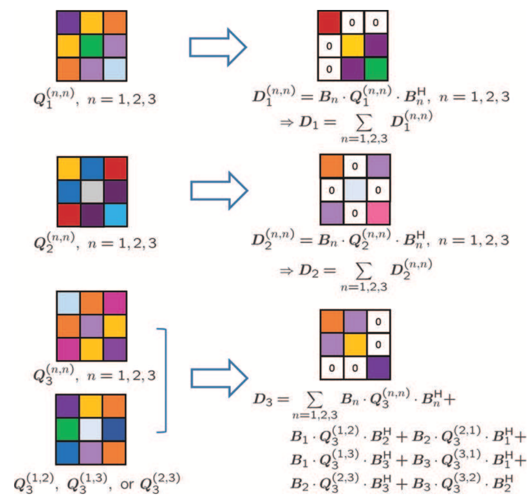

Fig. 3. Illustration of the proposed joint matrix transformation

3. SOLUTION OF THE PROPOSED MATRIX TRANSFORMATION

The aforementioned joint matrix transformation can be reformulated into ($\boldsymbol{D}_1$, $\boldsymbol{D}_2$, and $\boldsymbol{D}_3$ are also illustrated in Fig. 3)

$$\boldsymbol{D}_1 = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{Q}_1^{(1,1)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_1^{(2,2)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q}_1^{(3,3)} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{B}_1^\mathrm{H} \\ \boldsymbol{B}_2^\mathrm{H} \\ \boldsymbol{B}_3^\mathrm{H} \end{bmatrix}$$

$$\boldsymbol{D}_2 = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{Q}_2^{(1,1)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_2^{(2,2)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q}_2^{(3,3)} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{B}_1^\mathrm{H} \\ \boldsymbol{B}_2^\mathrm{H} \\ \boldsymbol{B}_3^\mathrm{H} \end{bmatrix}$$

$$\boldsymbol{D}_3 = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{Q}_3^{(1,1)} & \boldsymbol{Q}_3^{(1,2)} & \boldsymbol{Q}_3^{(1,3)} \\ \boldsymbol{Q}_3^{(2,1)} & \boldsymbol{Q}_3^{(2,2)} & \boldsymbol{Q}_3^{(2,3)} \\ \boldsymbol{Q}_3^{(3,1)} & \boldsymbol{Q}_3^{(3,2)} & \boldsymbol{Q}_3^{(3,3)} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{B}_1^\mathrm{H} \\ \boldsymbol{B}_2^\mathrm{H} \\ \boldsymbol{B}_3^\mathrm{H} \end{bmatrix},$$

where the k-th row and column of $\boldsymbol{D}_k$ ($k = 1, 2, 3$) are "drilled", i.e., except for the (k, k)-th element, the rest of the elements are zeros.

Let us define $\boldsymbol{T}_{i,j}^{(n)} \in \mathbb{C}^{3\times 3}$ ($i = 1, 2, i < j, n = 1, 2, 3$), such that

$$\boldsymbol{T}_{i,j}^{(n)}(i,i) = p_n,\ \boldsymbol{T}_{i,j}^{(n)}(i,j) = q_n, \tag{6}$$

$$\boldsymbol{T}_{i,j}^{(n)}(j,i) = r_n,\ \boldsymbol{T}_{i,j}^{(n)}(j,j) = s_n, \tag{7}$$

whereas the rest of the diagonal elements of $\boldsymbol{T}_{i,j}^{(n)}$ are ones, and the rest of the off-diagonal elements are zeros. Here for a matrix $\boldsymbol{A}$ we use $\boldsymbol{A}(i,j)$ to denote its (i,j)-th entry. Based on successive Jacobi rotations [11], the transformation matrices and the target matrices are updated via

$$\boldsymbol{B}_n \leftarrow \boldsymbol{T}_{i,j}^{(n)\mathrm{H}} \cdot \boldsymbol{B}_n,\ n = 1, 2, 3 \tag{8}$$

$$\boldsymbol{Q}_k^{(n,n)} \leftarrow \boldsymbol{T}_{i,j}^{(n)\mathrm{H}} \cdot \boldsymbol{Q}_k^{(n,n)} \cdot \boldsymbol{T}_{i,j}^{(n)},\ k = 1, 2, 3,\ n = 1, 2, 3 \tag{9}$$

$$\boldsymbol{Q}_3^{(n_1,n_2)} \leftarrow \boldsymbol{T}_{i,j}^{(n_1)\mathrm{H}} \cdot \boldsymbol{Q}_3^{(n_1,n_2)} \cdot \boldsymbol{T}_{i,j}^{(n_2)},\ (n_1, n_2) = (1,2), (1,3), (2,3). \tag{10}$$

Notice that the updated $\boldsymbol{Q}_k^{(n,n)}$ and $\boldsymbol{Q}_3^{(n_1,n_2)}$ resemble their old versions before the updating in (9) and (10) except for the i-th and j-th columns and rows. Then instead of the joint matrix transformation defined at the very beginning of this section, the following one with a reduced dimension is considered

$$\boldsymbol{D}_k' = \begin{bmatrix} p_1 & q_1 \\ r_1 & s_1 \\ p_2 & q_2 \\ r_2 & s_2 \\ p_3 & q_3 \\ r_3 & s_3 \end{bmatrix}^{\mathrm{H}} \cdot \boldsymbol{Q}_k' \cdot \begin{bmatrix} p_1 & q_1 \\ r_1 & s_1 \\ p_2 & q_2 \\ r_2 & s_2 \\ p_3 & q_3 \\ r_3 & s_3 \end{bmatrix} \in \mathbb{C}^{2\times 2}, \tag{11}$$

where for $k = 1, 2$

$$\boldsymbol{Q}_k' = \begin{bmatrix} \widetilde{\boldsymbol{Q}}_k^{(1,1)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \widetilde{\boldsymbol{Q}}_k^{(2,2)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \widetilde{\boldsymbol{Q}}_k^{(3,3)} \end{bmatrix} \in \mathbb{C}^{6\times 6} \tag{12}$$

with

$$\widetilde{\boldsymbol{Q}}_k^{(n,n)} = \begin{bmatrix} \boldsymbol{Q}_k^{(n,n)}(i,i) & \boldsymbol{Q}_k^{(n,n)}(i,j) \\ \boldsymbol{Q}_k^{(n,n)}(j,i) & \boldsymbol{Q}_k^{(n,n)}(j,j) \end{bmatrix},\ n = 1, 2, 3, \tag{13}$$

and $\boldsymbol{Q}_3' = \begin{bmatrix} \widetilde{\boldsymbol{Q}}_3^{(1,1)} & \widetilde{\boldsymbol{Q}}_3^{(1,2)} & \widetilde{\boldsymbol{Q}}_3^{(1,3)} \\ \widetilde{\boldsymbol{Q}}_3^{(2,1)} & \widetilde{\boldsymbol{Q}}_3^{(2,2)} & \widetilde{\boldsymbol{Q}}_3^{(2,3)} \\ \widetilde{\boldsymbol{Q}}_3^{(3,1)} & \widetilde{\boldsymbol{Q}}_3^{(3,2)} & \widetilde{\boldsymbol{Q}}_3^{(3,3)} \end{bmatrix} \in \mathbb{C}^{6\times 6}$. Each two-by-two element matrix of $\boldsymbol{Q}_3'$ is defined similarly as (13). In addition, $\boldsymbol{D}_k'$ in (11) is linked to $\boldsymbol{D}_k$ ($k = 1, 2, 3$) via

$$\boldsymbol{D}_k' = \begin{bmatrix} \boldsymbol{D}_k(i,i) & \boldsymbol{D}_k(i,j) \\ \boldsymbol{D}_k(j,i) & \boldsymbol{D}_k(j,j) \end{bmatrix} \in \mathbb{C}^{2\times 2}. \tag{14}$$

Note that $\boldsymbol{D}_k(k,k)$ is the desired component, and the rest of the elements on the k-th row as well as the k-th column should be nulled, where $k = 1, 2, \ldots, K$. Therefore, for a certain pair of (i,j), only $\boldsymbol{D}_i'$ and $\boldsymbol{D}_j'$ (i.e., $k = i$ or $k = j$) are relevant. It is then desired to find $\boldsymbol{T}_{i,j}^{(n)}$, i.e., p_n, q_n, r_n, and s_n ($n = 1, 2, 3$) to achieve the nulling of $\boldsymbol{D}_i(i,j)$ and $\boldsymbol{D}_j(i,j)$ as well as the maximization of $|\boldsymbol{D}_i(i,i)|$ and $|\boldsymbol{D}_j(j,j)|$. This is equivalent to

$$\max_{\substack{p_n, q_n, r_n, s_n \\ (n=1,2,3)}} \left(\left|\boldsymbol{D}_i'(1,1)\right|^2 + \left|\boldsymbol{D}_j'(2,2)\right|^2 \right), \tag{15}$$

and in the meantime minimizing the off-diagonal elements of $\boldsymbol{D}_i' \in \mathbb{C}^{2\times 2}$ and $\boldsymbol{D}_j' \in \mathbb{C}^{2\times 2}$.

For $i = 1$ and $j = 2$ (both "i" and "j" correspond to the indices of cell interior users), $\boldsymbol{T}_{i,j}^{(n)}$ ($n = 1, 2, 3$) are calculated separately, each similar to the case of the single-cell multi-user MIMO downlink in [7]. It is due to the fact that the cell interior users are only affected by the intra-cell interference, and the transmission from each base station to its corresponding cell interior users resembles that in a single-cell multi-user MIMO downlink setting. This also leads to the block diagonal structure of $\boldsymbol{Q}_1'$ and $\boldsymbol{Q}_2'$ in (12). The desired $\boldsymbol{T}_{i,j}^{(n)}$ is obtained by equivalently solving the channel diagonalization problem of a two-user system (with respect to $\boldsymbol{D}_i'$ and $\boldsymbol{D}_j'$) where the maximization of the signal-to-interference ratio is achieved [7]. First compute the eigenvalue decomposition (EVD) of $\widetilde{\boldsymbol{Q}}_j^{(n,n)} \in \mathbb{C}^{2\times 2}$

$$\widetilde{\boldsymbol{Q}}_j^{(n,n)} = \boldsymbol{V}_j^{(n,n)} \cdot \boldsymbol{\Lambda}_j^{(n,n)} \cdot \boldsymbol{V}_j^{(n,n)\mathrm{H}}, \tag{16}$$

and then define $\boldsymbol{W}_j^{(n,n)} = \boldsymbol{V}_j^{(n,n)} \cdot \boldsymbol{\Lambda}_j^{(n,n)^{-\frac{1}{2}}}$ as a "whitening" matrix. Further calculate the EVD of $\boldsymbol{W}_j^{(n,n)\mathrm{H}} \cdot \widetilde{\boldsymbol{Q}}_i^{(n,n)} \cdot \boldsymbol{W}_j^{(n,n)} \in \mathbb{C}^{2\times 2}$ and obtain $\boldsymbol{E}_i^{(n,n)}$ containing the eigenvectors that correspond to its eigenvalues sorted in a descending order. Finally, p_n, q_n, r_n, s_n are obtained via

$$\begin{bmatrix} p_n & q_n \\ r_n & s_n \end{bmatrix} = \boldsymbol{W}_j^{(n,n)} \cdot \boldsymbol{E}_i^{(n,n)} \cdot \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}, \tag{17}$$

where a_1 and a_2 are chosen to normalize the two columns of $\boldsymbol{W}_j^{(n,n)} \cdot \boldsymbol{E}_i^{(n,n)}$, respectively.

On the other hand, if $i = 1$ or 2 and $j = 3$ ("i" and "j" correspond to the indices of a cell interior user and a cell edge user, respectively), we follow the same philosophy described above for the case where $i = 1$ and $j = 2$ and propose a way of obtaining $\boldsymbol{T}_{i,j}^{(n)}$ ($n = 1, 2, 3$) jointly. We first compute the EVD of $\widetilde{\boldsymbol{Q}}_i^{(n,n)} \in \mathbb{C}^{2\times 2}$ for $n = 1, 2, 3$, respectively, and obtain $\boldsymbol{V}_i^{(n,n)} \in \mathbb{C}^{2\times 2}$ that contains the eigenvectors of $\widetilde{\boldsymbol{Q}}_i^{(n,n)}$. The eigenvectors correspond to the eigenvalues sorted in a descending order. Define

$$\boldsymbol{W}_i' = \begin{bmatrix} \boldsymbol{V}_i^{(1,1)} \\ \boldsymbol{V}_i^{(2,2)} \\ \boldsymbol{V}_i^{(3,3)} \end{bmatrix} \in \mathbb{C}^{6\times 2}, \tag{18}$$

and compute $\boldsymbol{\Sigma}_i = \boldsymbol{W}_i'^{\mathrm{H}} \cdot \boldsymbol{Q}_i' \cdot \boldsymbol{W}_i' \in \mathbb{C}^{2\times 2}$. Here $\boldsymbol{\Sigma}_i$ is a diagonal matrix whose diagonal elements are actually the sum of the corresponding eigenvalues of $\widetilde{\boldsymbol{Q}}_i^{(n,n)} \in \mathbb{C}^{2\times 2}$ ($n = 1, 2, 3$). The whitening matrix $\boldsymbol{W}_i$ is obtained as $\boldsymbol{W}_i = \boldsymbol{W}_i' \cdot \boldsymbol{\Sigma}_i^{-\frac{1}{2}} \in \mathbb{C}^{6\times 2}$. Then, we further calculate the EVD of $\boldsymbol{W}_i^{\mathrm{H}} \cdot \boldsymbol{Q}_3' \cdot \boldsymbol{W}_i \in \mathbb{C}^{2\times 2}$ and obtain $\boldsymbol{E}_3 \in \mathbb{C}^{2\times 2}$ that contains the eigenvectors (the corresponding eigenvalues are also sorted in a descending order). Finally, the unknown elements in $\boldsymbol{T}_{i,j}^{(n)}$ are computed as

$$\begin{bmatrix} p_1 & q_1 \\ r_1 & s_1 \\ p_2 & q_2 \\ r_2 & s_2 \\ p_3 & q_3 \\ r_3 & s_3 \end{bmatrix} = \boldsymbol{W}_i \cdot \boldsymbol{E}_3 \cdot \begin{bmatrix} 0 & a_1 \\ a_2 & 0 \end{bmatrix}. \tag{19}$$

Note that the two columns of $\boldsymbol{W}_i \cdot \boldsymbol{E}_3$ should be normalized such that the two-norm of each column is $\sqrt{3}$, as the number of cells considered in this example is three. In addition. the permutation of the columns of $\boldsymbol{W}_i \cdot \boldsymbol{E}_3$ in (19) results from the roles of $\boldsymbol{Q}'_i$ and $\boldsymbol{Q}'_j$ in the computation of $\boldsymbol{T}_{i,j}^{(n)}$ as compared to (17).

The transformation matrices are initialized as $\boldsymbol{B}_n = \boldsymbol{I}_3$ for $n = 1, 2, 3$, respectively. At the end of each sweep, the transformation matrices and the target matrices are updated via (8), (9), and (10). For the convergence, we can track either the residual interference or the following term

$$\epsilon = \sum_{n=1,2,3} |p_n - 1|^2 + |q_n|^2 + |r_n|^2 + |s_n - 1|^2. \tag{20}$$

When the convergence is achieved, ϵ is close to zero, and $\boldsymbol{T}_{i,j}^{(n)}$ ($n = 1, 2, 3$) are close to identity matrices.

Remarks:

(i) The proposed joint matrix transformation scheme is especially effective for the case where there is a single cell edge user, as shown in Section 4. In such scenarios, it provides a performance comparable with that of a CBF algorithm. Due to the limited spatial capacity of a base station, user scheduling is usually conducted before precoding. A single cell edge user can be scheduled for each transmission.

(ii) When there are more than one cell edge user, for a certain pair of indices (i, j), there exists a third case where both "i" and "j" correspond to the indices of cell edge users. To obtain $\boldsymbol{T}_{i,j}^{(n)}$ ($n = 1, \ldots, N$ with N denoting the number of cells), a whitening matrix is computed as $\boldsymbol{W}_j = \boldsymbol{V}_j \cdot \boldsymbol{\Lambda}_j^{-\frac{1}{2}}$, where the columns of $\boldsymbol{V}_j$ are the eigenvectors of $\boldsymbol{Q}'_j$, and $\boldsymbol{\Lambda}_j$ is a diagonal matrix whose diagonal elements are the corresponding eigenvalues. Finally, the stacking of $\begin{bmatrix} p_n & q_n \\ r_n & s_n \end{bmatrix}$ for $n = 1, \ldots, N$ (similar as the left-hand side of (19) for $N = 3$) is obtained as $\boldsymbol{W}_j \cdot \boldsymbol{E}_i \cdot \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$, where $\boldsymbol{E}_i$ contains the two eigenvectors of $\boldsymbol{W}_j^{\mathrm{H}} \cdot \boldsymbol{Q}'_i \cdot \boldsymbol{W}_j$ that correspond to its largest and smallest eigenvalues.

4. SIMULATION RESULTS AND CONCLUSIONS

In this section, we evaluate the performance of the proposed joint matrix transformation. Two multicast CoMP downlink settings are considered. The first is a three-cell case similar to the example scenario depicted in Fig. 2, while the number of cell interior users in each cell is three, leading to the fact that there are ten users in total. In addition, a six-user two-cell scenario is investigated, where each cell serves two cell interior users, and there are two cell edge users. In both scenarios, $M_{\mathrm{T}} = M_{\mathrm{R}} = 4$. The path loss of the cell edge user is assumed to be 10 times larger than that for the cell interior users [6]. Here the SNR is defined as $P_{\mathrm{T}}/\sigma_n^2$, where P_{T} represents the transmit power of each cell, and σ_n^2 denotes the noise variance. The global channel state information is available at all base stations. For each setting, both the Extended FlexCoBF algorithm [6] and the proposed extension of SeDJoCo are employed to compute the beamformer. The Frobenius norm square of $\boldsymbol{B}_n$ ($n = 1, \ldots, N$) is scaled to P_{T} to fulfill the total transmit power constraint and to guarantee a fair comparison of these two methods. The resulting comparison of the sum rate performance of these two schemes are presented in Fig. 4. It can be seen that for the scenario with a single cell edge user, the extension of SeDJoCo achieves a similar performance as Extended FlexCoBF, indicating that this new joint matrix transformation is a very promising linear algebraic tool for beamforming in the multicast CoMP downlink system. On the other hand, when there are two cell edge users, Extended FlexCoBF slightly outperforms the

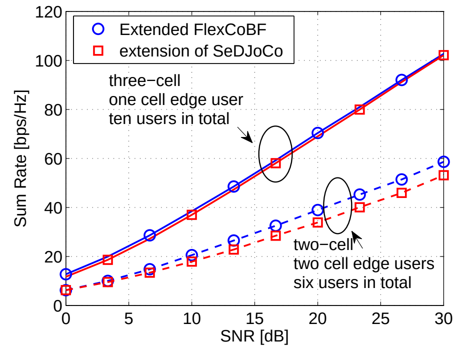

Fig. 4. Performance comparison between Extended FlexCoBF and the proposed joint matrix transformation (averaged over 500 trials)

extension of SeDJoCo. This observation leads to the conjecture that when both "i" and "j" in a certain sweep correspond to the indices of cell edge users, there might exist better ways of computing $\boldsymbol{T}_{i,j}^{(n)}$ ($n = 1, \ldots, N$) such that the performance of the proposed joint matrix transformation is improved.

Fig. 5 shows the convergence behavior of the proposed extension of SeDJoCo. For each multicast CoMP downlink setting described

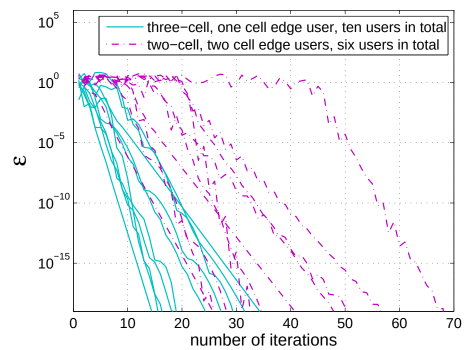

Fig. 5. Convergence behavior with respect to ϵ of the proposed joint matrix transformation (10 independent trials)

above, 10 independent trials have been simulated. We observe that especially in the three-cell case with a single cell edge user, the convergence of the extension of SeDJoCo is fast. To obtain the sum rate results shown in Fig. 4, it is enough to set a threshold of ϵ as 10^{-5}, i.e., once ϵ falls below 10^{-5}, no further iterations are needed. Note that as a CBF scheme, Extended FlexCoBF computes the transmit and receive beamformers jointly and iteratively [6]. For example, with a threshold for the residual interference set to 10^{-5} [6] and the maximum number of iterations set to 100, the mean number of iterations required by Extended FlexCoBF in the three-cell case is around 47. Due to limited space, detailed complexity comparison between these two schemes is not included in this paper. Still, it is worth mentioning that for each sweep (each pair of (i, j)), the extension of SeDJoCo considers joint matrix transformations with a reduced dimension (cf. (11)). Consequently, the increase of complexity caused by the increased number of cells or users is limited. In addition, for each iteration of Extended FlexCoBF, to update the receive beamforming vector for each cell edge user (e.g., (4) for User 3 in the example scenario depicted in Fig. 2), the base stations of neighboring cells have to exchange the precoded channels. By contrast, such an information exchange is not required for the proposed extension of SeDJoCo, leading to a smaller signaling overhead.

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