DOA ESTIMATION OF CLOSELY-SPACED AND SPECTRALLY-OVERLAPPED SOURCES BASED ON TIME-FREQUENCY SPARSE REPRESENTATION

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ABSTRACT

For the purpose of dealing with closely-spaced and spectrally-overlapped sources, a direction-of-arrival (DOA) estimation algorithm based on time-frequency (TF) sparse representation is proposed. Firstly a short-time Fourier transform (STFT) based single-source TF points selection method is briefly introduced. On this basis, we extract the STFT values corresponding to the single-source TF points of each source from the STFT values of array outputs to construct received data matrix. We then enforce sparsity by imposing the ℓ_1 norm based penalties on TF sparse signal representation and solve the optimization problem. Finally, the DOA of each source can be estimated from over-complete basis according to the peak of TF sparse signal vector. Simulation results demonstrate the advantages of the proposed algorithm in terms of dealing with closely-spaced and spectrally-overlapped sources and the flexibility in underdetermined cases.[1]

Index Terms— Closely-spaced and spectrally-overlapped sources, TF sparse representation, single-source TF points, DOA estimation

1. INTRODUCTION

Direction-of-arrival (DOA) estimation is a very important branch in array signal processing with many applications, involving radar, sonar, communication and biomedicine [1]. With the extensive research of DOA estimation methods, the resolution and precision have greatly improved. Subspace-based method MUSIC [1, 2], is considered effective to offer high-resolution on DOA estimation by using the orthogonality of the signal and noise subspaces obtained from the eigen-decomposition of correlation matrix. Nevertheless, MUSIC suffers from the ability to solve closely-spaced sources in low signal-to-noise ratio (SNR). Another kind of algorithms based on sparse signal representation [3, 4, 5, 6], can better deal with closely-spaced sources with a sharp spatial spectrum by introducing an over-complete basis, while it still cannot achieve desirable performance when the sources get very close. It is worth noting that, for all the above methods, the number of array sensors must be larger than the number of estimated sources.

In order to ease the above limitations, time-frequency (TF) analysis [7, 8] is introduced in the array signal processing [9, 10]. In [11], Wiger-Ville distribution of array output is firstly discovered to have a similar structure to the correlation matrix used in subspace-based methods, subsequently, spatial time-frequency distribution (STFD) is proposed. With STFDs [12, 13], TF-MUSIC with true instantaneous frequency (IF) information [14] is generated in [15] through substituting STFD matrices for correlation matrices. TF-MUSIC has greatly improved robustness to noise due to the property of localizing the signal around its IF while spreading the noise over the entire TF domain. More importantly, with the ability to separately construct STFD matrix for each source, TF-MUSIC can well resolve closely-spaced sources. Further, the use of less number of sensors is allowed in underdetermined cases. However, due to the inevitable cross-source TF points [16] in STFD matrices, TF-MUSIC has the limitation to resolve sources whose spectral contents are highly overlapped in TF domain. In addition, the required accurate IF information of each source which is assumed known in most of previous studies, may be unavailable in practice and hard to estimate. It can be seen from [10, 17, 18, 19], auto-source TF points are significant for precise DOA estimation, eliminating cross interference terms is the key to obtain an improved DOA estimation. Hence, a selection of appropriate auto-source TF points is crucial.

The main focus of this paper is that a TF sparse representation based DOA estimation algorithm aiming at solving closely-spaced and spectrally-overlapped sources is proposed. As a support of the proposed algorithm, a single-source TF points selection based on short-time Fourier transform (STFT) [20] is firstly introduced to eliminate the effects of cross-source and noise TF points. In the proposed algorithm, we separately consider the STFT values corresponding to single-source TF points of each source as received data matrix in TF domain, and formulate this problem in a sparse representation framework. For the sake of reducing computational complexity, we use the singular value decomposition (SVD) of the received data matrix. Here, ℓ_1 norm is adopted to enforce sparsity of the TF sparse representation and this leads to an optimization problem, which can be solved in a

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second-order cone (SOC) programming framework [4]. Once the TF sparse signal vector is computed, the DOA of each source can be estimated from over-complete basis. Simulation results have shown the efficiency and accuracy of the proposed algorithm.

2. DATA MODEL

Consider K narrowband far-field frequency-modulated (FM) sources, impinging on an uniform linear array with M omni-directional sensors from directions $\theta_1, \theta_2, \cdots, \theta_K$. The array output is represented as:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{d}(t) + \mathbf{n}(t), \tag{1}$$

$M \times 1$ array output vector $\mathbf{x}(t) = [x_1(t), \cdots, x_M(t)]^T$, signal vector $\mathbf{d}(t)$ is:

$$\begin{aligned}\mathbf{d}(t) &= [d_1(t),\ d_2(t),\ \cdots,\ d_K(t)]^T \\ &= [D_1 e^{j\psi_1(t)},\ D_2 e^{j\psi_2(t)},\ \cdots,\ D_K e^{j\psi_K(t)}]^T,\end{aligned} \tag{2}$$

where D_i and $\psi_i(t)$ respectively denote constant amplitude and time-varying phase of the i^{th} source. For one snapshot, $d_i(t)$ has an IF $f_i(t) = (1/2\pi)(d\psi_i(t)/dt)$. The $M \times K$ matrix $\mathbf{A}(\theta) = [\mathbf{a}_1, \cdots, \mathbf{a}_K]$ is the array manifold, where each element $\mathbf{a}_i, i \in 1, \cdots, K$ is the steering vector corresponding to θ_i. $\mathbf{n}(t)$ denotes the additive Gaussian noise vector with zero mean. $[.]^T$ denotes the transpose operator, $[.]^H$ denotes the conjugate transpose operator.

3. SINGLE-SOURCE TF POINTS SELECTION

For accurate DOA estimation of one source, the TF points only associated to this source are considered as the ideal TF points. However, selecting appropriate single-source TF points is a very intractable problem due to a large number of cross terms. STFT is a valuable tool for TF analysis because it is free of cross terms and easy to implement. Herein a single-source TF points selection method based on STFT is briefly introduced, which abides by the following assumptions: In real world scenarios, different sources may be closely spaced and highly overlapped in TF domain; The number of sources in each auto-source TF point is no more than two; For each source, there always exists TF points only associated to it.

Assuming $\mathbf{W}_{d_i}(t, f)$ is the STFT of the i^{th} source, the STFT of the array output $\mathbf{x}(t)$ without noise is expressed as:

$$\begin{aligned}\mathbf{W}_{\mathbf{x}}(t, f) &= \mathbf{A}(\theta)\mathbf{W}_{\mathbf{d}}(t, f) \\ &= [\mathbf{a}_1, \cdots, \mathbf{a}_K] \begin{bmatrix} \mathbf{W}_{d_1}(t, f) \\ \vdots \\ \mathbf{W}_{d_K}(t, f) \end{bmatrix}.\end{aligned} \tag{3}$$

The main steps of the single-source TF points selection method are outlined as follows:

1. Select a set of high-energy TF points $\in \Phi$:

   $$\frac{\|\mathbf{W}_{\mathbf{x}}(t, f)\|}{\max_v \|\mathbf{W}_{\mathbf{x}}(t, v)\|} > \zeta_0. \tag{4}$$

   Then compute the spatial vectors of the TF points $\in \Phi$:

   $$\boldsymbol{\iota}(t, f) = \begin{bmatrix} \frac{\mathbf{W}_{x_1}(t,f)}{\|\mathbf{W}_{\mathbf{x}}(t,f)\|} \\ \vdots \\ \frac{\mathbf{W}_{x_M}(t,f)}{\|\mathbf{W}_{\mathbf{x}}(t,f)\|} \end{bmatrix} \cdot \frac{\|\mathbf{W}_{x_1}(t, f)\|}{\mathbf{W}_{x_1}(t, f)}. \tag{5}$$

2. Apply k-means clustering method to classify the spatial vectors, and obtain K_0 ($K_0 > K$) direction vectors:

   $$\hat{\boldsymbol{\iota}}_k = \frac{1}{N_{\Upsilon_k}} \sum_{(t,f)\in\Upsilon_k} \boldsymbol{\iota}(t, f), \quad k = 1, \cdots, K_0. \tag{6}$$

   Υ_k denotes the TF points set of the k^{th} cluster. N_{Υ_k} is the number of TF points in set Υ_k. Matrix $\mathbf{A}_0 = [\hat{\boldsymbol{\iota}}_1\ \hat{\boldsymbol{\iota}}_2\ \cdots\ \hat{\boldsymbol{\iota}}_{K_0}]$ is formed with the direction vectors .

3. Based on $\mathbf{A}_0$, minimize the following subspace projection to determine the optimal $\mathbf{a}_{n_1}, \mathbf{a}_{n_2}$, which are the steering vectors of two most possible sources in each point $\in \Phi$.

   $$\{\mathbf{a}_{n_1}, \mathbf{a}_{n_2}\} = \arg\min_{\mathbf{a}_{n_1}, \mathbf{a}_{n_2}} \mathbf{Q}\mathbf{W}_{\mathbf{x}}(t, f), \quad (t, f) \in \Phi, \tag{7}$$

   where $\mathbf{Q} = \mathbf{I} - \widetilde{\mathbf{A}}_2(\widetilde{\mathbf{A}}_2^H\widetilde{\mathbf{A}}_2)^{-1}\widetilde{\mathbf{A}}_2^H$ represents the orthogonal projection matrix into the noise subspace of $\widetilde{\mathbf{A}}_2$ [21]. $\mathbf{I}$ is the identity matrix. $\widetilde{\mathbf{A}}_2 = [\mathbf{a}_{m_1}, \mathbf{a}_{m_2}]$, $\mathbf{a}_{m_1}, \mathbf{a}_{m_2}$ are two random columns of matrix $\mathbf{A}_0$.

4. Extract the single-source TF points $\in \Phi^*$ from Φ by setting appropriate thresholds:

   $$\begin{cases} \frac{\max\{|W_1|,|W_2|\}}{\min\{|W_1|,|W_2|\}} > \gamma_0 \\ \min\{|W_1|, |W_2|\} < \eta_0 \end{cases}, \tag{8}$$

   where W_1 and W_2 are two STFT values of each TF point in set Φ.

5. Apply k-means clustering method on set Φ^* to group the single-source TF points into K clusters $\{\Phi_i,\ i = 1, \cdots, K\}$.

4. TIME-FREQUENCY SPARSE REPRESENTATION

4.1. Sparse signal representation based on ℓ_1 norm

Assuming $\theta_1, \theta_2, \cdots, \theta_{N_\theta}$ is a sampling grid of all source locations of interest, and N_θ is the number of potential DOAs, which is much larger than the number of sensors M. Over-complete basis $\mathbf{A}$ [4] is introduced to represent steering vectors corresponding to the N_θ potential DOAs. $\mathbf{A}$ has the following form:

$$\mathbf{A} = [\mathbf{a}(\theta_1),\ \mathbf{a}(\theta_2),\ \cdots,\ \mathbf{a}(\theta_{N_\theta})]. \tag{9}$$

Different from $\mathbf{A}(\theta)$ in (1), the structure of the over-complete basis $\mathbf{A}$ is known and does not rely on the actual sources. The goal of sparse signal representation is to find a $N_\theta \times 1$ sparse signal vector $\mathbf{s}$, satisfying the equation $\mathbf{y} = \mathbf{A}\mathbf{s}$, $\mathbf{y}$ is the observation vector. In practice, noise is inevitable, therefore, a sparse signal representation with additive Gaussian noise is written as:

$$\mathbf{y} = \mathbf{A}\mathbf{s} + \mathbf{n}. \tag{10}$$

In order to enforce sparsity of the representation, ℓ_1 norm penalization is applied on the sparse signal vector $\mathbf{s}$. The appropriate objective function of the problem is:

$$\min \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 + \alpha\|\mathbf{s}\|_1. \tag{11}$$

The above optimization problem can be solved in a SOC programming framework. While the ℓ_2-penalization forces the residual $\mathbf{y} - \mathbf{A}\mathbf{s}$ to be small, parameter α controls the trade-off between the sparsity and the residual norm.

4.2. The TF sparse DOA estimation algorithm

Inspired by the sparse signal representation, herein we propose a new algorithm for the DOA estimation of closely-spaced and spectrally-overlapped sources with the aforementioned single-source TF points selection method. By separately dealing with the STFT values corresponding to single-source TF points of each source, the proposed algorithm offers the flexibility in underdetermined cases.

Firstly, the STFT values of each sensor output are taken to construct the TF matrix $\mathbf{D}_y$:

$$\mathbf{D}_y(m, t, f) = S_{y_m}(t, f), \quad m = 1, \cdots, M, \tag{12}$$

where $S_{y_m}(t, f)$ denotes the STFT values of each sensor output, $\mathbf{D}_y$ is a three-dimensional tensor which contains all the STFT values of M sensors.

Based on the single-source TF points selection, each single-source TF point belonging to set $\Phi_i, i \in 1, \cdots, K$ has a corresponding column coordinate in Φ^*. We define an index vector for each source:

$$\mathbf{q}_i = [q_{i_1}, \cdots, q_{i_{N_i}}], \quad i = 1, \cdots, K, \tag{13}$$

the element $q_{i_j}, j \in 1, \cdots, N_i$ denotes the corresponding column coordinate in Φ^* of the j^{th} single-source TF point $\in \Phi_i$. N_i is the number of TF points in set Φ_i. According to $\mathbf{q}_i$, the STFT values corresponding to the TF points in set Φ_i are selected from matrix $\mathbf{D}_y$, to construct a TF received data matrix $\mathbf{Y}_{tf}$. For each source, the process can be expressed in a mathematical way:

$$\mathbf{Y}_{tf}(m) = \mathbf{D}_y\{m\}(\mathbf{q}_i), \quad m = 1, \cdots, M. \tag{14}$$

Hence, the TF sparse signal representation of the $M \times N_i$ matrix $\mathbf{Y}_{tf}$ with over-complete basis $\mathbf{A}$ is:

$$\mathbf{Y}_{tf} = \mathbf{A}\mathbf{S}_{tf} + \mathbf{N}_{tf}. \tag{15}$$

To reduce computational complexity, we use the SVD of matrix $\mathbf{Y}_{tf}$, and keep one dimensional subspace

$$\mathbf{Y}_{tf} = \mathbf{U}\Sigma\mathbf{V}', \tag{16}$$

the reduced matrix is:

$$\mathbf{Y}_{SVD} = \mathbf{Y}_{tf}\mathbf{V}\mathbf{D}_K. \tag{17}$$

$\mathbf{D}_K = [\mathbf{I}_K \ \mathbf{0}^{K \times (N_i - K)}]$, $\mathbf{I}_K$ is $K \times K$ identity matrix. Here K is set to 1 (considering separately dealing with each source). Let $\mathbf{S}_{SVD} = \mathbf{S}_{tf}\mathbf{V}\mathbf{D}_K$, we can rewrite (15) as:

$$\mathbf{Y}_{SVD} = \mathbf{A}\mathbf{S}_{SVD} + \mathbf{N}_{SVD}. \tag{18}$$

Then, the objective function with ℓ_1 norm based penalties is:

$$\min \|\mathbf{Y}_{SVD} - \mathbf{A}\mathbf{S}_{SVD}\|_f^2 + \alpha\|\mathbf{s}_{SVD}^{(\ell_2)}\|_1, \tag{19}$$

where Frobenius norm $\|\mathbf{Y}_{SVD} - \mathbf{A}\mathbf{S}_{SVD}\|_f^2 = \|\mathrm{vec}(\mathbf{Y}_{SVD} - \mathbf{A}\mathbf{S}_{SVD})\|_2^2$, $\mathbf{s}_{SVD}^{(\ell_2)}$ is a $N_\theta \times 1$ TF sparse signal vector, and the i^{th} element of $\mathbf{s}_{SVD}^{(\ell_2)}$ is defined as $s_{i_{SVD}}^{(\ell_2)} = \sqrt{(s_{i_{SVD}}(1))^2}$.

The objective function in (19) has another constrained form as follows:

$$\min \|\mathbf{s}_{SVD}^{(\ell_2)}\|_1 \text{ subject to } \|\mathbf{Y}_{SVD} - \mathbf{A}\mathbf{S}_{SVD}\|_f^2 \leq \varrho, \tag{20}$$

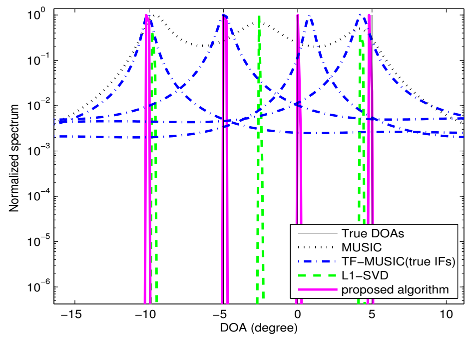
which can also be efficiently solved in the SOC framework. Parameter ϱ is related with α, denotes the noise level we allowed. Obviously, selecting the appropriate ϱ is much easier than α. $\mathbf{s}_{SVD}^{(\ell_2)}$ reflects the sparsity of the spatial spectrum. According to the peak of $\mathbf{s}_{SVD}^{(\ell_2)}$, the DOA of each source can be estimated from over-complete basis $\mathbf{A}$.

5. SIMULATION RESULTS

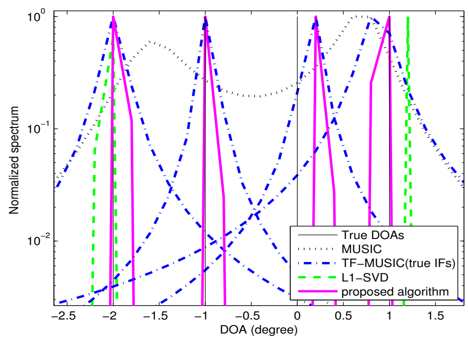
In this section, two experiments are performed to demonstrate the advantages of the proposed algorithm. The involved FM sources that are spectrally-overlapped in TF domain, are monopulse, linear FM, sinusoidal FM and frequency-shift-keying, respectively. We consider the sensors are half-wavelength spaced, all the sources have the same power.

In the first experiment, we compare the performance of the proposed algorithm with MUSIC [1], sparse signal representation based method ℓ_1-SVD [4], and true IF information based TF-MUSIC [15] in resolving closely-spaced and spectrally-overlapped sources. Fig. 1(a) gives the DOA estimation results of different algorithms with directions -10^o, -5^o, 0^o, 5^o at SNR = 5 dB. It is clear that MUSIC and ℓ_1-SVD are unable to provide accurate estimations for 5^o spaced sources, TF-MUSIC and the proposed algorithm can both achieve super resolution. Moreover, the proposed algorithm based on the selected single-source TF points outperforms the TF-MUSIC by mitigating the cross interference.

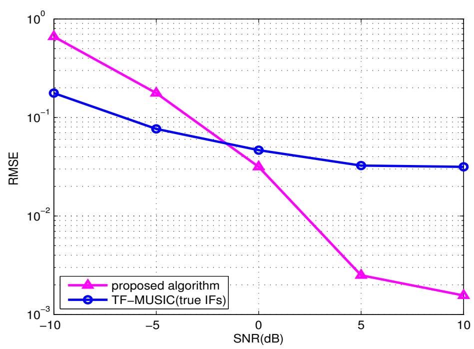
For comparison, we repeat the experiment with a set of



(a) SNR: 5 dB, sources spaced: 5^o



(b) SNR: 15 dB, sources spaced: 1^o



(c) RMSE at different SNR levels

Fig. 1. Estimated results of closely-spaced and spectrally-overlapped sources (M = 10, K = 4).

closer separation directions -2^o, -1^o, 0^o, 1^o in the presence of SNR = 15 dB. The estimated results in Fig. 1(b) show that the proposed algorithm still provides more accurate precision than the other methods, while MUSIC and ℓ_1-SVD cannot resolve the closely-spaced sources even at a high SNR level. Lastly, we compare the capability of the proposed algorithm and TF-MUSIC at different SNR levels. For illustrative purposes, the root-mean-square error (RMSE) of the two methods are shown in Fig. 1(c). It can be seen that, although TF-MUSIC based on known IF information outperforms the proposed algorithm at low SNR conditions, in practice, the required accurate IF information is unknown

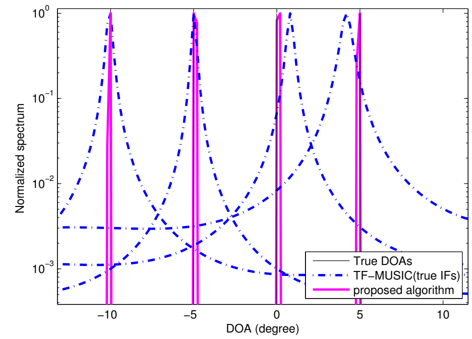


Fig. 2. Estimated results in underdetermined case (M = 3, K = 4).

and it is a intractable problem to estimate the IF information for the spectrally-overlapped sources at low SNR levels.

The second experiment demonstrates the flexibility of the proposed algorithm in an underdetermined case. We set SNR = 20 dB, the DOAs are -10^o, -5^o, 0^o, 5^o. The underdetermined case that the number of sensors M is smaller than the number of sources K is examined in Fig. 2. It is evident that MUSIC cannot work in the underdetermined case, whereas the other two methods are able to relax the well-known $M > K$ restriction with the ability to deal with sources separately. Noting that, with true IFs, TF-MUSIC suffers worse DOA estimation in underdetermined cases.

6. RELATION TO PRIOR WORK

There are different approaches for DOA estimation. The methods in [1] and [4] have no ability to resolve the closely-spaced sources and lack the flexibility in underdetermined cases. Another method in [15, 17] is limited to resolve the sources whose spectral contents are highly overlapped in TF domain. However, all the above shortcomings can be well overcome with the proposed algorithm, which is the novelty of this paper. The proposed algorithm shows the superiority of dealing with closely-spaced and spectrally-overlapped sources by applying the sparse signal representation on the STFT values corresponding to the single-source TF points. In addition, this algorithm is feasible in underdetermined cases.

7. CONCLUSION

In this paper, we propose a time-frequency sparse representation based algorithm for DOA estimation of closely-spaced and spectrally-overlapped sources. Based on the introduced single-source TF points selection method, the STFT values corresponding to the TF points where only one source exists are adopted in a sparse signal representation framework. From the simulation results, the superior performance of the proposed algorithm is verified.

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