SUBSPACE FITTING VIA SPARSE REPRESENTATION OF SIGNAL COVARIANCE FOR DOA ESTIMATION

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ABSTRACT

Based on the orthogonality between the signal subspace and the noise subspace, we propose a sparse recovery method for the direction of arrival (DOA) estimation. With the assumption of uncorrelated sources, signal covariance matrix fitting is achieved by embedding the MUSIC-like weights into a quadratic minimization, which is capable of prompting the sparsity of the solution. Numerical results show that the proposed method outperforms some other sparse recovery methods.

***Index Terms*—** Direction of arrival estimation, sparse recovery, subspace fitting, array signal processing

1. INTRODUCTION

Direction of arrival (DOA) estimation is a fundamental issue in array processing field, which can be applied in a wide range, including radar, sonar, radio astronomy, wireless communications, etc. [1]. Many DOA estimation algorithms have been presented to achieve high spatial resolution that is particularly useful for the limited sensor resources. The conventional subspace-like algorithms include MUSIC [2], ESPRIT [3], and Root-MUSIC [4], which have high spatial resolution when the sources are uncorrelated. In recent decade, the sparsity aware DOA estimation has been a very popular topic. An early work for DOA estimation based on sparse recovery is the so-called global matched filter (GMF) [5] that utilizes a model-fitting approach to improve upon the conventional beamformer. The classical ℓ_1-SVD algorithm [6] uses the ℓ_1 norm penalties and the ℓ_2 norm to enforce the sparsity and restrict the recovery error, respectively. By introducing the methodology of the weighted ℓ_1 minimization, a weighted GMF (WGMF) and an improved ℓ_1-SVD (subspace weighted $\ell_{2,1}$-SVD) were already proposed in [7] and [8] , respectively. The resolution and the accuracy of DOA estimates are improved because the weighted ℓ_1 minimization can further prompt the sparsity of the solution. In [9] ℓ_1-SVD is combined with the weighted subspace fitting and the Sparse Recovery for Weighted Subspace Fitting (SRWSF) method was proposed. It is noted that, the sparse recovery methods mentioned above require the regularization parameter that compromises the sparsity of the solution and the recovery error. Therefore, the regularization parameter need be carefully selected. However, determining proper values of the regularization parameter is a difficult task in practical applications [10, 11].

Some work have been made to develop the sparse recovery methods without the choice of the regularization parameter. Sparse Iterative Covariance-based Estimation (SPICE) does not require the regularization parameter and ensures reliable DOA estimates with global convergence [10]. However, it is reported in [12, 13] that the accuracy of SPICE is unsatisfactory. By employing the maximum likelihood principle, LIKelihood-based Estimation of Sparse parameters (LIKES) has been derived to improve SPICE. LIKES performs better than SPICE in term of accuracy and resolution at the cost of the increased computational burden [12, 14]. Another regularization parameter free Sparse Recovery algorithm via Covariance-like Fitting (SRCF) for DOA estimation under power constraints was proposed in [15], which has better performance than SPICE when the number of snapshots is large enough. Other spare recovery algorithms without regularization parameters including the Sparse Learning via Iterative Minimization (SLIM) [16], and the Iterative Adaptive Approach for Amplitude and Phase EStimation (IAA-APES) [17] can also be regarded as the modified versions of the adaptively reweighted SPICE method with different weighting schemes [18].

In this paper, we propose a Subspace Fitting algorithm via the Sparse Representation of the Signal Covariance (SF-SRSC) for DOA estimation under power constraints without determining the regularization parameter. By exploiting the orthogonality between the signal subspace and the noise subspace, a quadratic minimization is formulated to fit the signal covariance matrix in a sparse fashion. Then power constraints on the estimated spectrum is used to restrict the retrieved energy. It is worth mentioning that MUSIC-like weights are embedded into the objective function of the quadratic minimization to form a weighted sparse recovery and enhance the sparsity of the solution. Numerical examples demonstrate that the proposed algorithm outperforms some

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existing sparse recovery methods including SRWSF, WGMF, LIKES, and SRCF in terms of the DOA estimation accuracy and the spatial resolution.

This paper is organized as follows. In the next section, the signal model is introduced. In Section 3, based on the assumption of uncorrelated sources we formulate a quadratic minimization to resolve the DOA estimation problem. In Section 4, some numerical experiments are provided to illustrate the performances of the proposed method. A conclusion is given in Section 5.

2. SIGNAL MODEL

Take the problem of DOA estimation with M sensors and K far-field narrowband signals $\{s_k(t), k = 1, 2, \cdots, K\}$ as an example, where the sources impinging on the array from distinct directions $\{\theta_k, k = 1, 2, \cdots, K\}$. The signal model can be expressed as:

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), t = 1, 2, \cdots, T., \tag{1}$$

where $\mathbf{y}(t)$, $\mathbf{s}(t)$, $\mathbf{n}(t)$ denote the measurements, the signal vector, and the noise vector, respectively. The matrix $\mathbf{A} \in \mathbb{C}^{M\times K}$ is the array response matrix given by $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_K)]$ with

$$\mathbf{a}(\theta_k) = [e^{j2\pi f_0 x_1 sin(\theta_k)/c}, \cdots, e^{j2\pi f_0 x_M sin(\theta_k)/c}]^{\mathrm{T}}, \tag{2}$$

where f_0, c, and x_m, denote the center frequency, the propagation speed, and the the position of the mth sensor with $m = 1, \cdots, M$, respectively; $(\cdot)^{\mathrm{T}}$ denotes transpose. The vector $\mathbf{n}(t)$ is an additive noise vector with zero-mean and the variance σ^2. Without loss of generality, $\mathbf{n}(t)$ is assumed to be uncorrelated with $\mathbf{s}(t)$.

The covariance matrix of $\mathbf{y}(t)$ can be expressed as

$$\begin{aligned} \mathbf{R} &= E\{\mathbf{y}(t)\mathbf{y}^{\mathrm{H}}(t)\} \\ &= \mathbf{A}E\{\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)\}\mathbf{A}^{\mathrm{H}} + \sigma^2\mathbf{I}_M \\ &= \mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}} + \sigma^2\mathbf{I}_M, \end{aligned} \tag{3}$$

where $\mathbf{R}_s = E\{\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)\}$, $\mathbf{I}_M$ is a $M \times M$ identity matrix, $(\cdot)^{\mathrm{H}}$ denotes conjugate transpose.

Taking the eigenvalue decomposition of $\mathbf{R}$ yields

$$\mathbf{R} = \sum_{m=1}^{M} \lambda_m \mathbf{u}_m \mathbf{u}_m^{\mathrm{H}} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^{\mathrm{H}} + \sigma^2\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}, \tag{4}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K \geq \lambda_{K+1} = \cdots = \lambda_M = \sigma^2$ are the eigenvalues of $\mathbf{R}$, $\mathbf{\Lambda}_s = diag\{\lambda_1, \cdots, \lambda_K\}$, $\mathbf{\Lambda}_n = diag\{\lambda_{K+1}, \cdots, \lambda_M\}$, $\mathbf{U}_s$ and $\mathbf{U}_n$ span the signal subspace and the noise subspace, respectively, $diag\{\cdot\}$ denotes the diagonal matrix. From (3) and (4), we have

$$\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}} = \mathbf{U}_s\left(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K\right)\mathbf{U}_s^{\mathrm{H}}. \tag{5}$$

3. THE PROPOSED ALGORITHM FOR DOA ESTIMATION

3.1. Subspace fitting via sparse representation of signal covariance

Because of the orthogonality between the signal subspace and the noise subspace, we have

$$\begin{aligned} & \|\mathbf{U}_n^{\mathrm{H}}\mathbf{U}_s\left(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K\right)\mathbf{U}_s^{\mathrm{H}}\mathbf{U}_n\|_{\mathrm{F}}^2 \\ = & \|\mathbf{U}_n^{\mathrm{H}}\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\mathbf{U}_n\|_{\mathrm{F}}^2 \\ = & \ 0. \end{aligned} \tag{6}$$

Under the assumption of uncorrelated sources, the sparse representation of signal covariance can be written as

$$\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}} = \mathbf{\Phi}\mathbf{P}\mathbf{\Phi}^{\mathrm{H}}, \tag{7}$$

where the overcomplete basis matrix $\mathbf{\Phi} = [\phi_1, \cdots, \phi_N]$, $\phi_n = [e^{j2\pi f_0 x_1 sin(\alpha_n)/c}, \cdots, e^{j2\pi f_0 x_M sin(\alpha_n)/c}]^{\mathrm{T}}$, the angle set $\{\alpha_1, \cdots, \alpha_N\}$ denotes a sampling grid of all possible source locations, N is the number of the sampling grids, $\mathbf{P} = diag\{p_1, \cdots, p_N\}$. Ideally, if and only if $\alpha_n \in \{\theta_k, k = 1, \cdots, K\}$, $p_n > 0$; otherwise $p_n = 0$. Thus the support set can be defined as $\Gamma \triangleq \{n | p_n > 0\}$.

Substituting (7) into (6) yields

$$\begin{aligned} f(\mathbf{p}) &= \|\mathbf{U}_n^{\mathrm{H}}\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\mathbf{U}_n\|_{\mathrm{F}}^2 \\ &= \|\mathbf{U}_n^{\mathrm{H}}\mathbf{\Phi}\mathbf{P}\mathbf{\Phi}^{\mathrm{H}}\mathbf{U}_n\|_{\mathrm{F}}^2 \\ &= \mathrm{tr}\left(\mathbf{P}\mathbf{\Phi}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{\Phi}\mathbf{P}\mathbf{\Phi}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{\Phi}\right) \\ &= \mathrm{tr}\left(\mathbf{PBPB}\right) \\ &= \mathbf{p}^{\mathrm{T}}\mathbf{D}\mathbf{p}, \end{aligned} \tag{8}$$

where $\mathbf{B} = \mathbf{\Phi}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{\Phi}$, $\mathbf{p} = [p_1, \cdots, p_N]^{\mathrm{T}}$ corresponds to the diagonal elements of $\mathbf{P}$, $\mathbf{D} = \mathbf{B} \odot \mathbf{B}^*$, $\mathrm{tr}(\cdot)$, $\odot$, and $(\cdot)^*$ denote the trace of matrix, the Hadamart product and conjugate, respectively. The last equality in (8) is derived from the diagonal characteristics of $\mathbf{P}$.

Moreover, we have

$$\mathbf{1}^{\mathrm{T}}\mathbf{p} = \mathrm{tr}\left(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}_K\right)/M, \tag{9}$$

where $\mathbf{1}$ is the vector of all ones.

In practice, we have to replace the covariance $\mathbf{R}$ with the sampling covariance $\hat{\mathbf{R}}$, where $\hat{\mathbf{R}} = (1/T)\sum_{t=1}^{T}\mathbf{y}(t)\mathbf{y}^{\mathrm{H}}(t)$. The eigenvalue decomposition of $\hat{\mathbf{R}}$ can be expressed as

$$\hat{\mathbf{R}} = \sum_{m=1}^{M} \hat{\lambda}_m \hat{\mathbf{u}}_m \hat{\mathbf{u}}_m^{\mathrm{H}}, \tag{10}$$

where $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \cdots \geq \hat{\lambda}_M$. A maximum likelihood estimation of σ^2 can be written as $\hat{\sigma}^2 = \sum_{K+1}^{M}\hat{\lambda}_m/(M - K)$. Then, equation (8) and (9) need be replaced with the following equations, respectively,

$$f(\mathbf{p}) = \mathbf{p}^{\mathrm{T}}\hat{\mathbf{D}}\mathbf{p}, \tag{11}$$

and

$$\mathbf{1}^{\mathrm{T}}\mathbf{p} = \mathrm{tr}\left(\hat{\mathbf{\Lambda}}_s - \hat{\sigma}^2\mathbf{I}_K\right)/M, \quad (12)$$

where $\hat{\mathbf{D}} = \hat{\mathbf{B}} \odot \hat{\mathbf{B}}^*$, $\hat{\mathbf{U}}_n = [\hat{\mathbf{u}}_{K+1}, \cdots, \hat{\mathbf{u}}_M]$, $\hat{\mathbf{B}} = \mathbf{\Phi}^{\mathrm{H}}\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{\Phi}$, $\hat{\Lambda}_s = diag\{\hat{\lambda}_1, \cdots, \hat{\lambda}_K\}$.

Combining (11) and (12), the DOA estimation can be carried out by the following quadratic minimization:

$$\begin{aligned} \min_{p_n \geq 0} \quad & \mathbf{p}^{\mathrm{T}}\hat{\mathbf{D}}\mathbf{p} \\ s.t. \quad & \mathbf{1}^{\mathrm{T}}\mathbf{p} = \beta, \end{aligned} \quad (13)$$

where $\beta = \mathrm{tr}(\hat{\mathbf{\Lambda}}_s - \hat{\sigma}^2\mathbf{I}_K)/M$. Here, we employ the CVX package [19] to solve the quadratic optimization problem (13). Once $\hat{\mathbf{p}}$ is obtained, the DOA estimation can be achieved by determining the peaks of the estimated spatial spectrum $\hat{\mathbf{p}}$.

3.2. Remark on the objective function

In what follows, we will show that the objective function in (8) is equivalent to a MUSIC-like weighting scheme. We define the MUSIC-like weight $w_{i,j}$ as below:

$$w_{i,j} = |\phi_i^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\phi_j|. \quad (14)$$

If and only if $i = j$, $w_{i,j}$ becomes the MUSIC weight $w_{i,i}$. Furthermore, if and only if $i \in \Gamma$ or $j \in \Gamma$, $w_{i,j} = 0$ because of the orthogonality between the signal subspace and the noise subspace; otherwise $w_{i,j} > 0$.

Substituting (14) into (8), we have:

$$f(\mathbf{p}) = \mathbf{p}^{\mathrm{T}}\mathbf{D}\mathbf{p} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,j}^2 p_i p_j. \quad (15)$$

On one hand, from (15), it is noted that smaller weights $\{w_{i,j} = 0, i \in \Gamma \text{ or } j \in \Gamma\}$ and the signal-related items $\{p_i p_j, i \in \Gamma \text{ or } j \in \Gamma\}$ are linked together. Appointing smaller weights $\{w_{i,j} = 0, i \in \Gamma \text{ or } j \in \Gamma\}$ to all signal-related items further consolidates the priority of the spectrum peak $\{p_i, i \in \Gamma\}$ or $\{p_j, j \in \Gamma\}$ in the reconstruction process. On the other hand, those positions that are more likely corresponding to noise entries, i.e., $\{p_i p_j, i \notin \Gamma$ and $j \notin \Gamma\}$, are weighted by larger values $\{w_{i,j} > 0, i \notin \Gamma$ and $j \notin \Gamma\}$ so that they are encouraged to be close to zero in the recovery result. Therefore, the proposed SFSRSC method is in consistent with the methodology of the weighted sparse signal recovery in [20] that can promote the sparsity of the solution and improve the recovery performance. In practice, $\mathbf{D}$ is replaced with $\hat{\mathbf{D}}$ and the approximation of weights can be expressed as $\{w_{i,j} \approx 0\ , i \in \Gamma \text{ or } j \in \Gamma\}$. Therefore, the above analysis still holds.

4. NUMERICAL EXAMPLES

In this section, some numerical examples are presented to demonstrate the performance of the proposed method. The results of several sparse recovery methods, i.e., SRWSF [9], WGMF [7], LIKES [14], and SRCF [15], are also provided for comparison. In all experiments we consider a uniformly-spaced linear array (ULA) composed of $M = 10$ sensors with a spacing of $d = c/(2f_0)$. The angle interval $[-90°, 90°]$ is uniformly sampled with 1801 grids.

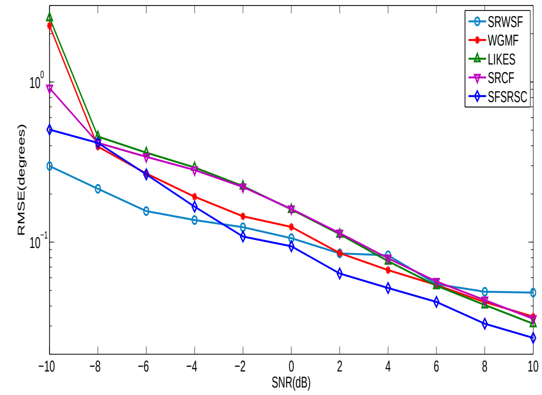

Fig. 1. RMSE versus SNR. The number of snapshots is 800, the number of sensors is 10, the sources is uncorrelated, and DOAs:$\{5.5°, 16.1°, 25.7°\}$, 200 Monte Carlo trials.

In the first experiment, the Root Mean Square Errors (RMSE) of the DOA estimation of these algorithms are compared. Three uncorrelated sources with the same amplitude impinging on the array from $\{5.5°, 16.1°, 25.7°\}$. As shown in Fig.1, the proposed method can obtain more accurate DOA estimates than other four methods when Signal Noise Ratio (SNR) is high enough. Although SRWSF has better performance when SNR is lower, it is noted that SRWSF requires the regularization parameter [1], while the proposed SFSRSC method does not. In addition, as can be seen from Fig.2, with the increasing number of snapshots, the performance of the proposed method exceeds that of other four methods. Furthermore, in contrast to LIKES and SRCF that also do not require the regularization parameters, the proposed method performs better when the number of snapshots becomes larger.

In the second experiment, the spatial resolution of the proposed algorithm is investigated. Two closely spaced sources are located at $\pm\Delta/2$, where Δ denotes the source separation and the units is the Rayleigh resolution (the Rayleigh resolution for our problem is $5.73°$ [21]). We follow the definition of the spatial resolution in [22] that the two sources are successfully resolved providing the estimate $\hat{\theta}_k$ is located in the neighborhood of the true position $U(\theta_k; \gamma)$.

[1] SRWSF and WGMF have to use the regularization parameter that is a user-dependent parameter (e.g., the confidence level of the estimated noise energy that is manually set by the user), which results in the performance degradation providing the confidence level is improper.

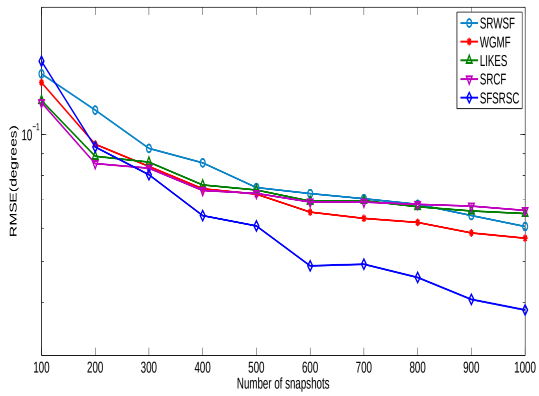

Fig. 2. RMSE versus number of snapshots. SNR is 5dB, the number of sensors is 10, the sources is uncorrelated, and DOAs:$\{5.5°, 16.1°, 25.7°\}$, 200 Monte Carlo trials.

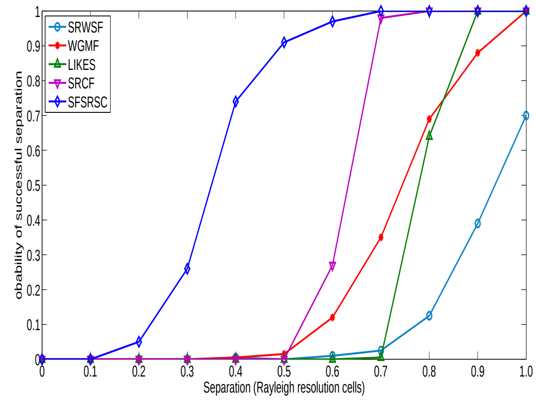

Fig. 3. The probability of resolution versus the separation between two uncorrelated sources. The number of snapshots is $T = 500$, the SNR is 5dB, and 200 Monte Carlo trials.

Here, the radius of the neighborhood γ is set to equal to a tenth of the Rayleigh resolution limitation, i.e., $\gamma = 0.573°$. As can be seen from Fig.3, SFSRSC has higher probability of successful separation than other four methods. The reason for the improved spatial resolution is that the sparsity of the solution of SFSRSC is enhanced by embedding the MUSIC-like weights into the objective function.

In the third experiment, the averaged spatial spectrum over 200 Monte carlo trials is plotted , where two correlated sources have the same amplitude with the correlation coefficient of 0.6. Fig. 4 shows that the proposed method is robust to the assumption of uncorrelated sources and works well in correlated source scenario.

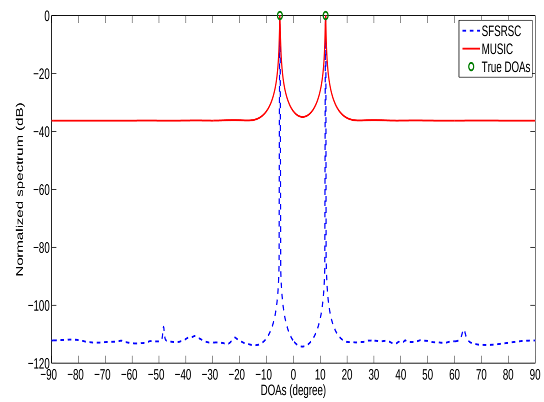

Fig. 4. Averaged spatial spectrum of the proposed method. The SNR is 10dB, the number of snapshots is $T = 500$, DOAs:$\{-5°, 12°\}$, the correlation coefficient of two sources is 0.6, and 200 Monte Carlo trials.

5. CONCLUSION

We present a DOA estimation method by a linearly constrained quadratic programming. By using the orthogonality between the signal subspace and the noise subspace, the MUSIC-like weights are embedded into the objective function, which can enhance the sparsity of the solution and improve the performance of the sparse recovery. Numerical experiments demonstrate that the proposed method has better performance than SRWSF, WGMF, LIKES, and SRCF for uncorrelated sources and works well for correlated sources.

6. REFERENCES

[1] H. Krim and M. Viberg, "Two decades of array signal processing research: the parametric approach," *IEEE Signal Processing Magazine*, vol. 13, no. 4, pp. 67–94, 1996.

[2] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, Mar. 1986.

[3] Richard Roy and Thomas Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, no. 7, pp. 984–995, 1989.

[4] A. Barabell, "Improving the resolution performance of eigenstructure-based direction-finding algorithms," in *1983 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*. IEEE, 1983, vol. 8, pp. 336–339.

[5] J-J Fuchs, "On the application of the global matched filter to doa estimation with uniform circular arrays," *IEEE Transactions on Signal Processing*, vol. 49, no. 4, pp. 702–709, 2001.

[6] D. Malioutov, M. Cetin, and A.S. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3010–3022, 2005.

[7] X. Xu, X. Wei, and Z. Ye, "Doa estimation based on sparse signal recovery utilizing weighted ℓ_1 norm penalty," *IEEE Signal Processing Letters*, vol. 19, no. 3, pp. 155–158, 2012.

[8] C. Zheng, G. Li, H. Zhang, and X. Wang, "An approach of doa estimation using noise subspace weighted ℓ_1 minimization," in *2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2011, pp. 2856–2859.

[9] N. Hu, Z. Ye, D. Xu, and S. Cao, "A sparse recovery algorithm for doa estimation using weighted subspace fitting," *Signal Processing*, vol. 92, no. 10, pp. 2566–2570, 2012.

[10] P. Stoica, P. Babu, and J. Li, "New method of sparse parameter estimation in separable models and its use for spectral analysis of irregularly sampled data," *IEEE Transactions on Signal Processing*, vol. 59, no. 1, pp. 35–47, 2011.

[11] A. Maleki and D.L. Donoho, "Optimally tuned iterative reconstruction algorithms for compressed sensing," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 330–341, 2010.

[12] P. Babu and P. Stoica, "Sparse spectral-line estimation for nonuniformly sampled multivariate time series: Spice, likes and msbl," in *2012 Proceedings of the 20th European Signal Processing Conference (EUSIPCO)*, Bucharest, Romania, August 27 - 31 2012, pp. 445–449.

[13] Stefano Fortunati, Raffaele Grasso, Fulvio Gini, Maria S Greco, and Kevin LePage, "Single-snapshot doa estimation by using compressed sensing," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, no. 1, pp. 1–17, 2014.

[14] P. Stoica and P. Babu, "Spice and likes: Two hyperparameter-free methods for sparse-parameter estimation," *Signal Processing*, vol. 92, no. 7, pp. 1580–1590, 2012.

[15] C. Zheng, G. Li, P. Li, and X. Wang, "Hyperparameter-free doa estimation under power constraints," in *2013 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2013, pp. 3991–3995.

[16] X. Tan, W. Roberts, J. Li, and P. Stoica, "Sparse learning via iterative minimization with application to mimo radar imaging," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1088–1101, 2011.

[17] T. Yardibi, J. Li, P. Stoica, M. Xue, and A.B. Baggeroer, "Source localization and sensing: A nonparametric iterative adaptive approach based on weighted least squares," *IEEE Transactions onAerospace and Electronic Systems*, vol. 46, no. 1, pp. 425–443, 2010.

[18] P. Stoica, D. Zachariah, and J. Li, "Weighted spice: A unifying approach for hyperparameter-free sparse estimation," *Digital Signal Processing*, vol. 33, pp. 1–12, 2014.

[19] M. Grant and S. Boyd, "Cvx: Matlab software for disciplined convex programming, version 1.22," May 2012.

[20] M. Amin Khajehnejad, W. Xu, A. Salman Avestimehr, and B. Hassibi, "Weighted ℓ_1 minimization for sparse recovery with prior information," in *2009 IEEE International Symposium on Information Theory (ISIT)*. IEEE, 2009, pp. 483–487.

[21] J.J. Fuchs, "Doa estimation in the presence of modeling errors, the global matched filter approach," in *17th European Signal Processing Conference (EUSIPCO)*, Glasgow, Scotland, August 24-28 2009, pp. 1963–1967.

[22] D.H. Johnson and D.E. Dudgeon, *Array Signal Processing: Concepts and Techniques*, Prentice-Hall, Englewood Cliffs, NJ, 1993.