SPARSITY-BASED LOCALIZATION OF SPATIALLY COHERENT DISTRIBUTED SOURCES

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ABSTRACT

In this paper, the localization of spatially distributed sources is considered. Based on the problem formulation of the Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS), a criterion based on a convex optimization under sparsity constraint is proposed to locate the sources. Also an original method is given to recover the angular distributions and the power of the sources. Simulations executed in the scenario of a mixture of distributed and point sources illustrate the validation of the proposed approach compared to other methods.

***Index Terms*—** distributed sources, sparsity method, angular dispersion, DAMAS, CMF

1. INTRODUCTION

Direction of arrival (DOA) estimation of source signals impinging on an array of sensors has been widely studied in the literature with the sources assumed to be far-field point transmitters or reflectors [1]. In many applications, the physical sources can no longer be considered as point sources and a spatially distributed model of the sources could be more appropriate. The influence of modeling errors due to the spatial distribution of the sources on the high resolution MUSIC [1] method has been recently analyzed [2] and shows the importance of taking into account the actual model of the sources.

Plenty of DOA estimation methods have been proposed in the literature. The commonly used Beamforming (or spatial filtering) combines the array elements to enhance the output at particular directions while weaken the output at other directions. Deconvolution algorithms such as DAMAS [3] improve the resolution of Beamforming, but require thousands of iterations of deconvolution to resolve the proposed criterion. Sparsity and convex optimization based deconvolution algorithms proposed in the literature have exhibited better performance than DAMAS. For instance, Sparsity constrained DAMAS (SC-DAMAS) [4] minimizes the DAMAS criterion with a sparsity constraint on the sources. Covariance Matrix Fitting (CMF) [4] minimizes the difference between the covariance matrix obtained by the sensor measurements and the modeled one, with a constraint that the trace of the source covariance matrix is not larger than the sum of the source powers. To locate coherent sources, DAMAS-C [5] and CMF-C [4] have been proposed as an extension of DAMAS and CMF. While the sources are distributed, an intra-task correlation within a same contiguous cluster is introduced as an *a-priori* knowledge in the convex criterion [6].

In this paper, we approximate the coherent distributed (CD) source as a compact cluster of coherent point sources weighted by the source distribution function, so as to formulate our problem in the scenario of DAMAS-C. We here extend the SC-DAMAS criterion with a constraint of minimizing the rank of the distribution function weighted source covariance matrix, to adapt to the distributed source scenario. After the DOA estimation step, we recover the source distribution functions and the power of the sources. The simulation results are compared with other methods to demonstrate the effectiveness of our proposed method.

The organization of this paper is as follows. The signal model is given in section 2. In section 3, we formulate mathematically the source localization problem and propose the criterion adapted to the CD sources. In section 4, we present the method to recover the source distribution functions and the source power. Numerical simulations are presented in section 5 to validate the proposed approach. Finally, conclusions are given is section 6.

2. SIGNAL MODEL

Let us consider q spatially CD far-field sources signals impinging at an array of M sensors. The q sources and the M signals received at the array at moment t are denoted by $\mathbf{s}(t) = [s_1(t), \ldots, s_q(t)]^T$ and $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T$, respectively, with:

$$\mathbf{y}(t) = \mathbf{C}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \quad (1)$$

where $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ represents the complex Gaussian distributed additive noise, $\mathbf{C}(\theta) = [\mathbf{c}_{h_1}(\theta_1), \ldots, \mathbf{c}_{h_q}(\theta_q)] \in$

$\mathbb{C}^{M\times q}$ is the array steering matrix composed of q steering vectors $\mathbf{c}_{h_i}(\theta)$ that can be written as proposed in [7] by:

$$\mathbf{c}_{h_i}(\theta_i) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{a}(\phi) h_i(\theta_i, \phi) d\phi, \tag{2}$$

where $i = 1 \ldots q$, and $\mathbf{a}(\phi)$ is the steering vector for a point source which arrives from the DOA ϕ. In the most general case, the steering vector $\mathbf{a}(\phi)$ is also a function of the array geometry, the sensor gains, the form of the wavefront, and other possible parameters which are supposed to be known. The function $h(\phi)$ is introduced to describe the angular spread distribution of the source (for instance, uniform or Gaussian distributions).

The angular sector of interest is discretized by L grid points such that $\phi_k = \phi_1 + (k-1)\delta$, $k = 1, \ldots, L$, where δ is the angular separation between two points of the grid, the value of δ can be determined by the required resolution. The steering matrix composed of L point source steering vectors corresponding to the grid can be given as:

$$\mathbf{A} = \left[\mathbf{a}(\phi_1), \ldots, \mathbf{a}(\phi_k), \ldots, \mathbf{a}(\phi_L)\right]. \tag{3}$$

Introducing a distribution function vector for source i: $\mathbf{h}_i = [h_i(\theta_i, \phi_1), \ldots, h_i(\theta_i, \phi_L)]^T \in \mathcal{R}^{L\times 1}$, and introducing the matrix $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_q] \in \mathcal{R}^{L\times q}$. Assuming that the value of δ is small enough, the distributed source steering vector for source i and the signals received at the sensor array can be approximated by:

$$\mathbf{c}_{h_i}(\theta_i) \approx \delta \sum_{k=1}^{L} \mathbf{a}(\phi_k) h_i(\theta_i, \phi_k) = \delta \mathbf{A}\mathbf{h}_i, \tag{4}$$

introducing (4) in (2), then in (1), the received signals $\mathbf{y}(t)$ can be approximated by:

$$\mathbf{y}(t) \approx \delta \mathbf{A}\mathbf{H}\mathbf{s}(t) + \mathbf{n}(t), \tag{5}$$

respectively. With (4) and (5) the CD source location problem can be transformed into a coherent point source localization problem.

The source signals and the additive noise are considered to be centered Gaussian independent random variables, and assuming that signals and noises are uncorrelated and the sources are uncorrelated with each other, considering (1), and (5), the correlation matrix is given by:

$$\begin{aligned} \mathbf{R} &= E[\mathbf{y}\mathbf{y}^H] \\ &\approx \delta^2 \mathbf{A}\mathbf{H} E[\mathbf{s}(t)\mathbf{s}^H(t)] \mathbf{H}^H \mathbf{A}^H + \sigma_b^2 \mathbf{I} \\ &= \delta^2 \mathbf{A}\mathbf{H}\mathbf{R}_s \mathbf{H}^H \mathbf{A}^H + \sigma_b^2 \mathbf{I}, \end{aligned} \tag{6}$$

where $\mathbf{R}_s = E[\mathbf{s}(t)\mathbf{s}^H(t)] = diag\{\sigma_{si}^2\}$ is a diagonal matrix, σ_{si}^2 is the power for source i, σ_b^2 is the noise variance. Now the covariance matrix is approximated as an explicit functions of the point source steering vectors and the clusters of the coherent point sources.

3. SOURCE LOCALIZATION ALGORITHM

In this section, we first present the problem formulation for applying the source localization approach, then we propose the optimization criterion.

Defining that $\mathbf{X} = \mathbf{H}\mathbf{R}_s\mathbf{H}^H$, and noting the element at k^{th} line and k'^{th} column of $\mathbf{X}$ as $\mathbf{X}_{kk'} = \gamma_{kk'}$, we employ a cross-beamform product:

$$\begin{aligned} g_{mn} &= \mathbf{a}_m^H \mathbf{R} \mathbf{a}_n = \delta^2 \mathbf{a}_m^H \mathbf{A}\mathbf{H}\mathbf{R}_s\mathbf{H}^H\mathbf{A}^H\mathbf{a}_n + \sigma_b^2 \mathbf{a}_m^H \mathbf{a}_n \\ &= \delta^2 \mathbf{a}_m^H \mathbf{A}\mathbf{X}\mathbf{A}^H \mathbf{a}_n + \sigma_b^2 \mathbf{a}_m^H \mathbf{a}_n \\ &= \delta^2 \sum_{k=1}^{L}\sum_{k'=1}^{L} \gamma_{kk'} \mathbf{a}_m^H \mathbf{a}_k \mathbf{a}_{k'}^H \mathbf{a}_n + \sigma_b^2 \mathbf{a}_m^H \mathbf{a}_n, \end{aligned} \tag{7}$$

where $\mathbf{a}_m$ and $\mathbf{a}_n$ are simplified notations for $\mathbf{a}(\phi_m)$ and $\mathbf{a}(\phi_n)$.

Assuming that the noise on the array can be ignored, an inverse problem can be given as:

$$\mathbf{g} = \delta^2 \mathbf{A}_D \mathbf{x}. \tag{8}$$

The details are given as:

$$\mathbf{g} = \left[g_{11}, g_{12}, \ldots, g_{1L}, \ldots, g_{mn}, \ldots, g_{LL}\right]^T \in \mathcal{R}^{L^2\times 1}, \tag{9}$$

$$\mathbf{A}_D = \begin{bmatrix} \mathbf{a}_1^H\mathbf{a}_1\mathbf{a}_1^H\mathbf{a}_1 & \ldots & \mathbf{a}_1^H\mathbf{a}_k\mathbf{a}_{k'}^H\mathbf{a}_1 & \ldots & \mathbf{a}_1^H\mathbf{a}_L\mathbf{a}_L^H\mathbf{a}_1 \\ \vdots & \ddots & \vdots & & \\ \mathbf{a}_m^H\mathbf{a}_1\mathbf{a}_1^H\mathbf{a}_n & \ldots & \mathbf{a}_m^H\mathbf{a}_k\mathbf{a}_{k'}^H\mathbf{a}_n & \ldots & \mathbf{a}_m^H\mathbf{a}_L\mathbf{a}_L^H\mathbf{a}_n \\ \vdots & \ddots & \vdots & & \\ \mathbf{a}_L^H\mathbf{a}_1\mathbf{a}_1^H\mathbf{a}_L & \ldots & \mathbf{a}_L^H\mathbf{a}_k\mathbf{a}_{k'}^H\mathbf{a}_L & \ldots & \mathbf{a}_L^H\mathbf{a}_L\mathbf{a}_L^H\mathbf{a}_L \end{bmatrix} \in \mathcal{R}^{L^2\times L^2}, \tag{10}$$

and

$$\mathbf{x} = \left[\gamma_{11}, \gamma_{12}, \ldots, \gamma_{1L}, \ldots, \gamma_{kk'}, \ldots, \gamma_{LL}\right]^T \in \mathcal{R}^{L^2\times 1}, \tag{11}$$

respectively. Notice that $\mathbf{x}$ can be obtained by vectorization of $\mathbf{X}$.

To resolve $\mathbf{x}$, DAMAS-C [5] has been proposed as an iterative approach based on the fact that all the elements are non-negative, the initial value of $\mathbf{x}$ can be set 0. If we know a priori that the sources are point and uncorrelated, then $\mathbf{x}$ shrinks to $[\gamma_{11}, \gamma_{22}, \ldots, \gamma_{kk}, \ldots, \gamma_{LL}] \in \mathcal{R}^{L\times 1}$, and $\mathbf{A}_D$ and $\mathbf{g}$ shrink correspondingly to a smaller size, where DAMAS [3] can be used. In many applications, the number of sources are much smaller than the number of grid points in the scanning region. Note e_i the number of non-zero coefficients of $\mathbf{h}_i$, so the number of non-zero coefficients of $\mathbf{X}$ is $\sum_{i=1}^{q} e_i^2$. One can assume that $\mathbf{X}$ is sparse if $\mathbf{h}_i$ are sparse, since $\sum_{i=1}^{q} e_i^2 \leq (\sum_{i=1}^{q} e_i)^2 \leq L^2$. Therefore, considering the sparsity property of the sources, a convex optimization criterion called SC-DAMAS [4] has been given as:

$$\begin{cases} \hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{g} - \delta^2 \mathbf{A}_D \mathbf{x}\|_2^2 + \lambda_1 \|\mathbf{x}\|_1, \\ \texttt{subject to} \quad \mathbf{X} \geq \mathbf{0}, \end{cases} \tag{12}$$

where λ_1 is the sparsity constraint weighting parameter, $\mathbf{X} \geq \mathbf{0}$ means that the matrix $\mathbf{X}$ is semi-definite. Note that (12) is the basis pursuit (or LASSO) problem with a semi-definite constraint on $\mathbf{X}$.

However, as we will see in section 5, SC-DAMAS can not estimate perfectly $\mathbf{x}$ in the scenario of distributed sources. To ameliorate the performance, recall that $\mathbf{X} = \mathbf{H}\mathbf{R}_s\mathbf{H}^H \in \mathcal{R}^{L \times L}$, but the rank of $\mathbf{X}$ equals to the number of the sources q which is much smaller than L, we here wish to add a low rank constraint of $\mathbf{X}$.

Rank minimization problem is in general non-deterministic polynomial-time hard (NP-hard). Fortunately, we have several relaxation solutions. For example, the trace-norm relaxation as in [8]. Introduce the trace-norm minimum constraint to (12):

$$\begin{cases} \hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{g} - \delta^2 \mathbf{A}_D \mathbf{x}\|_2^2 + \lambda_1 \|\mathbf{x}\|_1 + \lambda_2 Tr(\mathbf{X}), \\ \texttt{subject to} \quad \mathbf{X} \geq \mathbf{0}, \end{cases} \tag{13}$$

where λ_2 is the low-rank constraint weighting parameter. Both SC-DAMAS and the proposed approach can be resolved by CVX toolbox [9].

4. DISTRIBUTION SHAPE AND SOURCE POWER RECOVERY

We can get $\hat{\mathbf{X}}$ once $\hat{\mathbf{x}}$ is estimated, then the source locations and the number of sources can be deduced by $\hat{\mathbf{X}}$, if λ_1 and λ_2 are properly chosen. Most source localization algorithms stop here and have $\hat{\mathbf{X}}$ as a final result. Here, we continue to recover the source distribution shape and estimate the source power.

Let $\hat{\mathbf{X}} = \mathbf{V}\Lambda\mathbf{V}^H$ be the eigenvalue decomposition (EVD) of $\hat{\mathbf{X}}$, where the columns $\mathbf{v}_1, \ldots, \mathbf{v}_L$ of the unitary matrix $\mathbf{V}$ are the eigenvectors of $\hat{\mathbf{X}}$ and the diagonal elements $\lambda_1, \ldots, \lambda_L$ of the diagonal matrix Λ are the corresponding eigenvalues such that $\lambda_1 \geq \lambda_k \geq \lambda_L, 1 \leq k \leq L$. Recalling again that $\mathbf{X} = \mathbf{H}\mathbf{R}_s\mathbf{H}^H = \sum_{i=1}^{q} \sigma_{si}^2 \mathbf{h}_i \mathbf{h}_i^H$, the sources are assumed to be disjoints to each other, therefore $\mathbf{h}_i$ is orthogonal to $\mathbf{h}_j$, for $i \neq j$. Due to the noise and ambiguities in the case where several sources have same power, $\mathbf{h}_i$ are linear combinations to the eigenvectors of $\mathbf{X}$. Under the hypothesis that the sources are disjoint from each other, we seek to find vectors $\mathbf{v}_{h_i}$, $1 \leq i \leq q$, which are linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_q$: $\mathbf{v}_{h_i} = \alpha_{i1}\mathbf{v}_1 + \ldots + \alpha_{iq}\mathbf{v}_q$, such that the non-zero support of each vector $\mathbf{v}_{h_i}$ are disjoint so as to be proportional to the distribution vectors $\mathbf{h}_1, \ldots, \mathbf{h}_q$. We set the coefficients $\alpha_{11} = \alpha_{21} = \ldots = \alpha_{q1} = 1$. For $\mathbf{v}_{h_i}$, the other coefficients can be obtained as follows:

$$\{\alpha_{i2}, \ldots, \alpha_{iq}\} = \underset{\{\alpha_{i2}, \ldots, \alpha_{iq}\}}{\operatorname{argmin}} \|\mathbf{v}_1 + \ldots + \alpha_{iq}\mathbf{v}_q\|_1^1$$
$$\texttt{s.t.}: \mathbf{v}_{h_i}^H \mathbf{v}_{h_1} = \mathbf{v}_{h_i}^H \mathbf{v}_{h_2} = \quad \ldots, \quad \mathbf{v}_{h_i}^H \mathbf{v}_{h_{i-1}} = 0. \tag{14}$$

(14) can also be solved by the CVX toolbox [9]. The order of $\mathbf{v}_{h_1}, \ldots, \mathbf{v}_{h_q}$ should be adjusted according to the result of (13) to be identical to the actual sources. Notice that $\sum_{k=1}^{L} \delta h_{i_k} = 1$, $\mathbf{h}_i$ can be estimated by:

$$\hat{\mathbf{h}}_i = \frac{\mathbf{v}_{h_i}}{\delta \|\mathbf{v}_{h_i}\|_1^1}. \tag{15}$$

Finally, to recover the signal powers $\sigma_{s1}^2, \ldots, \sigma_{sq}^2$, we normalize $\mathbf{v}_{h_i}$ to simplify the following calculations as: $\tilde{\mathbf{v}}_i = \mathbf{v}_{h_i} / \sqrt{\mathbf{v}_{h_i}^H \mathbf{v}_{h_i}}$. Noting that $\hat{\mathbf{h}}_i = \rho_i \tilde{\mathbf{v}}_i$, we have $\rho_i^2 = \hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i$. With $\hat{\mathbf{X}}\tilde{\mathbf{v}}_i =$ $\sum_{j=1}^{q} \sigma_{sj}^2 \hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^H \tilde{\mathbf{v}}_i = \sum_{j=1}^{q} \sigma_{sj}^2 \rho_j^2 \tilde{\mathbf{v}}_j \tilde{\mathbf{v}}_j^H \tilde{\mathbf{v}}_i = \lambda_i \tilde{\mathbf{v}}_i$, the power of the i^{th} source can be estimated by:

$$\hat{\sigma}_{si}^2 = \frac{\lambda_i}{\rho_i^2} = \frac{\lambda_i}{\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i}. \tag{16}$$

5. NUMERICAL SIMULATIONS

In this section, numerical simulations are presented to illustrate the validity of the proposed approach and compared with other approaches. In all simulations, a uniform linear array composed of $M = 20$ sensors spaced by half-wavelength is considered, $SNR = 10dB$, and $N = 1000$ snapshots. The angular separation between two points in the grid $\delta = 1°$. Five sources are transmitted from far-field with different DOAs $(33°, 41°, 50°, 60°, 74°)$ and different distribution shapes (point, uniform, point, Gaussian, Gaussian) and the signal power : $\sigma_{s1}^2 = 1, \sigma_{s2}^2 = 5, \sigma_{s3}^2 = 1, \sigma_{s4}^2 = 4, \sigma_{s5}^2 = 5$. For SC-DAMAS and the proposed method, $\lambda_1 = 10$, $\lambda_2 = 50$. Retaking the same definition of $\mathbf{x}$ and $\mathbf{X}$ as in section 3, we here slightly modify the criterion CMF as:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\hat{\mathbf{R}} - \delta^2 \mathbf{A}\mathbf{X}\mathbf{A}^H\|_2^2 + \lambda_1 \|\mathbf{x}\|_1 + \lambda_2 Tr(\mathbf{X}), \tag{17}$$

where a sparsity constraint is added to the conventional criterion, $\hat{\mathbf{R}}$ is estimated by the sensor measurements, $\lambda_1 = \lambda_2 = 1$.

In Figure 1 we compare the estimated $\hat{\mathbf{X}}$ by DAMAS-C, SC-DAMAS, CMF and the proposed method to the real one. We can see that the source location result of DAMAS-C is damaged by the edge effects. SC-DAMAS can not recover the contiguous support of the sources. CMF has a better performance for source localization, except that the magnitude do not match the real values (especially for the first source s_1). In contrary the proposed approach has a better performance for recovering $\mathbf{X}$.

Figure 2a illustrates the eigenvectors of $\hat{\mathbf{X}}$ estimated by the proposed approach corresponding to the sources, we can see that the supports of sources have been found, but each support is composed of a mixture of all the sources, which motivates the unmixing step in section 4 to recover the source distribution functions. Results in Figure 2b shows that the sources are unmixed and $\mathbf{h}_i$ are estimated.

In Figure 3 we compare the results for the estimation of the source distribution functions by SC-DAMAS, CLEAN [10], CMF, and our proposed approach to the real ones. We can see that SC-DAMAS and CLEAN have a good estimation performance for point sources (s_1 and s_3), as to the distributed sources (s_2, s_4 and s_5), CLEAN can only find the DOA, and SC-DAMAS can only find discrete points within the source support, which is identical in Figure 1. CMF and the proposed method have better performances.

In table 1 we illustrate the mean square error (MSE) of the signal power estimation, based on the estimation results $\hat{\mathbf{X}}$ by SC-DAMAS, CMF and the proposed method , (16) is used to estimate the source power, while other methods can not achieve good estimation results of $\mathbf{X}$ to go on to this step. We can see that the MSE of CMF and the proposed method are of the same order, while SC-DAMAS gets a MSE slightly larger. In addition, in the process of the experiments, we have noticed that the performance of CMF is much more sensitive to the values of λ_1 and λ_2 than the proposed method.

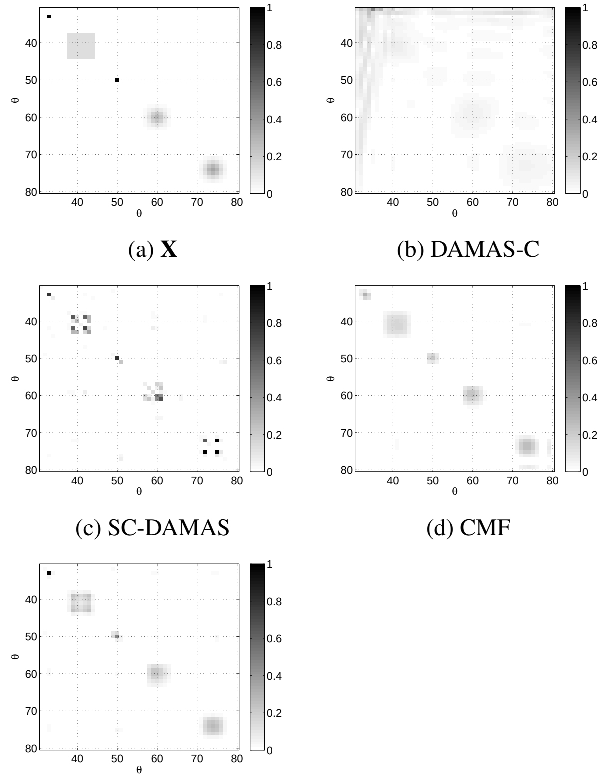
	s_1	s_2	s_3	s_4	s_5
SC-DAMAS	0.032	0.052	0.11	0.81	0.09
CMF	0.05	0.016	0.04	0.07	0.031
Proposed	0.023	0.04	0.023	0.09	0.025

Table 1: MSE of signal power estimation

6. CONCLUSION AND PERSPECTIVES

In this paper, an approach based on the sources' sparse property is proposed for the spatially CD source localization in far-field. In a first step, the supports of sources in the scanning region have been found; then, the distribution functions of each source have been separated and recovered; at last, the source powers have been estimated.

However, the computational time to resolve the convex criterion in the proposed approach augments rapidly as the grid number in the scanning region increases. In further study, we will focus on ameliorating the speed of the proposed approach, then apply this approach to real data.

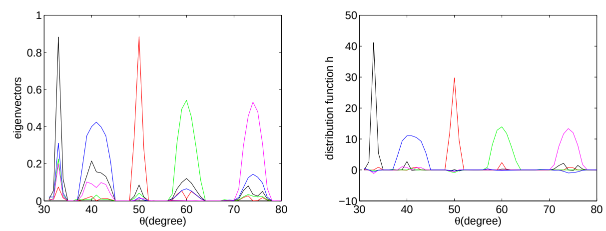


(a) $\mathbf{X}$ (b) DAMAS-C

(c) SC-DAMAS (d) CMF

(e) Trace-norm relaxation

Figure 1: Estimated source covariance matrix



(a) First five eigenvectors of $\hat{\mathbf{X}}$ (b) estimated $\hat{\mathbf{h}}_i$

Figure 2: Estimated sources distribution shapes

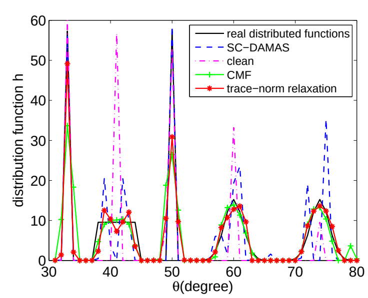


Figure 3: Real distribution functions and estimated distribution functions

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