LARGE REGION ACOUSTIC SOURCE MAPPING: A GENERALIZED SPARSE CONSTRAINED DECONVOLUTION APPROACH

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ABSTRACT

This paper presents a generalized multiple-point sparse constrained deconvolution approach for mapping acoustic noise sources in large regions using a movable array. Extended from our previous MPSC-DAMAS approach, we first derive a generalized inverse problem relating to the source powers and the array manifold using a generic beamformer and an explicit measurement noise model. We then propose a generalized MPSC-DAMAS (GMPSC-DAMAS) approach for resolving the inverse problem. A new parameter setting method based on a multiple-point minimum-variance-distortionless-response (MVDR) beamformer is also presented. The realizations of the GMPSC-DAMAS approach using the delay-and-sum (DAS) beamformer and the MVDR beamformer are evaluated. Simulation results show the proposed GMPSC-DAMAS approach achieves much lower absolute power estimation errors and processing time than the MPSC-DAMAS approach in terms of number of sources and robustness to measurement noise.

Index Terms— microphone arrays, source localization, acoustic source mapping, beamformer

1. INTRODUCTION

Environmental noise pollution in large urban areas becomes more and more serious and is known to produce significant adverse impact on health and longetivity. To address this problem, locating environmental noise sources and measuring their levels on a city- or even nation-scale are essential. However, deploying dense microphone arrays spanning the entire region of interest, or sequential noise measurements at thousands of locations on a dense grid on this scale would be prohibitively expensive [1]. In our previous study [2], we presented an acoustic measurement scheme using a small movable array to rapidly acquire measurements at many different locations, creating a kind of non-coherent virtual array of much larger aperture. The proposed multiple-point sparse-constrained deconvolution approach for mapping acoustic sources (MPSC-DAMAS) is more effective for mapping noise sources in large regions than the DAMAS [3], the SC-DAMAS [4] and the CMF [5] approaches that are originally proposed for aeroacoustic measurements of relatively small regions.

In this study, we further improve the MPSC-DAMAS approach by presenting a generalized MPSC-DAMAS (GMPSC-DAMAS) approach derived based on a generic beamformer and an explicit measurement noise model. Two implementations of the GMPSC-DAMAS approach are studied using the DAS beamformer and the minimum-variance-distortionless-response (MVDR) beamformer. We further present a new parameter-setting method based on a multiple-point MVDR (MP-MVDR) beamformer, which is not restricted to the normalized steering vectors as in the study for SC-DAMAS [4] and has no requirement of the number of sources as in the study for SC-RDAMAS [6]. Performance evaluations for the GMPSC-DAMAS approach are provided and compared with the MPSC-DAMAS approach in terms of number of sources, robustness to measurement noise, and processing time.

2. PROBLEM FORMULATION

Consider a wave field generated by a number of monopole acoustic sources, where the locations of the acoustic sources are considered to be sparse and inside the field. The wave field is divided into a dense grid of I "scanning" locations and every scanning location is considered as a potential source whose signal power is to be estimated, where the grid size determines the scanning resolution. Let the three-dimensional scanning location be denoted by $\mathbf{p}_i = [x_i, y_i, z_i]^T$ for $i = 1, ..., I$. A movable microphone array is used to sense the sound waveforms at K selected locations in the wave field, where the center of the microphone array corresponds to the

[*]This material is based on research/work supported in part by the research grant for the Human-Centered Cyber-physical Systems Programme at the Advanced Digital Sciences Center from Singapore's Agency for Science, Technology and Research (A*STAR) and in part by the Singapore Ministry of National Development and National Research Foundation under L2NIC Award No. L2NICCFP1-2013-7.

selected location. Let $\mathbf{p}_{k,m} = [x_{k,m}, y_{k,m}, z_{k,m}]^T$ denote the location of the mth microphone at the kth sensing location, where $m = 1, ..., M$ and M is the number of microphones in the array. Notice that in aeroacoustic measurements the microphone array is placed parallel to the scanning region. We present the acoustic measurements with the microphone array placed inside the scanning region. The M-dimensional frequency-domain array output vector at the kth sensing position can be expressed by the following signal model [2], [4]:

$$\mathbf{z}_k(n, \omega_l) = \mathbf{A}_k(\omega_l)\mathbf{s}_k(n, \omega_l) + \mathbf{v}_k(n, \omega_l), \quad (1)$$

where $n = 1, ..., N_k$ and N_k is the number of fast Fourier transform (FFT) segments recorded at the kth array position, ω_l denotes the lth interested frequency band, $\mathbf{s}_k(n, \omega_l) = [s_{k,1}(n, \omega_l), ..., s_{k,I}(n, \omega_l)]^T$ represents the vectorization of the source signals of all the scanning locations in the grid of interest at the kth array sensing position (note that not all scanning locations have source signals and the source signal is considered zero when there is no source signal at the scanning location), $(\cdot)^T$ denotes the transpose of the argument, $\mathbf{v}_k(n, \omega_l)$ is the additive spatially-white noise vector received by the array at the kth sensing position, $\mathbf{A}_k(\omega_l) \in \mathcal{C}^{M\times I}$ is the array manifold matrix at the kth sensing position and is defined as $\mathbf{A}_k(\omega_l) = [\mathbf{a}_k(\mathbf{p}_1, \omega_l), ..., \mathbf{a}_k(\mathbf{p}_I, \omega_l)]$, where $\mathcal{C}$ denotes the complex value space, and $\mathbf{a}_k(\mathbf{p}_i, \omega_l)$ is the steering vector corresponding to the ith scanning location and the kth sensing position and is modeled by

$$\mathbf{a}_k(\mathbf{p}_i, \omega_l) = \Big[\frac{1}{r_{i,k,1}} e^{-j\omega_l r_{i,k,1}/c}, ..., \frac{1}{r_{i,k,M}} e^{-j\omega_l r_{i,k,M}/c}\Big]^T \quad (2)$$

where $r_{i,k,m} = \|\mathbf{p}_i - \mathbf{p}_{k,m}\|$ is the Euclidean distance between the ith scanning location and the mth microphone at the kth array sensing position, and c is the speed of sound in air. In this work, we address the acoustic mapping problem of estimating the signal power level at each the scanning location i based on the observation vectors $\mathbf{z}_k(n, \omega_l)$ at different sensing positions. In the following, ω_l will be omitted from the presentation for simplicity.

3. GENERALIZED MULTIPLE-POINT SC-DAMAS

We assume that the acoustic sources and the additive noises are zero-mean mutually uncorrelated signals [2], [4]. Taking into account the signal model (1), the spatial covariance matrix at the kth sensing position is obtained by sample averaging as follows:

$$\mathbf{R}_k = \frac{1}{N_k}\sum_{n=1}^{N_k} \mathbf{z}_k(n)\mathbf{z}_k^H(n) = \mathbf{A}_k\mathbf{X}_k\mathbf{A}_k^H + \mathbf{V}_k, \quad (3)$$

where $\mathbf{R}_k \in \mathcal{C}^{M\times M}$, $\mathbf{X}_k = \frac{1}{N_k}\sum_{n=1}^{N_k} \mathbf{s}_k(n)\mathbf{s}_k^H(n) \in \mathcal{C}^{I\times I}$ represents the signal sample covariance matrix at the kth sensing position, and $\mathbf{V}_k = \frac{1}{N_k}\sum_{n=1}^{N_k} \mathbf{v}_k(n)\mathbf{v}_k^H(n) \in \mathcal{C}^{M\times M}$ represents the measurement noise covariance matrix at the kth sensing position, where $\mathcal{R}$ denotes the real value space and $(\cdot)^H$ denotes the conjugate transpose of the argument. When a sufficient number of segments is available ($N_k \gg 1$), we can have the following good approximations: $\mathbf{X}_k \approx \text{diag}\{x_{k,1}, \cdots, x_{k,I}\}$ and $\mathbf{V}_k \approx \text{diag}\{\sigma_{k,1}^2, \cdots, \sigma_{k,M}^2\}$ where $x_{k,i}$ denotes the source power and $\sigma_{k,m}^2$ denotes the noise power. In our previous study [2], we assumed $\mathbf{V}_k = \sigma_k^2\mathbf{I}$ where $\mathbf{I}$ is the identity matrix. Here, we consider a more general case of unequal additive noise powers. We further assume the sources are long-time stationary such that the signal covariance matrices $\mathbf{X}_k$ are the same at all the sensing positions and denoted by $\mathbf{X} = \text{diag}\{x_1, \cdots, x_I\}$. Then, $\mathbf{R}_k$ in (3) can be further written as

$$\mathbf{R}_k = \mathbf{A}_k\mathbf{X}\mathbf{A}_k^H + \mathbf{V}_k. \quad (4)$$

Let $\mathbf{w}_{k,i}, i = 1, ..., I, k = 1, ..., K$ denote the weight vector of a generic beamformer for the scanning location i and the array position k. The power estimate for the scanning location i and the array position k is given by

$$y_{k,i} = \mathbf{w}_{k,i}^H\mathbf{R}_k\mathbf{w}_{k,i}. \quad (5)$$

Substituting (4) into (5), we can obtain a convolution representation for $y_{k,i}$:

$$\begin{aligned} y_{k,i} &= \mathbf{w}_{k,i}^H\mathbf{A}_k\mathbf{X}\mathbf{A}_k^H\mathbf{w}_{k,i} + \mathbf{w}_{k,i}^H\mathbf{V}_k\mathbf{w}_{k,i} \\ &= \mathbf{u}_{k,i}^H\mathbf{X}\mathbf{u}_{k,i} + \tilde{\mathbf{w}}_{k,i}^T\boldsymbol{\sigma}_k \\ &= \mathbf{c}_{k,i}^T\mathbf{x} + \tilde{\mathbf{w}}_{k,i}^T\boldsymbol{\sigma}_k, i = 1, ..., I, \end{aligned} \quad (6)$$

where $\mathbf{u}_{k,i} = \mathbf{A}_k^H\mathbf{w}_{k,i}$, $\mathbf{c}_{k,i} = [\|u_{k,i,1}\|^2, ..., \|u_{k,i,I}\|^2]^T$ represents the convolution coefficients where $u_{k,i,j}, j = 1, ..., I$ is the jth element of $\mathbf{u}_{k,i}$, $\mathbf{x}$ is the vector of the diagonal elements of $\mathbf{X}$ and represents the signal powers of the potential sources at the scanning region of interest, $\tilde{\mathbf{w}}_{k,i} = [\|w_{k,i,1}\|^2, ..., \|w_{k,i,M}\|^2]^T$ where $w_{k,i,m}, i = m, ..., M$ is the mth element of $\mathbf{w}_{k,i}$, and $\boldsymbol{\sigma}_k$ is the vector of the diagonal elements of $\mathbf{V}_k$. Let $\mathbf{y}_k = [y_{k,i}, ..., y_{k,I}]^T$ denote the power estimate vector at the array position k. The vector form of (6) at the array position k is expressed as

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x} + \mathbf{W}_k\boldsymbol{\sigma}_k, k = 1, ..., K, \quad (7)$$

where $\mathbf{C}_k = [\mathbf{c}_{k,1}, ..., \mathbf{c}_{k,I}]^T \in \mathcal{R}^{I\times I}$ is the linear convolution matrix (also known as array's point spread function) and $\mathbf{W}_k = [\tilde{\mathbf{w}}_{k,1}, ..., \tilde{\mathbf{w}}_{k,I}]^T \in \mathcal{R}^{I\times M}$ is a linear matrix.

By stacking up all vectors $\mathbf{y}_k$, we obtain the following linear system of equations

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \vdots \\ \mathbf{C}_K \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{W}_1 & \cdots & \mathbf{0}_{I\times M} \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{I\times M} & \cdots & \mathbf{W}_K \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}_1 \\ \vdots \\ \boldsymbol{\sigma}_K \end{bmatrix} \quad (8)$$

or

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{W}\boldsymbol{\sigma}, \quad (9)$$

where $\mathbf{y} = [\mathbf{y}_1^T, ..., \mathbf{y}_K^T]^T$, $\mathbf{C} = [\mathbf{C}_1^T, ..., \mathbf{C}_K^T]^T$, $\mathbf{W} = \text{diag}\{\mathbf{W}_1, ..., \mathbf{W}_K\}$, and $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_1^T, ..., \boldsymbol{\sigma}_K^T]^T$. When $K = 1$, the inverse model (8) is considered as a generalization of the inverse model studied in DAMAS [3] and SC-DAMAS [4] using a generic beamformer and explicit noise model. When $K > 1$, the inverse model (8) is a generalization of the inverse model studied in our previous work [2].

Estimating the source power vector $\mathbf{x}$ in (8) is an inverse problem with $\mathbf{y}$, $\mathbf{C}$ and $\mathbf{W}$ known. We extend our previous MPSC-DAMAS approach [2] and propose a generalized MPSC-DAMAS (GMPSC-DAMAS) approach as follows:

$$\begin{cases} \min_{\mathbf{x},\boldsymbol{\sigma}} \mathcal{J}(\mathbf{x}, \boldsymbol{\sigma}) = ||\mathbf{y} - \mathbf{C}\mathbf{x} - \mathbf{W}\boldsymbol{\sigma}||_2^2 \\ \text{s.t. } \|\mathbf{x}\|_1 \leq \beta, x_i \geq 0, \sigma_{k,m}^2 \geq 0, \end{cases} \quad (10)$$

where i, k, m take values from 1 to I, K, M, respectively, $x_i \geq 0$ enforces every element of $\mathbf{x}$ to be nonnegative, and $\|\mathbf{x}\|_1$ is the ℓ_1 norm of $\mathbf{x}$. Here, the user parameter β is the upper bound of the total source power, which is to be discussed next. Note that the GMPSC-DAMAS approach jointly estimates the source power $\mathbf{x}$ and the additive noise power $\boldsymbol{\sigma}$. There are $M \times K$ unknown additive noise powers in (10), and the number reduces to K if the additive noise powers are equal at all the microphones and further reduces to 1 if the additive noise powers are equal at all sensing positions.

3.1. Parameter Setting using a Multiple-Point MVDR Beamformer

The determination of β is important for achieving sparse solution of (10). Due to the application constraints in the previous approaches [3], [4], [6], a new way of determining β has to be found for practicality. In this section, we present a multiple-point MVDR (MP-MVDR) beamformer for the determination of the parameter β.

Let $\mathbf{w}_{k,i}$ denote the weight vector of the MP-MVDR beamformer at the scanning location i and the array sensing position k. Under the uncorrelated assumption for the sources, the power estimate of the MP-MVDR beamformer is formulated as follows:

$$\hat{x}_i = \frac{1}{N_k} \sum_{n=1}^{N_k} \Big| \sum_{k=1}^{K} \mathbf{w}_{k,i}^H \mathbf{z}_k(n) \Big|^2 = \sum_{k=1}^{K} \mathbf{w}_{k,i}^H \mathbf{R}_k \mathbf{w}_{k,i} \quad (11)$$

where $i = 1, ..., I$, the spatial covariance matrix $\mathbf{R}_k$ is given in (3), and the weight vectors $\mathbf{w}_{k,i}, k = 1, ..., K$ are chosen to minimize the output power $\hat{x}_i$ by the following linearly constrained minimization problem:

$$\begin{cases} \min_{\mathbf{w}_{k,i}} \sum_{k=1}^{K} \mathbf{w}_{k,i}^H \mathbf{R}_k \mathbf{w}_{k,i}, k = 1, 2, ..., K \\ \text{s.t. } \sum_{k=1}^{K} |\mathbf{w}_{k,i}^H \mathbf{a}_k(\mathbf{p}_i)|^2 = 1, \end{cases} \quad (12)$$

where $\mathbf{a}_k(\mathbf{p}_i)$ is given in (2). Let us define $\alpha_k = \mathbf{w}_{k,i}^H \mathbf{a}_k(\mathbf{p}_i)$, then the constrained minimization problem (12) is equivalent to the following constrained minimization problem

$$\sum_{k=1}^{K} \Big(\min_{\mathbf{w}_{k,i}} \mathbf{w}_{k,i}^H \hat{\mathbf{R}}^{(k)} \mathbf{w}_{k,i} \text{ s.t. } \mathbf{w}_{k,i}^H \mathbf{a}^{(k)}(\mathbf{p}_i) = \alpha_k \Big), \quad (13)$$

where the constraint on α_k is $\sum_{k=1}^{K} |\alpha_k|^2 = 1$. The constrained minimization problem inside the bracket of (13) is the standard MVDR beamformer and it is straightforward to obtain

$$\hat{x}_i = \sum_{k=1}^{K} \frac{\alpha_k^2}{\mathbf{a}_k^H(\mathbf{p}_i) \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{p}_i)}. \quad (14)$$

The power expression in (14) implies that regardless of the relative scaling, α_k^2, between the various sensing positions, the optimal solution of the MVDR beamformer should be always used at each sensing position. The value of α_k in (14) is determined by the following constrained minimization problem

$$\begin{cases} \min_{\alpha_k} \sum_{k=1}^{K} \frac{\alpha_k^2}{\mathbf{a}_k^H(\mathbf{p}_i) \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{p}_i)}, k = 1, ..., K \\ \text{s.t. } \sum_{k=1}^{K} |\alpha_k|^2 = 1. \end{cases} \quad (15)$$

By the triangle inequality theorem [7], the optimal setting is

$$\alpha_{k_{min}} = 1 \text{ and } \alpha_{k \neq k_{min}} = 0, \quad (16)$$

where k_{min} is defined as the sensing position k where the MVDR beamformer produces the minimum output power for the scanning location i over all the K sensing positions. Now we can rewrite the expression (14) as

$$\hat{x}_i = \min\{\hat{x}_{i,1}, ..., \hat{x}_{i,K}\}, \quad (17)$$

where $\hat{x}_{i,k} = \frac{1}{\mathbf{a}_k^H(\mathbf{p}_i) \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{p}_i)}, k = 1, ..., K$. The power estimate in (17) is the output power of the proposed MP-MVDR beamformer at the scanning location i. The parameter β in (10) is therefore set to $\beta = \sum_{i=1}^{I} \hat{x}_i$.

3.2. Realization using DAS and MVDR

We now present the realizations of the GMPSC-DAMAS approach using the DAS and MVDR beamformers, which is quite straightforward. When the DAS beamformer is applied, the weight vector $\mathbf{w}_{k,i}$ in (5) has the following form [8]

$$\mathbf{w}_{k,i}^{\{DAS\}} = \frac{1}{M} \Big[r_{i,k,1} e^{-j\omega_l r_{i,k,1}/c}, ..., r_{i,k,M} e^{-j\omega_l r_{i,k,M}/c} \Big]^T, \quad (18)$$

and when the MVDR beamformer is applied, the weight vector $\mathbf{w}_{k,i}$ in (5) has the following form [9]

$$\mathbf{w}_{k,i}^{\{MVDR\}} = \frac{\mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{p}_i)}{\mathbf{a}_k^H(\mathbf{p}_i) \mathbf{R}_k^{-1} \mathbf{a}_k(\mathbf{p}_i)}. \quad (19)$$

Substituting the weight vectors (18) and (19) in (6), respectively, two different implementations of the GMPSC-DAMAS approach can be realized and the sparse solutions will be obtained via readily available interior-point methods with the free *Self-Dual Minimization* software package [10].

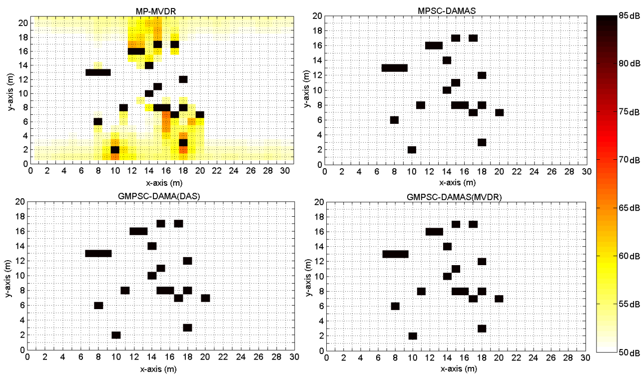

Fig. 1. Illustration of the tested scanning region and the noise-mapping results of MP-MVDR, MPSC-DAMAS, GMPSC-DAMAS(DAS) and GMPSC-DAMAS(MVDR).

4. EXPERIMENTAL RESULTS

In the simulation setup, we considered a $30m \times 20m$ environmental noise region, and the scanning locations were set on a $1m \times 1m$ grid. There are a total of 600 source locations as shown in Fig 1. The x-axis was considered as the horizontal direction and the y-axis was the vertical direction. A circular array consisting of 24 microphones with a radius of $0.72m$ was placed at $K = 7$ locations for data acquisition. The coordinates of the array sensing locations were $\{(0,10),(5,10),\cdots,(30,10)\}$ where the heights of the sources and the array were assumed the same and omitted. The acoustic sources and the additive noises were synthetic complex Gaussian zero-mean signals and the frequency of interest was 1kHz. A total of $N_k = 1000$ FFT segments were used at each sensing location. For all simulations, the source positions were randomly generated and the results were averaged over 500 instances. The measured absolute power error is defined as: $10\log\{\text{abs}(\|\hat{\mathbf{x}}\|_1 - \|\mathbf{x}\|_1)\}$.

Fig 1 shows the noise-mapping results of the tested algorithms for a setting of 20 noise sources with equal source power of 85 dB and the signal to noise ratio (SNR) of 35 dB. All algorithms have accurate location estimates and MP-MVDR has obviously noisy power estimation due to the convolution effect. Fig 2 shows the absolute power errors obtained by the four algorithms. The number of sources were 10, 20, 30, and 40 with equal source power of 85 dB and SNR of 35 dB. It is observed that MP-MVDR has the highest absolute power errors. The other three algorithms achieve lower absolute power errors than MP-MVDR. Compared to MPSC-DAMAS, GMPSC-DAMAS(DAS) and GMPSC-DAMAS(MVDR) achieve much lower absolute power errors. GMPSC-DAMAS(MVDR) has the lowest errors. Fig 3 shows the results of 20 sources with equal power of 85 dB and SNR ranging from 5 dB to 35 dB. Similar observations are obtained that GMPSC-DAMAS(DAS) and GMPSC-DAMAS(MVDR) achieve much lower absolute power errors than MPSC-DAMAS, and GMPSC-DAMAS(MVDR) has the lowest error. The above results also imply that GMPSC-DAMAS with adaptive beamformers can outperform GMPSC-DAMAS with fixed beamformers, and MP-MVDR works well for the parameter settings. The averaged computational times were $0.13s$ for MP-MVDR, $34.76s$ for MPSC-DAMAS, $17.28s$ for GMPSC-DAMAS(DAS), and $16.69s$ for GMPSC-DAMAS(MVDR) on a 64-bit personal computer with a 3.0 GHz processor and 10 Gbytes of random access memory (RAM) running MATLAB.

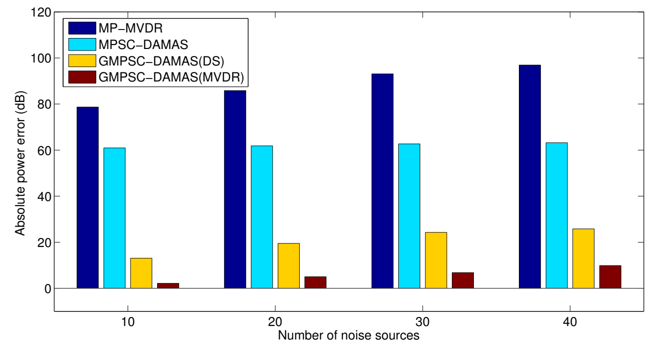

Fig. 2. Absolute power error comparison of noise mapping with different number of acoustic sources.

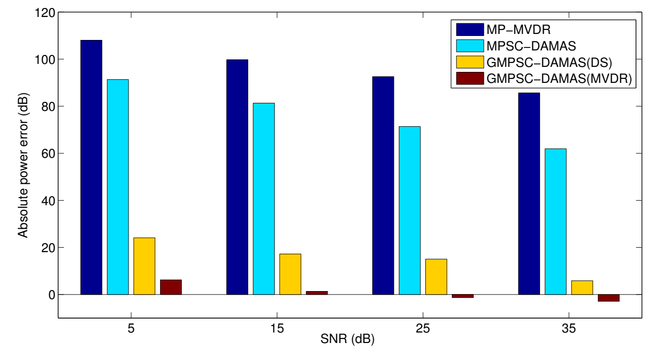

Fig. 3. Absolute power error comparison of noise mapping with different SNRs.

5. CONCLUSIONS

We have presented a generalized deconvolution approach of GMPSC-DAMAS for mapping environmental noise sources in large regions. Two implementations of GMPSC-DAMAS using DAS and MVDR have been evaluated and achieved much better power estimation than the previous MPSC-DAMAS approach. GMPSC-DAMAS has potential robustness to array mismatches with possible implementation of robust beamformers. Our future work is to evaluate GMPSC-DAMAS with array mismatches and test on real array data.

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