LUCKY RANGING WITH TOWED ARRAYS IN UNDERWATER ENVIRONMENTS SUBJECT TO NON-STATIONARY SPATIAL COHERENCE LOSS

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ABSTRACT

This paper presents our new results on lucky ranging utilizing a towed array in environments subject to unknown fluctuating spatial coherence losses. We derive a lucky maximum likelihood range estimator based on the probabilistic assumption that each collected data snapshot is either coherent or purely incoherent with some probability. Our lucky range estimator can be interpreted as first ranking the coherence quality of each data snapshot according to an array gain-like quantity during the parameter search, followed by accumulation of likelihood surfaces out of data snapshots of high spatial-coherence. This effectively avoids the wash-out or the smearing results encountered in the traditional processing procedures of utilizing a long integration time without a prior screening for the data spatial-coherence. An important advantage of the lucky approach is that it makes no prior assumptions about the signal spatial coherence loss model. This estimator has greatly improved robustness over the conventional estimator when coherence is low and time-varying.

Index Terms— Array Signal Processing, Passive Ranging, Spatial Coherence, Passive Sonar, Underwater Acoustics

1. INTRODUCTION

Utilizing a long array or a field of distributed sensors, a passive sonar system can estimate the range to a distant source by measuring its wave front curvature based on the assumption of spherical spreading [1]. This is due to the fact that under an ideal environment a distributed array of large aperture can sense the wave front curvature and hence can estimate the location (range and bearings) of an emission source. However, estimation of range in real underwater environments is often problematic because wave front curvature (WFC) ranging is highly sensitive to spatial coherence losses or equivalently, wave front distortions. Spatial coherence losses can result in large range estimation errors and biases and are usually of greater importance in WFC ranging systems than signal-to-noise ratio. These coherence losses are a consequence of wave front distortions induced by space and time-dependent fluctuations in the water temperature and hence sound speed from phenomena like internal waves, fronts, and random medium effects. One solution is to try explicitly incorporate a model of the spatial coherence losses into the maximum likelihood estimator to improve performance [2, 3, 4]. Along this line, we also developed different, non-coherent as well as coherent, solutions for passive ranging during the past few years [5, 6]. Applying such wide sense stationary (WSS) results to a real system in practice still faces challenges because the spatial coherence loss characteristics are usually *time-varying and not known beforehand*.

The possibility that the spatial coherence may actually fluctuate in ocean environments over short time scales, say even during the algorithm integration time, is typically ignored by array and distributed sensor algorithm designers. When testing solutions on real data sets, we often saw rapid variations in spatial coherence and consequently poor range estimation. Such non-stationary degradation in spatial coherence on distributed arrays, caused by wave front distortions, bears similarity to the atmospheric turbulence present in ground based telescopes that causes blurring of planets, stars, and galaxies. Astronomers have observed that if telescope images are recorded *at fast enough frame rates*, the atmospheric turbulence-induced blurring fluctuations become frozen and that in a few percent of images the blurring is momentarily small enough to yield high quality images [7]. This is the premise behind *lucky imaging*. By selecting the best images and combining them, an image can be generated that has much higher resolution than the normal seeing-limited image without resorting to expensive adaptive optics [7].

Motivated by the lucky imaging technique [7] used to overcome atmospheric turbulence in adaptive optics and/or ground-based telescopes in astronomy, in this work we propose a new paradigm for wavefront curvature range estimation and array processing in underwater environments subject to unknown and time-varying spatial coherence loss. Empirical evidence from real world underwater acoustic data analysis suggests that even in environments with apparently low spatial coherence, when the non-stationary data is analyzed *at a much finer time scales*, there are brief moments (a.k.a. lucky moments) when the wave front has little distortion. That is, rather than assuming a specific stochastic model for the spatial coherence for the whole data set, we simply assume that each short data frame is either good (with fully spatial coherent) or bad (with spatial coherence lost). In figure 1, we provide an illustration on the time-varying wavefront distortions using a sequences of simulated wavefronts from turbulence induced variations in water temperature. If these lucky moments can be detected, they can be utilized to estimate range more accurately, hence, to overcome coherence loss degradations typically experienced over long integration times.

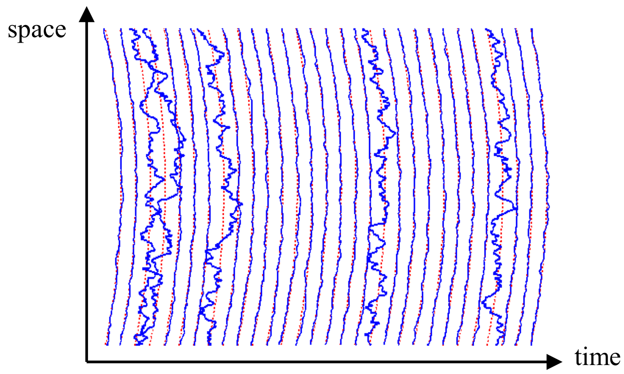

Fig. 1: Propagating wavefronts experience time-varying distortion due to the turbulence effect, resulting lucky as well as unlucky moments.

2. DATA FORMULATION FOR LUCKY MLE

To properly model the time-vary spatial coherence loss, we first introduce a probabilistic assumption that each collected data snapshot is either coherent, i.e., rank-1, or purely incoherent consisting of IID noise with some probability. Specifically, the received snapshot comprised of signal and noise, $\vec{x}_k = A_k \vec{s}_{r,\theta} + \vec{n}_k$, in a given kth time frame, is assumed as either being fully coherent with probability ε or incoherent (i.e., converted to IID noise spatially) with probability $1-\varepsilon$. Under the complex Gaussian assumption, we have come up with the following mixture distribution data model,

$$p(\vec{x}_k;\varepsilon) = \begin{cases} p(\vec{x}_k), & \text{with probability } 1-\varepsilon \\ p(\vec{x}_k; r,\theta), & \text{with probability } \varepsilon \end{cases} \quad (1)$$
$$= (1-\varepsilon)\cdot p(\vec{x}_k) + \varepsilon \cdot p(\vec{x}_k; r,\theta)$$

Here $p(\vec{x}_k) = CN(\vec{0}, (\sigma_s^2/N + \sigma_n^2) I_{N\times N})$, and $p(\vec{x}_k; r,\theta) = CN(\vec{0}, \sigma_s^2 \vec{s}_{r,\theta}\cdot \vec{s}_{r,\theta}^H + \sigma_n^2 I_{N\times N})$ stochastically model the received array data snapshot $\vec{x}_k$ under two extreme cases of spatial coherence, respectively. The vector $\vec{s}_{r,\theta}$ stands for array's steering vector towards the source of interest at the location (r,θ). Note that data vector has zero-mean, $E\{\vec{x}_k\} = \vec{0}$, and the structural covariance matrix, $\mathrm{Cov}\{\vec{x}_k\} = (1-\varepsilon)\cdot(\sigma_s^2/N + \sigma_n^2) I_{N\times N} + \varepsilon\cdot(\sigma_s^2 \vec{s}_{r,\theta}\cdot \vec{s}_{r,\theta}^H + \sigma_n^2 I_{N\times N})$. Due to the passive nature of system and data, the source information is embedded in the data covariance matrix. In the above distribution model we made further assumption: the total power from the received data perceived by the sensing array, $E\{\|\vec{x}_k\|^2\} = \sigma_s^2 + N\sigma_n^2$, remains to be the same, no matter whether or not the wave-front is coherent. In addition, the N dimensional steering vector is assumed to be normalized $\|\vec{s}_{r,\theta}\| = 1$. Given K frames of IID data, this model leads to a mixture-based maximum likelihood estimator (MLE) of the form,

$$\hat{r},\hat{\theta} = \arg\max_{r,\theta} \log\left(\prod_{k=1}^{K} p(\vec{x}_k;\varepsilon)\right)$$
$$= \arg\max_{r,\theta} \sum_{k=1}^{K} J_k(r,\theta) \quad (2)$$

With

$$J_k(r,\theta) = \log\left(1 + \frac{\varepsilon\cdot\beta}{1-\varepsilon} \frac{\exp\left\{-\vec{x}_k^H\left(\sigma_n^2 I + \sigma_s^2 \vec{s}_{r,\theta}\vec{s}_{r,\theta}^H\right)^{-1}\vec{x}_k\right\}}{\exp\left\{-\vec{x}_k^H \vec{x}_k / (\sigma_n^2 + \sigma_s^2/N)\right\}}\right) \quad (3)$$

Here, $\beta = \dfrac{(\sigma_n^2 + \sigma_s^2/N)^N}{\sigma_n^{2(N-1)}(\sigma_n^2+\sigma_s^2)} = \dfrac{(1+\rho_{SNR})^N}{1+N\rho_{SNR}}$, is related to array size N and data quality in terms of the SNR $\rho_{SNR} = \dfrac{\sigma_s^2}{N\sigma_n^2}$.

3. INTERPRETATION OF THE LUCKY MLE

To provide further simplifications as well as in-depth understandings on the MLE results in (3), let us exam some extreme cases of practical importance. In perfectly coherent environments ($\varepsilon \to 1$), this estimator becomes the conventional near-field beamformer as follows,

$$\hat{r},\hat{\theta} = \arg\max_{r,\theta} \log\left(\prod_{k=1}^{K} p(\vec{x}_k; r,\theta)\right)$$
$$= \arg\min_{r,\theta}\left(\sum_{k=1}^{K} \vec{x}_k^H\left(\sigma_n^2 I + \sigma_s^2 \vec{s}_{r,\theta}\vec{s}_{r,\theta}^H\right)^{-1}\vec{x}_k\right)$$
$$= \arg\min_{r,\theta}\left(\sum_{k=1}^{K} \vec{x}_k^H\left(\frac{1}{\sigma_s^2+\sigma_n^2}\vec{s}_{r,\theta}\vec{s}_{r,\theta}^H + \frac{1}{\sigma_n^2}P_{\vec{s}_{r,\theta}}^{\perp}\right)\vec{x}_k\right)$$
$$= \arg\max_{r,\theta}\left(\sum_{k=1}^{K} \vec{x}_k^H\left(\frac{\sigma_s^2}{\sigma_n^2(\sigma_s^2+\sigma_n^2)}\vec{s}_{r,\theta}\vec{s}_{r,\theta}^H - \frac{1}{\sigma_n^2} I\right)\vec{x}_k\right)$$
$$= \arg\max_{r,\theta} \sum_{k=1}^{K} \left| \vec{s}_{r,\theta}^H \vec{x}_k \right|^2$$

However, in environments with high SNR but poor chance of coherence, i.e., large ρ_{SNR} but small ε, it becomes,

$$\hat{r},\hat{\theta} = \arg\max_{r,\theta} \sum_{k=1}^{K} \frac{\exp\left\{-\vec{x}_k^H\left(\sigma_n^2 I + \sigma_s^2 \vec{s}_{r,\theta}\vec{s}_{r,\theta}^H\right)^{-1}\vec{x}_k\right\}}{\exp\left\{-\vec{x}_k^H\vec{x}_k/(\sigma_n^2+\sigma_s^2/N)\right\}}$$
$$= \arg\max_{r,\theta} \sum_{k=1}^{K} \frac{\exp\left\{-\vec{x}_k^H\left(\frac{1}{\sigma_n^2}\vec{s}_{r,\theta}\vec{s}_{r,\theta}^H - \frac{1}{\sigma_n^2}I\right)\vec{x}_k\right\}}{\exp\left\{-\vec{x}_k^H\vec{x}_k/(\sigma_n^2+\sigma_s^2/N)\right\}}$$
$$= \arg\max_{r,\theta} \sum_{k=1}^{K} \exp\left\{\frac{\left|\vec{s}_{r,\theta}^H\vec{x}_k\right|^2 - \vec{x}_k^H\vec{x}_k}{\sigma_n^2}\right\}$$

Recall our previous normalization assumption $\left\|\vec{s}_{r,\theta}\right\| = 1$, *the exponentiation operation term* in the above lucky MLE is actually connected to the spatial coherence,

$$\gamma_k \equiv \exp\left\{\frac{\left|\vec{s}_{r,\theta}^H\vec{x}_k\right|^2 - \vec{x}_k^H\vec{x}_k}{\sigma_n^2}\right\}$$
$$= \exp\left\{-\frac{\left\|\vec{x}_k\right\|^2}{\sigma_n^2}\left(1-\rho^2(\vec{s}_{r,\theta},\vec{x}_k)\right)\right\}.$$

Here, the quantity $\rho^2(\vec{s}_{r,\theta},\vec{x}_k) = \frac{\left|\vec{s}_{r,\theta}^H\vec{x}_k\right|^2}{\left\|\vec{x}_k\right\|^2\cdot\left\|\vec{s}_{r,\theta}\right\|^2}$ is simply the *magnitude squared coherence* [9, 10] between the kth data snapshot perceived by an array of sensors and the array's steering vector towards the source of interest. Therefore, the exponentiation operation *non-linearly enhances* the range-bearing surface contributions from snapshots with strong focusing, i.e., coherence, while suppressing ones with poor focusing, i.e., no coherence. Consequently, the lucky range estimator can be interpreted as first ranking the coherence quality of each data snapshot according to an array gain-like quantity during the parameter search, followed by accumulation of likelihood surfaces out of data snapshots of high spatial-coherence. An important advantage of the lucky approach is that it makes no prior assumptions about the signal spatial coherence loss model.

Without the knowledge of background noise power, we can translate the exponentiation term to an array-gain related quantity, $\left|\vec{s}_{r,\theta}^H\vec{x}_k\right|^2 - \vec{x}_k^H\vec{x}_k$, by ranking the array output power (when its input is properly normalized). In doing so, we can identify lucky moments for the task of lucky ranging.

The above practice can be justified from the following analysis by noticing the fact that,

$$E\left\{\left|\vec{s}_{r,\theta}^H\vec{x}_k\right|^2 - \vec{x}_k^H\vec{x}_k\right\} = \vec{s}_{r,\theta}^H \operatorname{cov}\{\vec{x}_k\}\vec{s}_{r,\theta} - (\sigma_s^2 + N\sigma_n^2)$$
$$= \left(\frac{1-\varepsilon}{N} - 1\right)\cdot(\sigma_s^2 + N\sigma_n^2) + \varepsilon(\sigma_s^2 + \sigma_n^2)$$

Using Jensen's inequality, we have,

$$E\left\{\exp\left(\frac{\left|\vec{s}_{r,\theta}^H\vec{x}_k\right|^2 - \vec{x}_k^H\vec{x}_k}{\sigma_n^2}\right)\right\}$$
$$\geq \begin{cases} \exp\{-(N-1)\}, & \text{for } \varepsilon = 1 \\ \exp\{-(N-1)(1+\rho_{\text{SNR}})\}, & \text{for } \varepsilon \approx 0 \end{cases}$$

Therefore, one can see that under the operation condition of a decent SNR, $\rho_{\text{SNR}} >> 1$, on average the contribution of the exponentiation terms γ_k's to the likelihood function in the lucky MLE from a low spatial-coherence data frame is significantly smaller than that from a high spatial-coherence data frame. This justifies our proposed lucky ranging procedure of ranking the frame-based likelihood surface according to the estimated spatial coherence based on an array gain-like quantity and only keep the significant terms.

4. CONNECTION TO THE KULLBACK-LEIBLER DIVERGENCE FOR COHERENCE TEST

It is interesting for us to point out an important observation that the above MLE based results can also be derived from the information theoretical perspective. In an effort to discriminate two different probability density functions (pdfs parameterized by ε) characterizing the presence or absence of spatial-coherence in data, i.e., *coherence present* with a pdf $p(\vec{x};r,\theta) \equiv p_1(\vec{x};\varepsilon \neq 0)$ versus *coherence absent* with a pdf $p(\vec{x}) \equiv p_0(\vec{x};\varepsilon = 0)$, a commonly used measure for such a task is the well-known Kullback-Leibler (KL) divergence [8]. For a given data vector $\vec{x}$ with two possible pdfs, the KL divergence is defined as,

$$D_{KL}\left(p_1(\vec{x};\varepsilon\neq 0)\middle\|p_0(\vec{x};\varepsilon=0)\right) \equiv E_{p_1}\left\{\log\frac{p_1(\vec{x};\varepsilon\neq 0)}{p_0(\vec{x};\varepsilon=0)}\right\}$$
$$= E_{p_1}\left\{\log\frac{(1-\varepsilon)\cdot p(\vec{x}) + \varepsilon\cdot p(\vec{x};r,\theta)}{p(\vec{x})}\right\}$$

Now, the task of coherence detection can be cast as maximizing the *estimated KL divergence* based on the given K IID data vectors/frames from population $p_1(\vec{x};\varepsilon \neq 0)$. Therefore, for the KL divergence test with respect to the baseline pdf $p_0(\vec{x};\varepsilon = 0)$, we adopt a sample-mean estimator (to replace the ensemble mean) as the *estimated* KL divergence, as follows,

$$\hat{D}_{KL}\left(p_1(\vec{x};\varepsilon \neq 0) \| p_0(\vec{x};\varepsilon = 0)\right) = \frac{1}{K}\sum_{k=1}^{K}\log\left(\frac{(1-\varepsilon)\cdot p(\vec{x}_k)+\varepsilon\cdot p(\vec{x}_k;r,\theta)}{p(\vec{x}_k)}\right). \quad (4)$$

Hence, if the spatial-coherence is present due to the emission source, we can estimate its location by maximizing the above estimated KL divergence in the range-bearing space. That is,

$$\hat{r},\hat{\theta} = \arg\max_{r,\theta} \hat{D}_{KL}\left(p_1(\vec{x};\varepsilon \neq 0) \| p_0(\vec{x};\varepsilon = 0)\right) = \arg\max_{r,\theta} \sum_{k=1}^{K}\log\left((1-\varepsilon)\cdot p(\vec{x}_k)+\varepsilon\cdot p(\vec{x}_k;r,\theta)\right), \quad (5)$$

which is equivalent to the MLE results in equations (2-3).

5. SIMULATION EXAMPLES

To illustrate the effective of the proposed lucky MLE for passive ranging application, we include some results from a computer simulated example in the following figure. In the fig.2, we show two range-bearing results, one from the conventional beamforming (CBF) processing and another from the lucky MLE solution, respectively. In the simulation, the chance of getting fully coherent data frame is set as 27%, that is $\varepsilon = 0.27$. In the conventional processing, we utilize a long integration time for the cross-spectral density matrices (CSDM) calculation, the smearing results in the integrated likelihood function (shown by the range-bearing plot) can be seen from the top figure. While with the lucky MLE, only the spatially coherent data frames of proper integration time are retained (through the coherence ranking and selection), a sharp peak can be seen with much reduced smearing effect in the range-bearing plot.

It should be pointed out that for passive ranging application, in order to obtain un-ambiguous range estimate, CSDM from multiple frequency bins with reasonable signal strength and high levels of spatial coherence need to be identified and utilized with a similar procedure as outlined in the lucky MLE above.

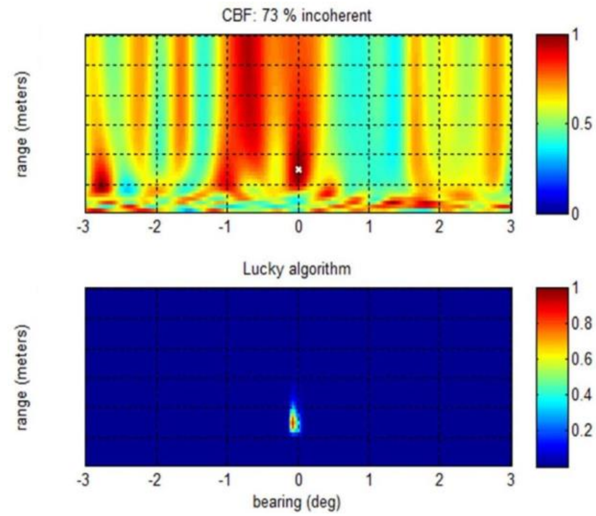

Fig. 2: Comparison of range-bearing results between a conventional array beamforming based approach and the lucky MLE based solution. The top figure shows range-bearing scan results using a conventional beamforming while the bottom figure shows the scan results using a lucky beamforming algorithm. The true location (range and bearing) of the emission source of interest is marked by a white cross.

6. CONCLUSIONS

To address the practical issue of non-stationary spatial coherence loss and to improve the performance of the WFC based passive ranging in underwater acoustic environments, we derived a lucky maximum likelihood estimate (MLE) based on a probabilistic model on the data quality in terms of spatial coherence. The exponentiation operation in the proposed lucky MLE automatically weights and accumulates data snapshots according to a spatial coherence measure. This can be translate into a practical procedure of first ranking data quality according to the array-gain like quantity in the estimation stage, followed by the accumulation of likelihood surfaces out of data snapshots of high spatial-coherence. By casting the problem as a hypothesis test for discriminating the case of spatial coherence present against the case of spatial coherence absent, we established an equivalence between the lucky MLE and the Kullback-Leibler (KL) divergence. We also tested our solution on real underwater acoustic data collected. Some illustrative simulation results are included to demonstrate the effectiveness of the MLE in reducing the smearing effect in range-bearing plots.

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