# COOPERATIVE JOINT SYNCHRONIZATION AND LOCALIZATION USING TIME DELAY MEASUREMENTS

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## ABSTRACT

In this paper a novel algorithm is proposed for joint synchronization and localization in ad hoc networks. The proposed algorithm is based on broadcast messaging, with number of messages linear to the number of nodes, versus quadratic for techniques based on two-way message exchange. The identifiability of network synchronization problem is improved by introducing localization constraints. Hence, the proposed algorithm does not require a full set of measurements. Numerical results are provided using a model based on wireless LAN specifications. In scenarios with missing data, the proposed algorithm significantly improves synchronization and localization performance compared to commonly used techniques.

Index Terms- Synchronization, localization, ad-hoc network

## 1. INTRODUCTION

In recent years, wireless services that utilize user's location became an integral part of mobile technology, hence there is a need to improve positioning accuracy in mobile devices [1]. Wireless ad hoc networks will play a central role in future communications [2]. A reliable ad hoc localization solution would be beneficial to many applications such as indoor positioning. Time-delay based ranging, which is a widely used technique for precise localization, has strict synchronization requirements. Moreover, precise network synchronization is crucial for increasing deployment of real-time services on packet switching networks. In this paper, we address the problem of synchronizing all the nodes in an ad hoc network, considering both clock offsets and clock rates, while estimating their unknown locations cooperatively using time delay measurements.

Synchronization and positioning in wireless networks have been extensively studied in recent years [3,4]. The GPS as a legacy solution for synchronization and localization has a limited availability in indoor environments. Commonly used network synchronization algorithms [5,6] mainly rely on two-way messaging (TWM) measurements, with number of messages quadratic to the number of nodes. This complexity in communication becomes linear using one-way messaging in a broadcast fashion. The broadcast-based algorithms proposed in [7-9] do not provide sufficient accuracy for localization. The scheduled broadcast synchronization (SBS) method proposed in [10] provides high precision for wireless localization. It requires all the nodes to transmit in their turn and receive the messages form all the other nodes. Joint localization and synchronization has the potential of relaxing this requirement. Several such algorithms have been proposed for underwater, acoustic and wireless networks [11–14], which do not use cooperative measurements, i.e., they are considering a single target node. Moreover, these methods do not compensate for clock skews, i.e, deviation of clock rate. Hence, they are not precise enough for wireless localization. A cooperative algorithm employs measurements between target nodes, in addition

to that between anchors and targets, to improve estimation performance and identifiability of the problem. The cooperative algorithms proposed in [15–17] do not compensate for clock skews either. The non-cooperative algorithms proposed in [18–21] are compensating for clock skews. They also require TWM measurements. Broadcastbased algorithms proposed in [22–24] are not cooperative either. The cooperative message passing techniques proposed in [25, 26] are expensive in communication and computation, and also require TWM measurements.

In this paper, a novel low complexity algorithm, called alternating synchronization and localization (ASL), is proposed for joint estimation of clock offsets, skews, and positions of the nodes in an ad hoc network. It is a cooperative technique based on broadcast messages transmitted by each node in a local neighborhood, and received by the other nodes. The proposed ASL algorithm improves the identifiability of network synchronization problem. Hence, it relaxes the requirement for having all the nodes transmitting and receiving messages. To the best knowledge of authors, there is no existing algorithm for cooperative joint synchronization and localization in wireless networks that has linear complexity in communication and is robust in the face of missing data.

The rest of this paper is organized as follows. Section 2 introduces a measurement model. A basic algorithm for synchronization and ranging is described in Section 3 which does not consider location constraints. The proposed ASL algorithm is described in Section 4. Some numerical results are presented in Section 5. Finally, Section 6 concludes the work.

## 2. MEASUREMENT MODEL

Assume a network comprised of *n* nodes with positions  $\mathbf{x}_i \in \mathbb{R}^2$ , *i* = 1,...,*n*, from which *m* are *anchor* nodes with known locations  $\mathbf{x}_i = \mathbf{a}_i$ , and n-m are target nodes with unknown locations to be estimated. Indices i = 1, ..., m are assigned to anchor nodes and i = m+1, ..., n to target nodes. The network nodes are transmitting messages to measure propagation delays between the nodes. The measurements are performed according to scheduled broadcast synchronization (SBS) protocol, proposed in [10]. Each node transmits a measurement message in a predetermined order, and others receive it and estimate its time-of-arrival (TOA). Every transmit time and estimated receive TOA are recorded by the nodes as timestamps. Messages are sent in a one-way broadcast fashion over the wireless channel, and can be received by the other nodes in the vicinity. Transmissions are scheduled in advance, and the procedure is controlled by each node. Measurements are continued for at least two complete rounds over all the nodes, since two sets of measurements are needed to estimate clock skews as well; and compensate for them. Note that, the proposed ASL algorithm relaxes the requirement of having all the timestamps available.

The propagation delay between two nodes *i*, *j* is modeled as an

unknown deterministic parameter denoted by  $d_{ij}$ . The signals travel with a fixed speed, i.e., speed of light for radio waves. We *normalize this speed to 1* in order to have the same scale for time and distance. Hence,  $d_{ij}$  is also the Euclidean distance between nodes *i*,*j*. If  $\tau_i$ and  $\tau_{ij}$  are receive and transmit time instances for a message from node *i* to *j*, we have

$$\tau_{ij} = \tau_i + d_{ij},\tag{1}$$

The times  $\tau_i$  and  $\tau_{ij}$  are actual times not clock readings of the nodes, i.e., they are times with respect to an ideal clock that is described as follows. Assuming a constant clock frequency, a linear model for the internal clock of a node is given by

$$c(t) = \theta + \rho t, \tag{2}$$

where  $\theta$  is the clock offset, and  $\rho$  is the clock rate. These offset and rate parameters are modeled as unknown deterministic constants. An ideal clock should have  $\theta = 0$  and  $\rho = 1$ . If any node is assigned as a reference clock, then its clock parameters are set to ideal values. The goal of network synchronization is to estimate clock offsets and rates for all the nodes with respect to a reference clock.

The estimation of clock offsets and rates is based on *transmit* and receive timestamps of the messages between nodes. A transmit timestamp  $T_i^k$  is the clock reading at node *i* for transmission time of its *k*-th message, which is given by

$$T_i^k = c_i(\tau_i^k) = \rho_i \tau_i^k + \theta_i.$$
(3)

The event of transmission is controlled by the same clock used for timestamping. Hence transmit timestamps contain no random error. A receive timestamp  $T_{ij}^k$  is the clock reading at node *j* for a message from node *i* in measurement round *k*. By using (1) and (2), a receive timestamp is given by

$$T_{ij}^{k} = c_{j}(\tau_{ij}^{k}) + \epsilon = \rho_{j}\tau_{ij}^{k} + \theta_{j} + \epsilon_{j}^{k}$$
  
$$= \rho_{j}(\tau_{i} + d_{ij}) + \theta_{j} + \epsilon_{j}^{k}, \qquad (4)$$

where  $\epsilon_i^k$  is the random error of delay estimation in receiver. Receive timestamp is the estimated TOA of signal corresponding to lineof-sight (LOS) propagation path. It contains a random estimation error, which is related to measurement noise, multipath propagation of signals, and the resolution of the estimation. We assume an i.i.d. Gaussian model for delay estimation error, i.e.  $\epsilon_i^k \sim \mathcal{N}(0, \sigma_{\epsilon}^2)$ . In wireless networks, the assumption is valid for high-resolution delay estimation if LOS path is detectable [27-29]. If K rounds of measurements are performed in a fully connected network, then Kn(n-1) pairs of transmit and receive timestamps are collected. Note that, TOA estimation problem is not considered in this paper, but it is assumed to be estimated reliably. Obtaining such estimates in a multipath propagation environment is a challenging task, which may be done using high-resolution delay estimation techniques [30,31]. Furthermore, the internal delays that a signal experiences in a mobile device may be significantly larger than the required precision of time synchronization. Constant delays in transceiver front-end can be measured and compensated for. Delays in digital circuitry are inversely proportional to the clock rate of the node, hence can be considered as a part of clock offset [10].

## 3. ESTIMATING CLOCK PARAMETERS AND DISTANCES

### 3.1. Estimating clock rates

The clock rate of node *i* can be directly estimated from the time difference of messages received from it at different time instances,

independently from the other parameters. At least two rounds of measurements are required, but better estimates can be obtained by acquiring more measurements. The difference between two consecutive receive timestamps (4) is given by

$$T_{ij}^{k+1} - T_{ij}^{k} = \rho_j \left( T_i^{k+1} - T_i^k \right) + \delta_j^k.$$
 (5)

By substituting (3) in (5), we get

$$T_{ij}^{k+1} - T_{ij}^k = \rho^j \left( \frac{T_i^{k+1} - \theta_i}{\rho_i} - \frac{T_i^k - \theta_i}{\rho_i} \right) + \delta_j^k, \tag{6}$$

where  $\delta_i^k \sim \mathcal{N}(0, 2\sigma_{\epsilon}^2)$  is an error term. After re-arranging we have

$$\rho_j = \rho_i \left( T_{ij}^{k+1} - T_{ij}^k \right) / \left( T_i^{k+1} - T_i^k \right) + \delta_{ji}^k, \quad k = 1, \dots, K-1, \quad (7)$$

where  $\delta_{ji}^k \sim \mathcal{N}\left(0, 2\sigma_\epsilon^2 \rho_i / (T_i^{k+1} - T_i^k)\right)$  is an error term. If node *i* transmits at regular intervals, the distribution of the error term does not depend on *k*. The system of equations in (7), with *n*-1 degrees of freedom and *n* unknown parameters, is not identifiable. One needs to introduce an extra constraint that defines a reference clock in the network. The equation  $\rho_1 = 1$  sets node 1 as a reference clock. Then, all other clock rates can be found by a successive substitution, as

$$\hat{\rho}_j = \hat{\rho}_i \sum_{k=1}^{K-1} \left( T_{ij}^{k+1} - T_{ij}^k \right) / \left( T_i^{k+1} - T_i^k \right).$$
(8)

Note that, the choice of a reference for clock rates affects the distance estimation results. If no reference clock is given, one may use an average clock rate of the network as a reference, e.g., by setting the geometric mean of clock rates to one. Equation (7), along with a geometric mean constraint, may be linearized by a logarithmic transformation to derive a better estimator.

#### 3.2. Estimating clock offsets and distances

Assuming that clock rates are already estimated as described above, we formulate a problem to estimate clock offsets and pairwise distances. By substituting  $\tau_i^k$  from (3) in (4), we get

$$T_{ij}^{k} = \rho_{j}(T_{i}^{k} - \theta_{i}) / \rho_{i} + \rho_{j}d_{ij} + \theta^{j} + \epsilon_{j}^{k}, \quad i \neq j,$$
  
$$\Rightarrow \quad \rho_{j}d_{ij} + \theta_{j} - \rho_{ji}\theta_{i} = T_{ij}^{k} - \rho_{ji}T_{i}^{k} + \epsilon_{j}^{k}, \quad i \neq j,$$
(9)

where  $\rho_{ji} = \rho_j / \rho_{ji}$  is a relative clock rate of two nodes. Theses equations are not linearly independent in parameters  $\theta_i$ . One needs to add an extra constraint for clock offsets in order to define a reference clock, e.g.,  $\theta_1 = 0$  if node 1 is a reference clock. The choice of reference node for clock offsets does not affect the solution. The parameters  $d_{ij}$  and  $\theta_i$  may be found by solving a system of linear equations, given by

$$\theta_{j} - \hat{\rho}_{ji}\theta_{i} + \hat{\rho}_{j}d_{ij} = y_{ij}^{k} + \epsilon_{j}^{k}, \quad (i,j,k) \in I$$
  

$$\theta_{1} = 0,$$
  

$$d_{ij} = ||\mathbf{a}_{i} - \mathbf{a}_{j}||_{2}, \quad i,j \in \{1,...,m\}$$
(10)

where  $y_{ij}^k = T_{ij}^k - \hat{\rho}_{ji}T_i^k$  is a time measurement between nodes *i*, *j* at round *k*, and *I* is a set of index 3-tuples (*i*, *j*, *k*) for available measurements. The last row of (10) indicates known distances between anchor nodes. Each entry in y represents a timing measurement between a pair of nodes in one direction, ordered lexicographically by indices (*i*, *j*, *k*)  $\in I$ . The number of unknown distances between target nodes is (n-m)(n-m-1)/2. The number of unknown

clock offsets is n-1. Hence, the number of parameters in (10) is at most p=n(n+1)/2-1 for m=0. The number of measurements  $y_{ij}^k$  is q=n(n-1) for one round of measurements, which is larger than the number of parameters, p < q, and the system of equations becomes overdetermined. Since the error term  $\epsilon$  obeys zero-mean Gaussian distribution, an optimal solution to (10) may be found using the least square minimization, given by

$$\widehat{\boldsymbol{\vartheta}} = \arg\min \|\mathbf{A}\boldsymbol{\vartheta} - \mathbf{y}\|_2, \tag{11}$$

where  $\vartheta = [\theta_2,...,\theta_n,d_{12},d_{13},...,d_{(n-1)n}]^T$  contains *p* unknown parameters. The coefficients matrix  $\mathbf{A}^{q \times p}$  can be written as  $\mathbf{A} = [\mathbf{B} | \mathbf{C}]$ , where  $\mathbf{B} \in \mathbb{R}^{q \times n-1}$  contains the coefficients of  $\theta_i$ , and  $\mathbf{C} \in \mathbb{R}^{q \times n(n-1)/2}$  the coefficients of  $d_{ij}$ . If a complete round of measurements is performed in a fully connected network, then **B** and **C** are both full rank with linearly independent rows [10]. However, **A** can easily become rank-deficient if some measurements are missing. In this case, a so-called basic solution to (11) may be found using QR factorization with column pivoting [32, chap. 5]. Since such a solution is not unique, it is not necessarily an exact solution. This basic solution can be improved by ASL algorithm proposed in next section.

## 4. JOINT ESTIMATION OF CLOCK OFFSETS, DISTANCES AND LOCATIONS

The equations given in (10) do not constrain the distance estimates to conform with Euclidean geometry. That is, the estimated distances may not correspond to any valid point configuration on a plane, especially if **A** is rank-deficient. The Euclidean distance between target nodes may be added to (10) as constraints, which results in a new optimization problem given by

$$\begin{array}{ll} \underset{\{d_{ij},\theta_{i}\}}{\text{minimize}} & \sum_{i,j,k\in \mathcal{I}} (\hat{\rho}_{j}d_{ij} + \theta_{j} - \hat{\rho}_{ji}\theta_{i} - y_{ij}^{k})^{2} \\ \text{subject to} & \sum_{i=1}^{n} \theta_{i} = 0, \\ & \|\mathbf{x}_{i} - \mathbf{x}_{j}\|_{2} = d_{ij}, \quad \forall i \neq j \\ & \mathbf{x}_{i} = \mathbf{a}_{i}, \quad i = 1, ..., m \end{array}$$

$$(12)$$

In this joint synchronization and localization problem,  $\mathbf{x}_i$  are also optimization variables. The quadratic equality constraints make (12) a difficult non-convex problem. Hence, we are going to solve it in an alternating manner. That is, we fix clock offset parameters and estimate distances, and vice versa.

#### 4.1. Estimating distances given clock offsets

The constraints  $\|\mathbf{x}_i - \mathbf{x}_j\|_2 = d_{ij}$  in (12) imply that the matrix of squared distances  $\mathbf{D} = [d_{ij}^2]$  is an Euclidean distance matrix, i.e.,  $\mathbf{D} \in \mathbb{EDM}^N$  where  $\mathbb{EDM}^N$  is the cone of Euclidean distance matrices. Moreover, it implies that the affine rank of  $\mathbf{D}$  is no larger than two, i.e. rank( $(\mathbf{VDV}) \leq 2$ , where the orthogonal matrix  $\mathbf{V} = \mathbf{I}_n - \frac{1}{n} \mathbf{11}^T$  is a geometric centering matrix [33, chap. 5]. Assume that an estimate of  $\hat{\mathbf{\theta}}$  is given, e.g., obtained from (11). A new symmetric matrix of squared distance measurements  $\mathbf{H} = [h_{ij}^2]$  is defined as

$$h_{ij} = \frac{1}{2K} \sum_{k=1}^{K} (w_{ij}^k + w_{ji}^k), \quad (i, j, k), (j, i, k) \in I,$$
(13)

where  $w_{ij}^k = (y_{ij}^k - \hat{\theta}_j + \hat{\rho}_{ji}\hat{\theta}_i)/\hat{\rho}_j$ . Multiple measurements corresponding to each pairwise distance in both directions are averaged, since

the error obeys zero-mean Gaussian distribution. If a measurement  $w_{ij}^k$  is not available then only the measurement in other direction is used. If both  $w_{ij}^k$  and  $w_{ji}^k$  are not available, the corresponding entries  $h_{ij}$  and  $h_{ji}$  are set to zero. A new optimization problem may be written as

$$\begin{array}{ll} \underset{D}{\text{minimize}} & \|\mathbf{V}(\mathbf{D}-\mathbf{H})\mathbf{V}\|_{\mathrm{F}}^{2},\\ \text{subject to} & \operatorname{rank}(\mathbf{V}\mathbf{D}\mathbf{V}) \leq 2,\\ & \mathbf{D} \in \mathbb{E}\mathbb{D}\mathbb{M}^{N}. \end{array}$$
(14)

By solving this problem one finds the closest Euclidean distance matrix (EDM) to **H** for centered positions in the sense of Frobenius norm. This problem is equivalent to (12) with known clock offsets [33, chap. 7]. The problem (14) is not convex. However, with non-negative symmetric matrix **H**, it is equivalent to Euclidean projection on a rank-2 subset of a positive semidefinite (PSD) cone [33, chap. 7]. Hence, it has a well-known solution given by

$$\widehat{\mathbf{G}} = -\frac{1}{2}\mathbf{V}\mathbf{D}^{*}\mathbf{V} = -\frac{1}{2}\mathbf{U}_{2}\mathbf{\Lambda}_{2}\mathbf{U}_{2}^{\mathrm{T}} \in \mathbf{S}^{N-1},$$
(15)

where  $U_2 \Lambda_2 U_2^T$  is a truncated eigenvalue decomposition of VHV, and  $D^* \in \mathbb{EDM}^N$  is an optimal EDM solution to (14). A PSD matrix  $\hat{G} \in \mathbb{S}^N_+$  is an estimated Gramian matrix, defined as  $G \triangleq X^T X$ , for centered locations. The truncated eigenvalues matrix  $\Lambda_2$  is given by

$$\mathbf{\Lambda}_2 = \begin{bmatrix} \max(0, \lambda_1) & 0\\ 0 & \max(0, \lambda_2) \end{bmatrix}, \tag{16}$$

where  $\lambda_i$  are eigenvalues of **VHV** in a non-increasing order. The matrix of corresponding eigenvectors is  $U_2 = [u_1, u_2]$ . A linear map from a Gramian matrix to an EDM is given by

$$\widehat{\mathbf{D}} = \operatorname{diag}(\mathbf{G}^*)\mathbf{1}^{\mathrm{T}} + \mathbf{1}\operatorname{diag}(\mathbf{G}^*)^{\mathrm{T}} - 2\mathbf{G}^*,$$
(17)

where diag(.) denotes a vector of diagonal entries [33, chap. 5].

#### 4.2. Estimating clock offsets given distances

With given delay estimates  $\hat{d}_{ij}$ , the equations in (10) can be written as a new least-squares problem, as

$$\boldsymbol{\theta} = \arg\min \|\mathbf{B}\boldsymbol{\theta} - \mathbf{g}\|_2, \tag{18}$$

where **B** is a subset of **A** containing the coefficients of  $\theta_i$ . A new measurement vector **g** contains  $g_{ij}^k = y_{ij}^k - \hat{\rho}_j \hat{d}_{ij}$ , ordered lexicographically by indices  $(i,j,k) \in I$ . If **B** is rank-deficient a basic solution may be obtained using QR factorization with column pivoting.

## 4.3. Estimating locations given distances

By using (17), the locations can be found in a similar way to multidimensional scaling with map matching (MDS-MAP) algorithm [34]. The relationship between Gramian matrix and EDM is given by  $VGV = -\frac{1}{2}VDV$ . Hence, we have

$$\mathbf{V}\hat{\mathbf{X}}^{\mathrm{T}}\hat{\mathbf{X}}\mathbf{V} = -\frac{1}{2}\mathbf{V}\mathbf{D}^{*}\mathbf{V} = \frac{1}{2}\mathbf{U}_{2}\mathbf{\Lambda}_{2}\mathbf{U}_{2}^{\mathrm{T}}$$
(19)

An estimate of the centered locations  $\widetilde{\mathbf{X}}$  is obtained by

$$\widetilde{\mathbf{X}} \stackrel{\text{\tiny def}}{=} \mathbf{V} \widehat{\mathbf{X}} = \frac{1}{\sqrt{2}} \mathbf{\Lambda}_2^{\frac{1}{2}} \mathbf{U}_2.$$
(20)

With sufficient number of anchor nodes, at least three in 2D, it is possible to transform  $\widetilde{\mathbf{X}}$  to an absolute coordinate system based on anchor positions. This rigid-body transformation is given by

$$\widehat{\mathbf{X}} = \mathbf{W}\widetilde{\mathbf{X}} + \mathbf{\mu}_A,\tag{21}$$

where  $\mathbf{W}^{2\times 2}$  is an orthogonal transformation matrix, i.e., rotation and reflection. The translation vector  $\boldsymbol{\mu}_A = \frac{1}{m} X_A \mathbf{1}$  is the geometric center of anchor nodes, where  $\mathbf{X}_A \in \mathbb{R}^{2\times m}$  contains true anchor positions. The orthogonal matrix  $\mathbf{W}$  is obtained by solving a mapmatching problem, given by

minimize 
$$\|\widetilde{\mathbf{X}}_A - \mathbf{W}\mathbf{X}_A\|_F^2$$
, subject to  $\mathbf{W}\mathbf{W}^{\mathrm{T}} = \mathbf{I}$ , (22)

where  $\widetilde{\mathbf{X}}_A$  is a subset of  $\widetilde{\mathbf{X}}$  corresponding to anchor nodes. The solution is given by  $\mathbf{W} = \mathbf{U}_A \mathbf{V}_A^{\mathsf{T}}$ , where  $\mathbf{U}_A, \mathbf{V}_A$  are left and right singular vectors of  $\widetilde{\mathbf{X}}_A \mathbf{X}_A^{\mathsf{T}}$ .

## 4.4. Proposed Algorithm

An alternating maximization algorithm can be constructed using the estimators for parameters  $\theta$ , $\tau$ ,X. The proposed ASL algorithm is given in Table 1. The non-convex optimization problem (12) is solved as systems of linear equations and eigenvalue decompositions. Hence, the computational complexity of the algorithm is  $O(n^3)$ . Note that, the complexity in communication is linear to the number of nodes due to one-way measurement.

Table 1 Alternating synchronization and localization (ASL) algorithm

- 1: Find clock rates by successive equations given in (8).
- 2: Find an initial estimate of distances and clock offsets by solving a system of linear equations in (11).
- 3: Estimate pairwise distances, by eigenvalue decomposition given in (15), and then applying (17).
- 4: Find clock offsets by solving a system of linear equations in (18).
- 5: Go to step 3 until convergence, normally very few iterations.
- 6: Find relative locations by eigenvalue decomposition given in (20) and map it to the anchor coordinates using (21).

## 5. RESULTS

Numerical results are produced using a simulated network of 10 nodes randomly placed in a 10×10 meters area, from which 4 are anchor nodes. The values of clock parameters are randomly drawn from uniform distributions,  $\theta_i \sim \mathcal{U}(-25\mu s, +25\mu s)$  and  $\rho_i \sim \mathcal{U}(1-25\times10^{-6},1+2\times10^{-6})$ , conforming to wireless LAN specifications [35]. The error in receive timestamps, i.e., TOA estimation error, is drawn form a zero-mean Gaussian distribution. Figure 1 shows the synchronization and localization errors versus number of silent nodes. A silent node does not transmit any message, hence all corresponding measurements are missing. The clock rates of silent nodes are set to one. The root mean square (RMS) error in TOA estimates is 1 meters. The proposed ASL algorithm is compared with results from (11) and MDS-MAP algorithm, denoted by "Separate". The performance of both methods is similar with a full dataset. However, the ASL algorithm is robust since it significantly improves the estimation performance when there is missing data. Figure 2 shows the synchronization and localization errors versus the error in TOA estimates. The number of silent nodes is two. The results for a case with full data is also shown. The proposed ASL algorithm is closely matching the results obtained using the full dataset. Figure 3 shows the synchronization error of the ASL algorithm versus the number of iterations. The algorithm converges quickly, and may be typically stopped after five iterations.



Fig. 1: (a) RMS error of synchronization (clock offsets). (b) RMS error of localization. With increasing number of silent nodes, the proposed joint algorithm outperforms the initial estimates. RMS error of TOA estimates is 1 meters.



**Fig. 2**: (a) RMS error of synchronization (clock offsets). (b) RMS error of localization. Two nodes are silent. The proposed ASL algorithm is closely matching the results with a full dataset.



Fig. 3: The RMS error of synchronization (clock offsets) versus number of iterations. The ASL algorithm converges quickly, i.e., after four iterations. The RMS error in TOA estimates is 1 meters, and number of silent nodes is two.

## 6. CONCLUSIONS

In this paper a novel algorithm, called ASL, is proposed for joint synchronization and localization in ad hoc networks. A broadcast measurement protocol is used with number of required transmissions linear to the number of nodes. The non-convex problem of joint synchronization and localization is broken down into two simple sub-problems, which are solved in an alternating manner. The ASL algorithm improves the identifiability of network synchronization problem by introducing localization constraints. Hence, it can be reliably applied to the scenarios with missing data. Numerical results are provided using a network model based on wireless LAN specifications, showing robustness in the face of missing data.

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