# LONG-CPI MULTI-CHANNEL SAR BASED GROUND MOVING TARGET INDICATION

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# ABSTRACT

The detection of ground moving targets with arbitrary linear motion from an airborne multi-channel radar via a long coherent processing interval is considered. A reparameterization of the target's linear motion is developed which allows for maximizing the target signal energy in synthetic aperture radar images, and decouples the array processing from the image formation process. The algorithm generates estimates of the target's along-track and cross-track velocity components, making a unique determination of the target's motion parameters possible.

*Index Terms*— Ground moving target indication (GMTI), synthetic aperture radar (SAR)

# 1. INTRODUCTION

The problem of detecting ground moving targets from an airborne radar platform has been studied regularly for over 60 years. The main objectives of the ground moving target indication (GMTI) problem are to detect ground moving targets, and to accurately estimate their relevant parameters, i.e., their location and velocity parameters. Several methods using either a single, or multiple antennas, have been described in the literature.

Single-channel GMTI algorithms have been developed which exploit a moving target's Doppler signature over a large coherent processing interval (CPI) [1, 2, 3], use a series of short CPI synthetic aperture radar (SAR) images to estimate target motion [4, 5], or attempt to focus the moving target signature via compensating for the target's motion in a parametric [6, 7, 8, 9] or non-parametric [10] fashion. Of particular relevance to the current work is the realization, described in [3, 11, 6, 12, 7], that a target with linear motion, which can be described by four parameters, namely, x location, ylocation, x velocity, and y velocity, may have its SAR image signature described via *three* parameters, apparent x location, apparent ylocation, and relative velocity (see Section 3). Algorithms utilizing this fact exploit the relative velocity difference between the stationary ground clutter, the moving target, and the radar platform to enhance the target signal-to-clutter ratio, making them more sensitive to along-track moving targets than cross-track moving targets.

The set of multi-channel GMTI algorithms may be divided into those which require two channels, and those that are applicable to systems with two or more channels. Different from the previously described single-channel GMTI focusing algorithms, multi-channel GMTI approaches typically exploit the Doppler shift caused by the moving target's radial velocity. Therefore, multi-channel GMTI algorithms are more sensitive to cross-track moving targets than alongtrack moving targets. The well-known displaced phase center antenna (DPCA) and along-track interferometry (ATI) algorithms [13, 14, 15, 16] are examples of GMTI techniques designed for two channel SAR systems. The ATI approach was generalized to arrays with more than two channels for the purpose of ocean surface velocity estimation in [17]. While these algorithms may provide satisfactory detection performance, they suffer from a cross-range / velocity ambiguity problem, and are therefore unable to uniquely determine the target location and velocity parameters. A novel dual-channel long-CPI GMTI approach was presented in [11]. Compared to its singlechannel counterpart, this method can significantly improve detection performance, especially for cross-track moving targets, but placed stringent requirements on the platform motion.

Many types of multi-channel GMTI algorithms designed for systems equipped with more than two antennas have been developed, such as the velocity SAR [18, 19]. Perhaps the most well-known of these algorithms is the space-time adaptive processing (STAP) [20, 21] approach and its reduced dimension variants [22, 23], for example, post-Doppler STAP. More recently, approaches tailored to multi-channel SAR GMTI have been developed. An extension of the moving target signature reparameterization from the single-channel case to the multi-channel case was provided in [24]. Additionally, two approaches were put forth in [25] and [26] called imaging STAP (ISTAP) and extended DPCA (EDPCA), respectively. Different from the single-channel or multi-channel GMTI algorithms, these methods seek to both match the target motion profile as well as leverage an array of antennas to gain sensitivity to both along-track and cross-track moving targets. However, while the algorithms entail matching the target signature explicitly, no exact relationships between the target parameters and SAR image formation algorithms are provided.

The algorithm presented in this paper aims to combine the advantages of the previously described approaches to perform long-CPI multi-channel SAR GMTI. The algorithm leverages the SAR image signature reparameterization detailed in [11, 6] to maximize target signal power and increase sensitivity to along-track linearly moving targets via a long CPI. Furthermore, the array of antennas is used to suppress the ground clutter and estimate the target's radial velocity component. Because both the relative velocity and radial velocity of the target are estimated by the algorithm, a unique determination of the target's true location, speed, and heading is possible. An implementation of the backprojection (BP) [6, 12] SAR imaging algorithm for forming focused SAR imagery with respect to a particular relative velocity is also described.

The remainder of the paper is structured as follows. Section 2 introduces the geometry and associated data model of the proposed long-CPI multi-channel SAR GMTI algorithm. Section 3 discusses the target signature reparameterization, as well as describes the GMTI algorithm in detail. Section 4 presents applications of the long-CPI multi-channel SAR GMTI algorithm. Algorithm performance is investigated by examining simulated point spreading functions generated by the approach using the parameters of the

This work was supported in part by the National Science Foundation (NSF) CCF-1218388.

GOTCHA SAR GMTI data set [27], as well as an example of processing field collected GOTCHA SAR GMTI data.

## 2. GEOMETRY AND DATA MODEL

Consider an *M*-element along-track GMTI system with the distance between the *m*th  $(m = 0, 1, \dots, M - 1)$  antenna and the first antenna denoted by  $\Delta_m$ , i.e.,  $\Delta_0 = 0$ . As shown in Figure 1, the platform is assumed to move with a speed of v (in units of meters per pulse repetition interval (PRI)) along the x direction at a height of h. Note that the x direction is the along-track direction, and the y direction is the cross-track direction. The location of the *m*th antenna at the *n*th pulse  $(n = -\frac{N}{2}, -\frac{N}{2} + 1, \dots, \frac{N}{2} - 1)$  can be written as  $(nv - \Delta_m, 0, h)$ . Consider a ground moving target at a position with coordinates (x, y, 0) at pulse n = 0, and velocity (av, bv) (|a| < 1 and |b| < 1). Hence, the target location at the *n*th pulse will be (x + avn, y + bvn, 0).



Fig. 1. Geometry of an airborne multi-channel SAR system.

Denote the radar carrier frequency by  $f_c$ , let f denote the instantaneous frequency of a fast-time sample, and assume that the first antenna is used for transmission. Then, the de-chirped signal at the *m*th antenna and *n*th pulse can be written as

$$r_m(f,n) = \beta e^{-j2\pi \frac{f[d_0(n)+d_m(n)]}{c}} + z_m(f,n), \qquad (1)$$

where

$$d_m(n) = \sqrt{[(x + avn) - (vn - \Delta_m)]^2 + (y + bvn)^2 + h^2},$$
(2)

denotes the distance between the *m*th antenna and the target at the *n*th pulse,  $\beta$  represents the complex-valued reflection coefficient proportional to the target radar cross section (RCS), and  $z_m(f, n)$  denotes the clutter, interference and noise. It should be noted that the algorithm does not depend on the de-chirp processing, and can be easily extended to systems radiating arbitrary waveforms and using standard matched filter pulse compression. The problem of interest is to detect the moving target, and estimate its location and velocity, i.e., (x, y, a, b).

#### 3. GROUND MOVING TARGET INDICATION

#### 3.1. Geometry Manipulation

The reparameterization of the target's linear motion parameters is applied to the data model. First, by using the technique described in [11, 6] (see Appendix A), the distance between the first antenna and the target at the *n*th pulse can be rewritten as

$$d_0(n; \tilde{x}, \tilde{y}, \alpha) = \sqrt{(\alpha v n - \tilde{x})^2 + \tilde{y}^2 + h^2},$$
(3)

where

$$\tilde{x} = \frac{(1-a)x - by}{\alpha},\tag{4}$$

$$\tilde{y} = \frac{bx + (1 - a)y}{\alpha},\tag{5}$$

and

$$\alpha = \sqrt{(1-a)^2 + b^2}.$$
 (6)

In (3),  $\alpha v$  represents the relative speed between the radar and the moving target, and  $(\tilde{x}, \tilde{y})$  denotes the target location in a new coordinate system. This equation shows that the target motion trajectory with respect to (w.r.t.) the radar platform is equivalent to that of a stationary target at  $(\tilde{x}, \tilde{y})$  w.r.t. a radar platform moving at a speed of  $\alpha v$ .

Second, the distance between the mth antenna and the moving target can be rewritten approximately as (see Appendix B for the derivation)

$$d_m(n; \tilde{x}, \tilde{y}, \alpha) \approx d_0 \left( n - \frac{\Delta_m}{v}; \tilde{x}, \tilde{y}, \alpha \right) + \eta \Delta_m, \qquad (7)$$

where

$$\eta = \frac{ax + by}{\sqrt{x^2 + y^2 + h^2}}\tag{8}$$

represents the radial velocity of the target w.r.t. the radar platform. As expected, when m = 0, (7) reduces to (3).

Inserting (7) into (1), the data model can be rewritten approximately as

$$r_{m}(f,n) = \beta e^{-j2\pi \frac{f_{c} \Delta_{m} \eta}{c}} e^{-j2\pi \frac{f[d_{0}(n;\bar{x},\bar{y},\alpha)+d_{0}(n-\frac{\Delta_{m}}{v};\bar{x},\bar{y},\alpha)]}{c}} + z_{0}(f,n).$$
(9)

Note that in (9), the target's location and velocity are now represented by  $(\tilde{x}, \tilde{y}, \alpha, \eta)$ , whose relationship to the original parameters (x, y, a, b) is defined by (4), (5), (6) and (8). As mentioned above,  $(\tilde{x}, \tilde{y})$  represents the target location in the new coordinate system, while  $\alpha v$  and  $\eta v$  represent the platform's speed relative to the target and the target's radial velocity, respectively. Moving target detection is carried out in the  $(\tilde{x}, \tilde{y}, \alpha, \eta)$  domain. Henceforth, (x, y) and  $(\tilde{x}, \tilde{y})$  are referred to as the actual and apparent locations of the target, respectively. Once  $(\tilde{x}, \tilde{y}, \alpha, \eta)$  is obtained, the estimate of the target's parameters in the original coordinates, i.e., (x, y, a, b), can be calculated via the following equations (see Appendix C for the derivations)

$$x = \eta \sqrt{\tilde{x}^2 + \tilde{y}^2 + h^2} + \alpha \tilde{x},\tag{10}$$

$$y = \sqrt{\tilde{x}^2 + \tilde{y}^2 - x^2},$$
 (11)

$$a = 1 - \alpha \frac{x\tilde{x} + y\tilde{y}}{\tilde{x}^2 + \tilde{y}^2},\tag{12}$$

and

$$b = \alpha \frac{x\tilde{y} - \tilde{x}y}{\tilde{x}^2 + \tilde{y}^2}.$$
(13)

It should be noted that the two complex exponential terms in (9) are related to the array aperture and the synthetic aperture, respectively. With this new data model, the temporal and spatial processing may be performed separately. Specifically, for a given channel and value of  $\alpha$ , the data model of (9) reduces to a SAR imaging problem with a platform movement speed of  $\alpha v$ . Thus, existing SAR imaging algorithms can be adopted after some minor modifications.

## 3.2. 3D SAR Imaging

For notational simplicity, define

$$g_m(\tilde{x}, \tilde{y}, \alpha) = \beta e^{-j2\pi \frac{Jc\Delta m\eta}{c}}.$$
(14)

Then, the data model (9) can be rewritten as

$$r_{m}(f,n) = g_{m}(\tilde{x}, \tilde{y}, \alpha) e^{-j2\pi \frac{f[d_{0}(n; \tilde{x}, \tilde{y}, \alpha) + d_{0}(n - \frac{\Delta m}{v}; \tilde{x}, \tilde{y}, \alpha)]}{c}} + z_{0}(f, n),$$
(15)

where  $d_0(n; \tilde{x}, \tilde{y}, \alpha)$  and  $d_0(n - \frac{\Delta_m}{v}; \tilde{x}, \tilde{y}, \alpha)$  are functions of  $(\tilde{x}, \tilde{y}, \alpha)$  defined in (3) and (7). The task in this step is to estimate  $g_m(\tilde{x}, \tilde{y}, \alpha)$  for each candidate  $(\tilde{x}, \tilde{y}, \alpha)$ , i.e., to form a 3D SAR image from each receive channel.

The BP algorithm can be applied to (15) as follows

$$\hat{g}_{m}(\tilde{x}, \tilde{y}, \alpha) = \sum_{n} \sum_{f} r_{m}(f, n) e^{j2\pi \frac{f[d_{0}(n; \tilde{x}, \tilde{y}, \alpha) + d_{0}(n - \frac{\Delta_{m}}{v}; \tilde{x}, \tilde{y}, \alpha)]}{c}}$$
(16)

#### 3.3. Array Processing

The formed SAR images from various channels are stacked into a column vector as follows

$$\mathbf{g}(\tilde{x}, \tilde{y}, \alpha) = [\hat{g}_0(\tilde{x}, \tilde{y}, \alpha), \cdots, \hat{g}_{M-1}(\tilde{x}, \tilde{y}, \alpha)]^T.$$
(17)

By (14), the column vector  $\mathbf{g}(\tilde{x}, \tilde{y}, \alpha)$  can be modeled as follows

$$\mathbf{g}(\tilde{x}, \tilde{y}, \alpha) = \mathbf{a}(\eta)\beta + \breve{\mathbf{z}}(\tilde{x}, \tilde{y}, \alpha), \tag{18}$$

where

$$\mathbf{a}(\eta) = \begin{bmatrix} 1 & e^{-j2\pi \frac{f_c \Delta_0 \eta}{c}} & \cdots & e^{-j2\pi \frac{f_c \Delta_{\mathbf{M}} \eta}{c}} \end{bmatrix}^T$$
(19)

represents the array steering vector for a target with radial velocity  $\eta$ , with  $(\cdot)^T$  denoting the matrix transpose, and  $\breve{z}(\tilde{x}, \tilde{y}, \alpha)$  denotes the clutter, noise, and interference after 3D SAR imaging.

First, the ground clutter, which has parameters  $\alpha = 1$  and  $\eta = 0$ , can be estimated by applying the standard delay-and-sum (DAS) [28] algorithm to  $\mathbf{g}(\tilde{x}, \tilde{y}, 1)$ , as

$$\beta_{\text{Ground}}(\tilde{x}, \tilde{y}) = \frac{\mathbf{a}(0)^H \mathbf{g}(\tilde{x}, \tilde{y}, 1)}{\|\mathbf{a}(0)\|_F^2},$$
(20)

with  $\|\cdot\|_F$  denoting the Frobenius norm. Second, to estimate the power at various radial velocities, the minimum variance distortion-less response (MVDR) [28] is used as follows

$$\beta(\tilde{x}, \tilde{y}, \alpha, \eta) = \frac{\mathbf{a}(\eta)^H \hat{\mathbf{R}}^{-1} \mathbf{g}(\tilde{x}, \tilde{y}, \alpha)}{\mathbf{a}(\eta)^H \hat{\mathbf{R}}^{-1} \mathbf{a}(\eta)},$$
(21)

where

$$\hat{\mathbf{R}} = \frac{1}{K_x K_y} \sum_{\tilde{x}, \tilde{y}} \mathbf{g}(\tilde{x}, \tilde{y}, 1) \mathbf{g}^H(\tilde{x}, \tilde{y}, 1)$$
(22)

is the noise and interference array covariance matrix estimated from the ground clutter images ( $\alpha = 1$ ), with  $K_x$  and  $K_y$  denoting the image pixel size in the  $\tilde{x}$  and  $\tilde{y}$  dimensions, respectively. That is, the estimate of the noise and interference array covariance matrix is formed using data from the entire ground clutter image.

From (21), a 4D amplitude image is obtained. The peaks of  $\beta(\tilde{x}, \tilde{y}, \alpha, \eta)$  indicate the existence of moving targets, with associated parameters  $(\tilde{x}, \tilde{y}, \alpha, \eta)$ . Once  $(\tilde{x}, \tilde{y}, \alpha, \eta)$  are obtained, the true target location and velocity (x, y, a, b) can be computed by (10)-(13) explicitly.

#### 4. NUMERICAL EXAMPLES

In this section, the performance of the proposed moving target detection method is investigated by analyzing a point spreading function (PSF) generated by the algorithm. The GOTCHA radar parameters, including carrier frequency (9.6 GHz), bandwidth (640 MHz), PRF (2171.6 Hz), antenna spacing (0.476 meters), etc., are used to facilitate this analysis [27]. An application of the algorithm to the field collected SAR GOTCHA GMTI data set is also presented.

#### 4.1. Point Spreading Function

In order to demonstrate the algorithm's sensitivity to target motion in the along-track and cross-track directions, the PSF of a target located at (-100, 0) meters moving in both the along-track and crossdirections at a speed of 5 m/s is presented. Using (4), (5), (6) and (8), the target apparent location is computed to be approximately (-252.1, -3.8) m, the relative speed  $\alpha \approx 0.95$ , and radial speed  $\eta \approx 0.034$ . Figure 2(a) shows the target response as a function of  $(\tilde{x}, \tilde{y}, \alpha)$ , and contains three isosurfaces. The values from the darkest to the lightest are -3 dB, -10 dB, and -20 dB. Inspecting the upper image ( $\alpha = 1$ ) displayed in Figure 2(b), the well-known phenomenon of target smearing [29, 6] caused by along-track target motion is seen. Further, note the excellent focus obtained by properly compensating the data for the target's along-track motion in Figure 2(b) ( $\alpha = 0.953$ ). Figure 2(c) depicts the PSF as a function of  $\tilde{x}$ and  $\alpha$  at the correct  $\tilde{y} = -3.8$  and  $\eta = 0.034$ . Finally, Figure 2(d) demonstrates the PSF at the true  $\tilde{x}$ ,  $\tilde{y}$ , and  $\alpha$  as a function of  $\eta$  formed using the DAS beamforming algorithm. Due to the large antenna spacing in the GOTCHA array, grating lobes are present in the beamformer output. Note that the radial velocity  $\eta$ , outside of the ambiguity problem, correctly identifies the true value of  $\eta$ .



**Fig. 2.** Point spreading function of a target with an along-track and cross-track velocity of 5 m/s. (a) 3D image, (b) x-y images at  $\alpha = 1$  and  $\alpha = 0.953$ , (c) x- $\alpha$  image, and (d) beampattern at the true  $(\tilde{x}, \tilde{y}, \alpha)$ .

## 4.2. GOTCHA Data

In this subsection, the field collected GOTCHA data is used to demonstrate the proposed GMTI method. The CPI under consideration is from [45, 47.5] seconds, which yields a cross-range resolution



Fig. 3. (a) Single-channel ground clutter SAR image with  $\alpha = 1$ . (b) Beamformed target image formed via MVDR algorithm with  $\alpha = 0.976$  and  $\eta = 0.09$ . (c) Moving target detection image.

of approximately 0.87 m. Based on the auxiliary information provided with the data set, the target's parameters during this CPI are  $\tilde{x} = -625.6 \text{ m}, \tilde{y} = -24.5 \text{ m}, \alpha = 0.976, \text{ and } \eta = 0.090.$  Figure 3(a) displays a conventional SAR image formed from channel 0 focused to the ground clutter ( $\alpha = 1$ ). The ground target's location in this image is indicated by the white circle. Inspecting this region, the target's signature is faint. Further, note the high dynamic range of the images (100 dB), which yields a slightly apparent target. As mentioned previously, SAR images focused to all values of  $\alpha$  considered are generated in the first step of the algorithm, and the 4D moving target image is then generated via beamforming these data for a particular  $(\tilde{x}, \tilde{y}, \alpha)$  using the MVDR algorithm. In this work, the entire SAR image of the ground clutter ( $\alpha = 1$ ) is used for estimating the covariance matrix. Figure 3(b) displays a 2D slice of the 4D moving target image formed with the MVDR beamformer from (21) with relative speed  $\alpha = 0.976$  and radial velocity  $\eta = 0.09$ . In this image, the moving target's signature is stronger, whereas the ground clutter is suppressed and spread. Figure 3(c) contains a detection image formed by normalizing the 4D MVDR amplitude images generated from (21) by the estimated clutter sidelobes at each  $\tilde{x}, \tilde{y}$ , and  $\alpha$ . This 4D detection image is then collapsed to a 2D representation by taking the maximum over the  $\alpha$  and  $\eta$  dimensions. The clutter sidelobe contributions are computed by estimating the ground clutter via (20) and using the point spreading function to calculate the contribution at a particular  $\tilde{x}, \tilde{y}, \alpha$ . Clearly, the moving target is greatly enhanced, and may be easily detected from this image.

#### 5. CONCLUSION

A long-CPI multi-channel approach to SAR GMTI was developed and described. This SAR GMTI algorithm is sensitive to both alongtrack and cross-track moving targets with linear motion parameters, and maximizes the target signal via focusing prior to clutter cancellation and radial velocity estimation. The performance of the algorithm was demonstrated using simulated data to form a PSF using the well-known GOTCHA radar parameters. An example of processing field collected data using the GOTCHA SAR GMTI data set was presented.

#### 6. APPENDIX A: PROOF OF (3)

From (2) and using the fact that  $\Delta_0 = 0$ , we have

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$$\begin{aligned} l_0^2(n) &= (x + avn - nv)^2 + (y + bvn)^2 + h^2 \\ &= \left[ (1 - a)^2 + b^2 \right] v^2 n^2 - 2nv \left[ (1 - a)x - by \right] \\ &+ x^2 + y^2 + h^2 \\ &= \left[ \alpha vn - \frac{(1 - a)x - by}{\alpha} \right]^2 + \left[ \frac{bx + (1 - a)y}{\alpha} \right]^2 + h^2 \\ &= (\alpha vn - \tilde{x})^2 + \tilde{y}^2 + h^2, \end{aligned}$$

where  $\tilde{x}$ ,  $\tilde{y}$  and  $\alpha$  are defined in (4), (5) and (6), respectively. From (23), Equation (3) follows directly.

#### 7. APPENDIX B: PROOF OF (7)

From (2), after some algebraic manipulations, we have:

$$d_m(n) = \sqrt{\zeta + \chi + h^2}, \qquad (24)$$

$$\zeta = \left[ (1-a) \left( n - \frac{\Delta_m}{v} \right) v - x - a\Delta_m \right]^2, \qquad \chi = \left[ y + b \left( n - \frac{\Delta_m}{v} \right) v + b\Delta_m \right]^2.$$

Applying Taylor expansion w.r.t. the minor terms  $a\Delta_m$  and  $b\Delta_m$  yields:

$$d_{m}(n) \approx d_{0} \left(n - \frac{\Delta_{m}}{v}\right) + \frac{\left[x - (1 - a)\left(n - \frac{\Delta_{m}}{v}\right)v\right]a + \left[y + b\left(n - \frac{\Delta_{m}}{v}\right)v\right]b}{d_{0}\left(n - \frac{\Delta_{m}}{v}\right)} \Delta_{m}$$
$$\approx d_{0} \left(n - \frac{\Delta_{m}}{v}\right) + \eta\Delta_{m}, \tag{25}$$

where  $\eta$  is defined in (8).

# 8. APPENDIX C: PROOF OF (10)-(13)

From (4) and (5), we have:

$$\tilde{x}^{2} + \tilde{y}^{2} = \frac{\left[(1-a)x - by\right]^{2} + \left[bx + (1-a)y\right]^{2}}{\alpha^{2}}$$
$$= x^{2} + y^{2},$$
(26)

where we have used (6) for the second equality in (26). Then, (8) can be rewritten as:

$$ax + by = \eta \sqrt{\tilde{x}^2 + \tilde{y}^2 + h^2}.$$
 (27)

Inserting (27) into (4) yields:

$$\tilde{x} = \frac{x - (ax + by)}{\alpha} = \frac{x - \eta\sqrt{\tilde{x}^2 + \tilde{y}^2 + h^2}}{\alpha}.$$
 (28)

From (28), (10) follows immediately. Once x is obtained, y can be computed using (26), which can be rewritten as (11). Furthermore, given x and y, the target speed a and b can be obtained via solving the linear equations (4) and (5), with solutions provided in (12) and (13).

#### 9. REFERENCES

- S. Barbarossa, "Detection and imaging of moving objects with synthetic aperture radar Part 1: Optimal detection and parameter estimation theory," *IEE Proceedings of Radar and Signal Processing*, vol. 139, pp. 79–88, Februrary 1992.
- [2] S. Barbarossa and A. Farina, "Detection and imaging of moving objects with synthetic aperture radar Part 2: Joint timefrequency analysis by Wigner-Ville distribution," *IEE Proceedings of Radar and Signal Processing*, vol. 139, pp. 89–97, February 1992.
- [3] H. Yang and M. Soumekh, "Blind-velocity SAR/ISAR imaging of a moving target in a stationary background," *IEEE Transactions on Image Processing*, vol. 2, pp. 80–95, January 1993.
- [4] M. Kirscht, "Detection and velocity estimation of moving objects in a sequence of single-look SAR images," *International Geoscience and Remote Sensing Symposium*, Lincoln, Nebraska, vol. 1, pp. 333–335, May 1996.
- [5] M. Kirscht, "Detection and imaging of arbitrarily moving targets with single-channel SAR," *IEE Proceedings - Radar*, *Sonar and Navigation*, vol. 150, pp. 7–11, February 2003.
- [6] J. K. Jao, "Theory of synthetic aperture radar imaging of a moving target," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 39, pp. 1984–1992, September 2001.
- [7] M. J. Minardi, L. A. Gorham, and E. G. Zelnio, "Ground moving target detection and tracking based on generalized SAR processing and change detection," *Proceedings of SPIE Conference on Algorithms for Synthetic Aperture Radar Imagery*, Orlando, Florida, vol. 12, pp. 156–165, June 2005.
- [8] G. Li, X. G. Xia, J. Xu, and Y. N. Peng, "A velocity estimation algorithm of moving targets using single antenna SAR," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, pp. 1052–1062, July 2009.
- [9] V. T. Vu, T. K. Sjogren, M. I. Pettersson, A. Gustavsson, and L. M. H. Ulander, "Detection of moving targets by focusing in UWB SAR - Theory and experimental results," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, pp. 3799– 3815, October 2010.
- [10] J. R. Fienup, "Detecting moving targets in SAR imagery by focusing," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 37, pp. 794–798, July 2001.
- [11] M. Soumekh, "Moving target detection in foliage using along track monopulse synthetic aperture radar imaging," *IEEE Transactions on Image Processing*, vol. 6, pp. 1148–1163, August 1997.
- [12] A. F. Yegulalp, "Analysis of SAR image formation equations for stationary and moving targets," MIT Lincoln Lab., Lexington, MA, Technical Report FPR-14, June 2002.
- [13] C. Gierull and I. Sikaneta, "Raw data based two-aperture SAR ground moving target indication," in 2003 IEEE International Geoscience and Remote Sensing Symposium Proceedings (IGARSS '03), vol. 2, pp. 1032–1034, July 21-25 2003.
- [14] I. Sikaneta and C. Gierull, "Ground moving target detection for along-track interferometric SAR data," in 2004 IEEE Aerospace Conference Proceedings, vol. 4, pp. 2227–2235, March 6-13 2004.

- [15] C. Livingstone, I. Sikaneta, C. Gierull, S. Chiu, A. Beaudoin, J. Campbell, S. Gong, and T. A. Knight, "An airborne synthetic aperture radar (SAR) experiment to support RADARSAT-2 ground moving target indication (GMTI)," *Canadian Journal* of *Remote Sensing*, vol. 28, no. 6, pp. 794–813, 2002.
- [16] S. Chiu and C. Livingstone, "A comparison of displaced phase centre antenna and along-track interferometry techniques for RADARSAT-2 ground moving target indication," *Canadian Journal of Remote Sensing*, vol. 31, no. 1, pp. 37–51, 2005.
- [17] F. Lombardini, F. Bordoni, F. Gini, and L. Verrazzani, "Multibaseline ATI-SAR for robust ocean surface velocity estimation," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 40, pp. 417–433, April 2004.
- [18] B. Friedlander and B. Porat, "VSAR: A high resolution radar system for detection of moving targets," *IEE Proceedings Radar, Sonar and Navigation*, vol. 144, pp. 1350–2395, August 1997.
- [19] B. Friedlander and B. Porat, "VSAR: A high resolution radar system for ocean imaging," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, pp. 755–776, July 1998.
- [20] J. Ward, "Space-time adaptive processing for airborne radar," MIT Lincoln Lab., Lexington, MA, Technical Report 1015, December 1994.
- [21] J. H. G. Ender, "Space-time processing for multichannel synthetic aperture radar," *Electronics and Communication Engineering Journal*, vol. 11, no. 1, pp. 29–38, 1999.
- [22] C. D. Peckham, A. M. Haimovich, T. F. Ayoub, J. S. Goldstein, and I. S. Reed, "Reduced-rank STAP performance analysis," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, pp. 664–676, April 2000.
- [23] J. R. Guerci, J. Goldstein, and I. S. Reed, "Optimal and adaptive reduced-rank STAP," *IEEE Transactions on Aerospace* and Electronic Systems, vol. 36, pp. 647–663, April 2000.
- [24] J. K. Jao and A. Yegulalp, "Multichannel synthetic aperture radar signatures and imaging of a moving target," *Inverse Problems*, vol. 29, no. 5, 2013.
- [25] D. Cerutti-Maori, I. Sikaneta, and C. H. Gierull, "Optimum SAR/GMTI processing and its application to the radar satellite RADARSAT-2 for traffic monitoring," *IEEE Transactions* on Geoscience and Remote Sensing, vol. 50, pp. 3868–3881, October 2012.
- [26] D. Cerutti-Maori and I. Sikaneta, "A generalization of DPCA processing for multichannel SAR/GMTI radars," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 51, pp. 560– 572, January 2013.
- [27] S. M. Scarborough, C. H. Casteel, L. Gorham, M. J. Minardi, U. K. Majumder, M. G. Judge, E. Zelnio, and M. Brya, "A challenge problem for SAR-based GMTI in urban environments," in *Algorithms for Synthetic Aperture Radar Imagery XVI, Proceedings of the SPIE*, 2009.
- [28] H. L. V. Trees, Detection, Estimation, and Modulation Theory, Part IV, Optimum Array Processing. New York: John Wiley and Sons, 2002.
- [29] R. K. Raney, "Synthetic aperture imaging radar and moving targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 7, pp. 499–505, May 1971.