ROBUST WAVEFORM DESIGN OF WIDEBAND COGNITIVE RADAR FOR EXTENDED TARGET DETECTION

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ABSTRACT

This paper investigates the waveform design problem of wideband cognitive radar for the detection of extended targets in which the knowledge of target impulse response and interference is imprecise. We resort to a maximin approach to design the waveform that is robust to the model uncertainties, i.e., we optimize the waveform to maximize the worst-case signal to interference plus noise ratio (SINR) over the uncertainty region. We show that the maximin waveform design problem can be formulated into a convex optimization problem. Results indicating the robustness of the proposed method are provided via numerical simulations.

***Index Terms*—** Waveform design, extended target, model uncertainty, robust design, signal to interference plus noise ratio (SINR).

1. INTRODUCTION

Cognitive radar is a new concept that has received considerable interests in recent years (see, e.g., [1, 2] and the references therein). Cognitive radar system can adjust its transmit waveform and receive filter adaptively based on the prior knowledge of targets and the environment, thus has great potentials in enhancing the performance of current radar systems. There are two key ingredients in cognitive radar systems: waveform design in the transmitter and adaptive processing in the receiver. Herein, we focus on the waveform design problem of cognitive radar system.

In order to enhance the detection performance of extended targets for wideband cognitive radar, we consider the waveform optimization to maximize the signal to interference plus noise ratio (SINR). Similar methodologies have been adopted in several studies (but with different signal models, see, e.g., [3–9] and the references therein). However, current design methods based on maximizing SINR mostly assume precise knowledge of both target and interference, while in practice the *a priori* knowledge of them might be inaccurate. As a consequence, the mismatch of prior knowledge might degrade the performance of the designed waveform.

In this paper, we consider the waveform design problem of wideband cognitive radar system for the detection of extended targets, in which the prior knowledge of the target impulse response and statistics of the interference is assumed to be imprecise. In order to design the waveform that is robust to the model mismatch of the target impulse response and interference covariance matrix, we formulate a waveform design problem based on maximizing the worst-case SINR over the uncertainty region. We show that, we can efficiently find the globally optimal solution of the robust waveform design problem through solving a convex optimization problem.

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2. SIGNAL MODEL AND PROBLEM FORMULATION

Consider a wideband cognitive radar system with $s(t)$ denoting the transmit waveform. For an extended target, which occupies multiple range cells, its received signal after down-conversion can be written as [10]

$$y(t) = \int_{\tau_1}^{\tau_2} h(\tau)s(t-\tau)\mathrm{d}\tau + n(t), \tag{1}$$

where $h(\tau)$ is the target impulse response, τ_1 and τ_2 are the minimum and maximum two-way propagation delay of the target, respectively (i.e., the target size in the line of sight is about $\Delta L = c(\tau_2 - \tau_1)/2$, where c is the speed of light), and $n(t)$ is a signal-independent interference in the receiver (including thermal noise, possible intentional jamming and external disturbance).

For simplicity, we digitalize $y(t)$ and consider the following discrete-time model

$$\mathbf{y} = \mathbf{h} * \mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{h} = [h_1, \cdots, h_P]^T$ with the p^{th} element representing the target scattering coefficient in the p^{th} range cell, P is the number of range cells that the target occupies ($P \approx \Delta L/B$ with B denoting the bandwidth), $*$ denotes the operator of convolution, $\mathbf{s} = [s_1, \cdots, s_L]^T$, L is the code length. We can also write (2) in an equivalent matrix form

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{3}$$

where $\mathbf{H}$ is a Toeplitz matrix sharing the following expression

$$\mathbf{H} = \begin{pmatrix} h_1 & 0 & \cdots & 0 \\ h_2 & h_1 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ h_P & h_{P-1} & \cdots & 0 \\ 0 & h_P & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & h_P \end{pmatrix} \in \mathbb{C}^{(L+P-1)\times L}. \tag{4}$$

The target detection can be formulated as the following binary hypothesis testing problem

$$\begin{cases} \mathcal{H}_0 : \mathbf{y} = \mathbf{n} \\ \mathcal{H}_1 : \mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \end{cases}. \tag{5}$$

Let $\mathbf{R} = \mathrm{E}(\mathbf{n}\mathbf{n}^H)$ be the interference covariance matrix. Assume that $h_1, \cdots, h_P$ are deterministic (i.e., not random), then it can be easily verified that the probability of false alarm and the probability of detection is given by

$$P_{\text{FA}} = \frac{1}{2}\text{erfc}(T/\sqrt{\mathbf{s}^H\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}\mathbf{s}}), \tag{6}$$

$$P_{\text{D}} = \frac{1}{2}\text{erfc}(\frac{T}{\sqrt{\mathbf{s}^H\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}\mathbf{s}}} - \sqrt{\mathbf{s}^H\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}\mathbf{s}}), \tag{7}$$

where $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \mathrm{e}^{-t^2}\mathrm{d}t$ is the complementary error function and T is the detector threshold.

Note that P_{D} can also be written as

$$P_{\text{D}} = \frac{1}{2}\text{erfc}\left\{\text{erfc}^{-1}(2P_{\text{FA}}) - \sqrt{\mathbf{s}^H\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}\mathbf{s}}\right\} \tag{8}$$

Since the complementary error function $\text{erfc}(x)$ is monotonically decreasing with x, then for a fixed probability of false alarm, the optimal waveform that maximizes the SINR also achieves the largest probability of detection. As a consequence, the waveform design for the optimal detection of extended targets can be formulated as

$$\begin{aligned} &\max_{\mathbf{s}} \text{SINR} = \mathbf{s}^H\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}\mathbf{s} \\ &\text{s.t. } \mathbf{s}^H\mathbf{s} \le P_0, \end{aligned} \tag{9}$$

where P_0 is the total available energy.

It is straightforward to obtain that the optimal waveform under the energy constraint is given by

$$\mathbf{s}_{\text{opt}} = \sqrt{P_0}\mathcal{P}(\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}), \tag{10}$$

where $\mathcal{P}(\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H})$ denotes the eigenvector of $\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}$ associated with its largest eigenvalue.

We can observe from (10) that, the design of waveform for the optimal detection of extended target requires the knowledge of the second-order statistics of the interference $\mathbf{R}$ as well as the target impulse response vector $\mathbf{h}$. In practice, $\mathbf{R}$ and $\mathbf{h}$ can be obtained by some cognitive methods (e.g., $\mathbf{R}$ can be obtained by setting the radar operating in the passive mode so that the radar can collect the signals which only contain thermal noise and possible interference, and $\mathbf{h}$ (or equivalently, $\mathbf{H}$) can be obtained by previous estimations). However, both estimation of $\mathbf{R}$ and $\mathbf{h}$ might be inaccurate. The consequence of the use of mismatched prior knowledge in the waveform design in (10) is the performance degradation. Therefore, when the precise knowledge of target impulse response and interference covariance matrix is unavailable, robust techniques have to be developed to improve the detection performance.

3. ROBUST WAVEFORM DESIGN METHODS

First we introduce two uncertainty sets to describe the uncertainty of $\mathbf{H}$ and $\mathbf{R}$, respectively, i.e.,

$$\mathbf{H} \in \mathcal{S} = \{\mathbf{H} \in \mathcal{T} | \|\mathbf{H} - \mathbf{H}_0\|_{\text{F}} \le \varepsilon\}, \tag{11}$$

where $\mathcal{T}$ denotes the set of $(L+P-1)\times L$ Toeplitz matrices defined similarly to (4) and $\mathbf{H}_0$ is the presumed target scattering matrix associated with the prior target impulse response $\mathbf{h}_0$. In addition, $\mathbf{R}$ lies in an uncertainty set which is given by

$$\mathbf{R} \in \mathcal{R} = \{\mathbf{R} \in \mathcal{H}^+_{L+P-1} | \|\mathbf{R} - \mathbf{R}_0\|_{\text{F}} \le \eta\}, \tag{12}$$

with $\mathcal{H}^+_{L+P-1}$ denoting the set of $(L+P-1)\times(L+P-1)$ positive definite matrices and $\mathbf{R}_0$ the presumed interference covariance matrix. In order to find a meaningful robust solution, we assume that $\varepsilon < \|\mathbf{H}_0\|_{\text{F}}$ and $\eta \le \|\mathbf{R}_0\|_{\text{F}}$ so that $\mathcal{S}$ and $\mathcal{R}$ include nonzero elements.

Similar to that in [11], we resort to a maximin approach to design the robust waveform, i.e., we optimize the waveform to maximize the worst-case SINR in the uncertainty region defined by $\mathcal{S}$ and $\mathcal{R}$, so that the optimized waveform is robust to both the target and interference model uncertainties. The corresponding robust waveform design problem can be formulated as the following maximin optimization problem

$$\begin{aligned} &\max_{\mathbf{s}} \min_{\mathbf{H}\in\mathcal{S},\mathbf{R}\in\mathcal{R}} \mathbf{s}^H\mathbf{H}^H\mathbf{R}^{-1}\mathbf{H}\mathbf{s} \\ &\qquad\text{s.t. } \mathbf{s}^H\mathbf{s} \le P_0. \end{aligned} \tag{13}$$

Obviously, the output SINR by using the optimal robust waveform $\mathbf{s}^\star$ (i.e., the solution of (13)) is guaranteed to be at least $p^\star$ in the uncertainty sets $\mathcal{S}$ and $\mathcal{R}$, where $p^\star$ is the optimal value of (13).

In order to find $\mathbf{s}^\star$, we can observe that, the optimization problem in (13) is equivalent to the following joint optimization problem:

$$\max_{\mathbf{s},\mathbf{w}} \min_{\mathbf{H}\in\mathcal{S},\mathbf{R}\in\mathcal{R}} \frac{|\mathbf{w}^H\mathbf{H}\mathbf{s}|^2}{\mathbf{w}^H\mathbf{R}\mathbf{w}} \quad \text{s.t. } \mathbf{s}^H\mathbf{s} \le P_0, \tag{14}$$

where $\mathbf{w}$ is the weight in the receiver side and for fixed $\mathbf{s}, \mathbf{R}$ and $\mathbf{H}$, the optimal weight is given by

$$\mathbf{w}^{\text{opt}} = \mathbf{R}^{-1}\mathbf{H}\mathbf{s}. \tag{15}$$

Next we consider the inner minimization problem of (14) with respect to (w.r.t.) $\mathbf{R} \in \mathcal{R}$, or equivalently, we consider

$$\max_{\mathbf{R}\in\mathcal{R}} \mathbf{w}^H\mathbf{R}\mathbf{w} \tag{16}$$

Note that for $\mathbf{R} \in \mathcal{R}$, we have

$$\mathbf{w}^H\mathbf{R}\mathbf{w} = \mathbf{w}^H\mathbf{R}_0\mathbf{w} + \mathbf{w}^H\mathbf{\Delta R}\mathbf{w}, \tag{17}$$

where $\mathbf{\Delta R}$ is a matrix satisfying $\|\mathbf{\Delta R}\|_{\text{F}}^2 \le \eta^2$. As a result,

$$\mathbf{w}^H\mathbf{R}\mathbf{w} \le \mathbf{w}^H\mathbf{R}_0\mathbf{w} + \eta\mathbf{w}^H\mathbf{w} = \mathbf{w}^H(\mathbf{R}_0 + \eta\mathbf{I})\mathbf{w}, \tag{18}$$

where the equality holds if and only if $\mathbf{\Delta R} = \eta\mathbf{w}\mathbf{w}^{\mathbf{H}}/\|\mathbf{w}\|^{\mathbf{2}}$. Thus the optimization problem in (14) can be reformulated as

$$\max_{\mathbf{s},\mathbf{w}} \min_{\mathbf{H}\in\mathcal{S}} \frac{|\mathbf{w}^H\mathbf{H}\mathbf{s}|^2}{\mathbf{w}^H(\mathbf{R}_0 + \eta\mathbf{I})\mathbf{w}} \quad \text{s.t. } \mathbf{s}^H\mathbf{s} \le P_0. \tag{19}$$

Interestingly, we only need diagonal loading on the interference covariance matrix $\mathbf{R}$ to improve the robustness when the knowledge of $\mathbf{R}$ is inaccurate.

By using the equivalence between (13) and (14), we can similarly rewrite (19) equivalently into

$$\max_{\mathbf{s}} \min_{\mathbf{H}\in\mathcal{S}} \mathbf{s}^H\mathbf{H}^{-1}(\mathbf{R}_0 + \eta\mathbf{I})^{-1}\mathbf{H}\mathbf{s} \quad \text{s.t. } \mathbf{s}^H\mathbf{s} \le P_0, \tag{20}$$

Let $\widetilde{\mathbf{R}} = \mathbf{R}_0 + \eta\mathbf{I}$. Note that adding a constant term to the cost function of (20) will not change its optimal solution. Therefore, we consider the following cost function

$$\mathbf{s}^H\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}\mathbf{s} - \gamma P_0, \tag{21}$$

where γ is a constant larger than the largest eigenvalue of $\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}$, denoted by $\lambda_{\max}(\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H})$. By using the result that the optimal waveform has to satisfy $\mathbf{s}^H\mathbf{s} = P_0$, we can also write (21) by

$$\begin{aligned}\mathbf{s}^H\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}\mathbf{s} - \gamma P_0 &= \mathbf{s}^H\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}\mathbf{s} - \gamma\mathbf{s}^H\mathbf{s} \\ &= \mathbf{s}^H(\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H} - \gamma\mathbf{I})\mathbf{s}.\end{aligned} \tag{22}$$

Lemma 1 (Sion's minimax theorem [12]) *Let $\mathcal{U}$ be a compact convex space and $\mathcal{V}$ be a convex space. Let $f(\mathbf{u}, \mathbf{v})$ denote a real-value function defined on $\mathcal{U} \times \mathcal{V}$. If f is quasi-convex w.r.t. $\mathbf{u}$, for $\forall\mathbf{v} \in \mathcal{V}$, and quasi-concave w.r.t. $\mathbf{v}$, for $\forall\mathbf{u} \in \mathcal{U}$, then*

$$\min_{\mathbf{u}} \max_{\mathbf{v}} f(\mathbf{u}, \mathbf{v}) = \max_{\mathbf{v}} \min_{\mathbf{u}} f(\mathbf{u}, \mathbf{v}). \tag{23}$$

since $\gamma > \lambda_{\max}(\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H})$, then $\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H} - \gamma\mathbf{I}$ is negative definite. Consequently, based on the definition of convexity [13], (22) is a convex function of $\mathbf{H}$ when $\mathbf{s}$ is fixed while a concave function of $\mathbf{s}$ for fixed $\mathbf{H}$. Using Sion's minimax theorem as well as the equivalence between (22) and the cost function of (20), we can interchange the order of the minimization and maximization in (20), i.e., we can reformulate the robust waveform design problem by

$$\min_{\mathbf{H}\in\mathcal{S}} \max_{\mathbf{s}} \mathbf{s}^H\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}\mathbf{s}. \quad \text{s.t. } \mathbf{s}^H\mathbf{s} \le P_0. \tag{24}$$

We can observe that the inner maximization of (24) is achieved when $\mathbf{s}$ is an eigenvector of $\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}$ corresponding to its largest eigenvalue. As a result, we can recast (24) as

$$\min_{\mathbf{H}\in\mathcal{S}} \lambda_{\max}(\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}), \tag{25}$$

with the optimal robust waveform given by

$$\mathbf{s}^\star = \sqrt{P_0}\mathcal{P}((\mathbf{H}^\star)^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}^\star), \tag{26}$$

where $\mathbf{H}^\star$ is the optimal solution of (25). Next we show we can formulate (25) into a convex optimization problem. By introducing an auxiliary variable t, (25) can be written as

$$\min_{t,\mathbf{H}\in\mathcal{S}} t \quad \text{s.t. } t \ge \lambda_{\max}(\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H}). \tag{27}$$

It is also equivalent to

$$\min_{t,\mathbf{H}\in\mathcal{S}} t \quad \text{s.t. } \mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H} \preceq t\mathbf{I}. \tag{28}$$

Lemma 2 (Schur complement theorem [14]) *Let $\mathbf{M}$ be a matrix which can be partitioned as*

$$\mathbf{M} = \begin{bmatrix}\mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{12}^H & \mathbf{M}_{22}\end{bmatrix}, \tag{29}$$

then $\mathbf{M} \succ \mathbf{0}$ if and only if $\mathbf{M}_{22} \succ \mathbf{0}$ and $\mathbf{M}_{11} - \mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{12}^H \succ \mathbf{0}$.

According to Schur complement theorem, $\mathbf{H}^H\widetilde{\mathbf{R}}^{-1}\mathbf{H} \preceq t\mathbf{I}$ holds if and only if

$$\begin{bmatrix} t\mathbf{I} & \mathbf{H}^H \\ \mathbf{H} & \widetilde{\mathbf{R}} \end{bmatrix} \succeq \mathbf{0}. \tag{30}$$

Therefore, we can further write (28) by

$$\min_{t,\mathbf{H}\in\mathcal{S}} t$$
$$\text{s.t.} \begin{bmatrix} t\mathbf{I} & \mathbf{H}^H \\ \mathbf{H} & \widetilde{\mathbf{R}} \end{bmatrix} \succeq \mathbf{0}. \tag{31}$$

Since $\mathcal{S}$ is a convex set and the constraint in (31) is also convex, (31) is a convex optimization problem and its globally optimal solution can be obtained efficiently with polynomial time [13], by public domain software, e.g., CVX [15, 16].

4. NUMERICAL EXAMPLES

Consider a cognitive radar system with the length of the transmitted waveform $L = 20$. The extended target occupies $P = 3$ range bins, with presumed target impulse response vector $\mathbf{h}_0 = [3, 20, 1]^T$. The prior interference covariance matrix is modeled by an AR(1) process with the one-lag coefficient $\rho = 0.5$.

Fig.1 shows the worst-case output SINR of the orthogonal waveform, the designed waveform based on prior knowledge of target scattering coefficients and interference covariance matrix, and the proposed robust waveform against different transmit energy, where $\varepsilon = \epsilon_1\|\mathbf{H}_0\|_{\text{F}}$, $\epsilon_1 = 0.3$, $\eta = \eta_1\|\mathbf{R}_0\|_{\text{F}}$, $\eta_1 = 0.25$, the orthogonal waveform is generated by a phase-coded waveform of random phases. We can observe that, when the prior knowledge is imprecise, the performance of the waveform designed based on prior knowledge degrades and is even worse than that of orthogonal waveform. However, the proposed robust waveform outperforms both the waveform designed based on prior knowledge and the orthogonal waveform.

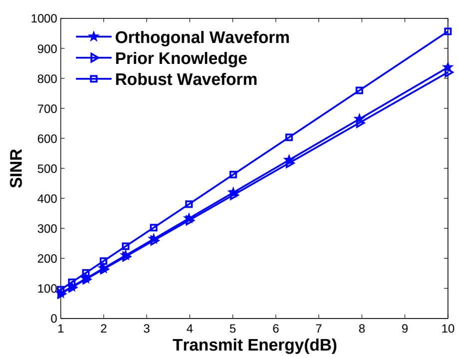

Fig. 1. SINR of three types of waveform against transmit energy. $\epsilon_1 = 0.3$, $\eta_1 = 0.25$.

Fig.2 presents the worst-case output SINR of the three types of waveform versus different size of target uncertainty, where $P_0 = 1$ and η_1 is fixed to be 0.25. Fig.3 shows the worst-case output SINR of the three types of waveform versus different size of uncertainty of $\mathbf{R}$, where $P_0 = 1$ and ϵ_1 is fixed to be 0.3. Both figures illustrate that the performance of all three waveforms degrades with increasing size of uncertainty region. In addition, owing to the robustness of the proposed algorithm with respect to the model uncertainties, the proposed waveform shows clear superiority over the other two waveforms.

5. CONCLUSION

We proposed a robust waveform design method for wideband cognitive radar, in which the prior knowledge of the target and interference is imprecise. We considered the waveform optimization to maximize the worst-case SINR over the uncertainty region of target impulse response and interference covariance matrix. We showed that, the diagonal loading of the prior interference covariance matrix contributed to the robustness of the proposed waveform. Moreover, the optimal waveform could be obtained through solving a convex optimization problem. Numerical examples demonstrated the effectiveness of the proposed method.

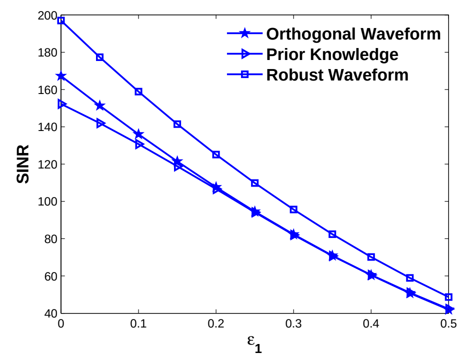

Fig. 2. SINR of three types of waveform against different model uncertainty. $P_0 = 1$, $\eta_1 = 0.25$.

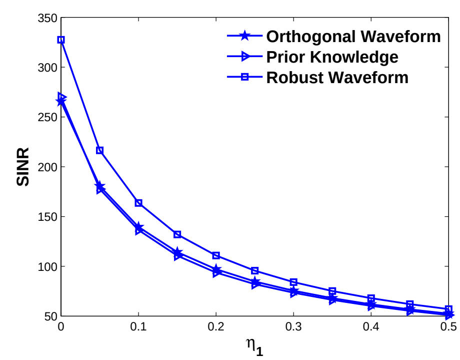

Fig. 3. SINR of three types of waveform against different model uncertainty. $P_0 = 1$, $\epsilon_1 = 0.3$.

6. REFERENCES

[1] Simon Haykin, "Cognitive radar: a way of the future," *IEEE Signal Processing Magazine*, vol. 23, no. 1, pp. 30–40, 2006.

[2] Joseph R Guerci, *Cognitive Radar: The Knowledge-Aided Fully Adaptive Approach*, Artech House, 2010.

[3] Fulvio Gini, A. De Maio, and Lee Patton, *Waveform design and diversity for advanced radar systems*, The Institution of Engineering and Technology, 2012.

[4] A. De Maio, S. De Nicola, Yongwei Huang, Shuzhong Zhang, and A. Farina, "Code design to optimize radar detection performance under accuracy and similarity constraints," *IEEE Transactions on Signal Processing*, vol. 56, no. 11, pp. 5618–5629, 2008.

[5] JD Zhang, XH Zhu, and HQ Wang, "Adaptive radar phase-coded waveform design," *Electronics Letters*, vol. 45, no. 20, pp. 1052–1053, 2009.

[6] P. Stoica, H. He, and J. Li, "Optimization of the receive filter and transmit sequence for active sensing," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1730–1740, 2012.

[7] M. Soltanalian, B. Tang, Jian Li, and P. Stoica, "Joint design of the receive filter and transmit sequence for active sensing," *IEEE Signal Processing Letters*, vol. 20, no. 5, pp. 423–426, 2013.

[8] Bo Tang, Jun Li, Yu Zhang, and Jun Tang, "Design of mimo radar waveform covariance matrix for clutter and jamming suppression based on space time adaptive processing," *Signal Processing*, vol. 121, pp. 60–69, 2016.

[9] S. M. Karbasi, A. Aubry, A. De Maio, and M. H. Bastani, "Robust transmit code and receive filter design for extended targets in clutter," vol. 63, no. 8, pp. 1965–1976, 2015.

[10] Harry L Van Trees, *Detection, Estimation, and Modulation Theory Part III*, John Wiley & Sons, Inc., New Jersey, 2001.

[11] C. Y. Chen and P. P. Vaidyanathan, "MIMO radar waveform optimization with prior information of the extended target and clutter," *IEEE Transactions on Signal Processing*, vol. 57, no. 9, pp. 3533–3544, 2009.

[12] Maurice Sion et al., "On general minimax theorems," *Pacific J. Math*, vol. 8, no. 1, pp. 171–176, 1958.

[13] Stephen Boyd and Lieven Vandenberghe, *Convex optimization*, Cambridge university press, 2004.

[14] Roger A Horn and Charles R Johnson, *Matrix analysis*, Cambridge university press, 2012.

[15] Michael Grant and Stephen Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," `http://cvxr.com/cvx`, Mar. 2014.

[16] Michael Grant and Stephen Boyd, "Graph implementations for nonsmooth convex programs," in *Recent Advances in Learning and Control*, V. Blondel, S. Boyd, and H. Kimura, Eds., Lecture Notes in Control and Information Sciences, pp. 95–110. Springer-Verlag Limited, 2008, `http://stanford.edu/~boyd/graph\_dcp.html`.