ADAPTIVE RADAR DETECTION IN THE PRESENCE OF GAUSSIAN CLUTTER WITH SYMMETRIC SPECTRUM

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ABSTRACT

In this paper, we address the problem of detecting the signal of interest in the presence of Gaussian clutter with symmetric spectrum. To this end, we exploit the spectral properties of the clutter to transfer the binary hypothesis test problem from complex domain to real domain. Then, we devise and assess a detection strategy based on the so-called two-step Generalized Likelihood Ratio Test (GLRT) design procedure. Finally, a preliminary performance assessment, conducted by resorting to simulated data, has confirmed the effectiveness of the newly proposed detector compared with the traditional state-of-the-art counterparts which ignore the spectrum symmetry.

***Index Terms*—** Adaptive Detection, Ground Clutter, Generalized Likelihood Ratio Test (GLRT), Symmetric Spectrum.

1. INTRODUCTION

Adaptive detection of a signal, known up to a scaling factor, in the presence of homogeneous Gaussian clutter with unknown spectral properties, has received an increased attention in radar signal processing community. This is a problem of composite hypothesis testing in which the GLRT is the most widely accepted method of solution [1, 2]. Starting from the lack of a Uniformly Most Powerful (UMP) test for the quoted problem, other design criteria have been investigated as an alternative to the GLRT in [3–5]. For instance, in [3] the Rao test is used to derive a detector that exhibits enhanced rejection capabilities of mismatched signals [6]. All the above solutions suppose that a set of secondary data, free of signal components and sharing the same spectral properties of the data under test (primary data), is available to estimate the clutter covariance matrix. However, secondary data are often contaminated by power variations over range, clutter discretes, and other outliers, which drastically reduce number of homogeneous secondary data. Adaptive detection of signals buried in clutter environments for which the secondary data volume is not large is referred to as *sample-starved problem* [7–9].

Strategies conceived to cope with such situations exhibit a common denominator that consists in incorporating the available a priori information into the detector design (knowledge-aided paradigm). Precisely, in [10–12] the authors show that significant performance improvements can be achieved exploiting the structure information about the clutter covariance matrix. Another example is provided in [13], where the Bayesian approach is employed assuming that the unknown covariance matrix of the clutter obeys a suitable distribution. More recently, the Bayesian framework is also used together with the structural information on the clutter covariance matrix as shown in [14], where the clutter is modeled as a multi-channel auto-regressive process with a random cross-channel covariance matrix.

Another source of a priori information, which can be exploited in the design of adaptive algorithms, is the possible symmetry in the clutter spectral characteristics. In fact, it is well-known that ground clutter observed by a stationary monostatic radar often exhibits a symmetric Power Spectral Density (PSD) centered around the zero-Doppler frequency and whose integral (clutter power) depends on the type of illuminated background [15]. This property has been corroborated by diverse statistical analyses on experimentally measured data [16, 17] and implies that clutter autocorrelation function is real-valued and even. This represents an important structure which would reduce the number of nuisance parameters to estimate and can be exploited at the design stage.

Following the above guideline, in this work we focus on ground clutter dominated environments and design an adaptive decision scheme which leverage on the symmetric PSD constraint for the clutter. We first transform the problem from the complex domain to the real domain and then solve the new hypothesis test resorting to the so-called two-step design procedure [2]. Precisely, this design procedure consists in eval-

The work was in part supported by the National Natural Science Foundation of China under Grant Nos. 61172166 and 61571434.

uating the GLRT assuming that the clutter covariance matrix is known and maximizing over the other unknown parameters. An appropriate estimate of the clutter covariance matrix based on the secondary data data alone is then substituted into this test. The preliminary performance analysis confirms the superiority of the considered architecture over their conventional counterparts which do not capitalize on the real and even PSD of the clutter.

The remainder of this paper is organized as follows. Section 2 addresses the problem formulation, Section 3 deals with the design of the detector, and Section 4 provides illustrative examples. Finally, Section 5 contains some concluding remarks.

1.1. Notation

In the sequel, vectors and matrices are denoted by boldface lower-case and upper-case letters, respectively. Symbols $\det(\cdot)$ and $\mathrm{Tr}(\cdot)$ denote the determinant and the trace of a square matrix, respectively. The Euclidean norm of a vector is denoted by $\|\cdot\|$. Symbol $\boldsymbol{I}_N$ represents the $(N \times N)$-dimensional identity matrix. As to the numerical sets, $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^{N\times M}$ is the set of $(N \times M)$-dimensional real matrices (or vectors if $M = 1$), $\mathbb{C}$ is the set of complex numbers, and $\mathbb{C}^{N\times M}$ is the set of $(N \times M)$-dimensional complex matrices (or vectors if $M = 1$). The real and imaginary parts of a complex vector or scalar are denoted by $\Re(\cdot)$ and $\Im(\cdot)$, respectively. Symbols $(\cdot)^T$, and $(\cdot)^\dagger$ stand for transpose, and conjugate transpose, respectively. Finally, the acronym iid means independent and identically distributed while the symbol $E[\cdot]$ denotes statistical expectation.

2. PROBLEM FORMULATION

In this section, we introduce the detection problem at hand and show that, under the assumption of a symmetric spectrum for the clutter, it is equivalent to another decision problem dealing with real vectors and matrices. To this end, let us begin by formulating the initial problem in terms of a binary hypothesis test. Specifically, we assume that the considered sensing systems acquires data from $N \geqslant 2$ channels which can be spatial and/or temporal. The echoes from the cell under test are properly pre-processed, namely, the received signals are downconverted to baseband or an intermediate frequency; then, they are sampled and organized to form a N-dimensional vector, $\boldsymbol{r}$ say. We want to test whether or not $\boldsymbol{r}$ contains useful target echoes assuming the presence of a set of K secondary data. Summarizing, we can write this decision problem as follows

$$\begin{cases} H_0 : \begin{cases} \boldsymbol{r} = \boldsymbol{n}, \\ \boldsymbol{r}_k = \boldsymbol{n}_k, \quad k = 1, \dots, K, \end{cases} \\ \\ H_1 : \begin{cases} \boldsymbol{r} = \alpha \boldsymbol{v} + \boldsymbol{n}, \\ \boldsymbol{r}_k = \boldsymbol{n}_k, \quad k = 1, \dots, K, \end{cases} \end{cases} \tag{1}$$

where

- $\boldsymbol{v} = \boldsymbol{v}_1 + j\boldsymbol{v}_2 \in \mathbb{C}^{N\times 1}$ with $\|\boldsymbol{v}\| = 1$, $\boldsymbol{v}_1 = \Re\{\boldsymbol{v}\}$, and $\boldsymbol{v}_2 = \Im\{\boldsymbol{v}\}$ is the nominal steering vector;
- $\alpha = \alpha_1 + j\alpha_2 \in \mathbb{C}$ with $\alpha_1 = \Re\{\alpha\}$ and $\alpha_2 = \Im\{\alpha\}$ represents the target response which is modeled in terms of an unknown deterministic factor accounting for target reflectivity and channel propagation effects;
- $\boldsymbol{n} = \boldsymbol{n}_1 + j\boldsymbol{n}_2 \in \mathbb{C}^{N\times 1}$ and $\boldsymbol{n}_k = \boldsymbol{n}_{1k} + j\boldsymbol{n}_{2k} \in \mathbb{C}^{N\times 1}$, $k = 1, \dots, K$, with $\boldsymbol{n}_1 = \Re\{\boldsymbol{n}\}$, $\boldsymbol{n}_2 = \Im\{\boldsymbol{n}\}$, $\boldsymbol{n}_{1k} = \Re\{\boldsymbol{n}_k\}$, and $\boldsymbol{n}_{2k} = \Im\{\boldsymbol{n}_k\}$, are iid complex normal random vectors with zero mean and unknown positive definite covariance matrix $\boldsymbol{M}_0 \in \mathbb{R}^{N\times N}$; it is important to observe here that, since the clutter has zero mean and exhibits a power spectral density with symmetric symmetry, $\boldsymbol{M}_0$ is real-valued.

Now, recall that a zero-mean complex Gaussian vector $\boldsymbol{x} = \boldsymbol{x}_1 + j\boldsymbol{x}_2 \in \mathbb{C}^N$, $\boldsymbol{x}_1 = \Re\{\boldsymbol{x}\}$ and $\boldsymbol{x}_2 = \Im\{\boldsymbol{x}\}$, is said to be complex normal [6] if

$$E[\boldsymbol{x}_1\boldsymbol{x}_1^T] = E[\boldsymbol{x}_2\boldsymbol{x}_2^T], \tag{2}$$

$$E[\boldsymbol{x}_1\boldsymbol{x}_2^T] = -E[\boldsymbol{x}_2\boldsymbol{x}_1^T], \tag{3}$$

and, under the above assumption, the covariance matrix of $\boldsymbol{x}$ can be written as

$$E[\boldsymbol{x}\boldsymbol{x}^\dagger] = 2(E[\boldsymbol{x}_1\boldsymbol{x}_1^T] - jE[\boldsymbol{x}_1\boldsymbol{x}_2^T]) \in \mathbb{C}^{N\times N}. \tag{4}$$

In (1), we have modeled the clutter in terms of complex normal random vectors with zero mean and real covariance matrix, which, in turn, implies that the cross-covariances between the real and imaginary parts of $\boldsymbol{n}$ and $\boldsymbol{n}_k$, $k = 1, \dots, K$, are zero. Thus, we can claim that $\boldsymbol{n}_1$, $\boldsymbol{n}_2$ and $\boldsymbol{n}_{1k}$, $\boldsymbol{n}_{2k}$, $k = 1, \dots, K$, are iid Gaussian vectors with zero mean and covariance matrix

$$\boldsymbol{M} = \frac{1}{2}\boldsymbol{M}_0 \in \mathbb{R}^{N\times N}. \tag{5}$$

As a consequence, (1) is equivalent to the following problem

$$\begin{cases} H_0 : \begin{cases} \boldsymbol{z}_1 = \boldsymbol{n}_1, \ \boldsymbol{z}_2 = \boldsymbol{n}_2, \\ \boldsymbol{z}_{1k} = \boldsymbol{n}_{1k}, \ \boldsymbol{z}_{2k} = \boldsymbol{n}_{2k}, \quad k = 1, \dots, K, \end{cases} \\ \\ H_1 : \begin{cases} \boldsymbol{z}_1 = (\alpha_1\boldsymbol{v}_1 - \alpha_2\boldsymbol{v}_2) + \boldsymbol{n}_1, \\ \boldsymbol{z}_2 = (\alpha_1\boldsymbol{v}_2 + \alpha_2\boldsymbol{v}_1) + \boldsymbol{n}_2, \\ \boldsymbol{z}_{1k} = \boldsymbol{n}_{1k}, \ \boldsymbol{z}_{2k} = \boldsymbol{n}_{2k}, \quad k = 1, \dots, K. \end{cases} \end{cases} \tag{6}$$

The above problem is formally equivalent to (1). As a matter of fact, for the latter problem, the relevant parameter to decide for the presence of a target is α, or, equivalently, the pair (α_1, α_2). After transformation leading to (6), the formal structure of the decision problem is again

$$H_0 : (\alpha_1, \alpha_2) = (0, 0), \quad H_1 : (\alpha_1, \alpha_2) \neq (0, 0). \tag{7}$$

3. DETECTOR DESIGN

A possible way to solve problem (6) is to resort to the two-step GLRT-based design criterion [2]. The rationale of the design procedure is the following: first assume that the covariance matrix $\boldsymbol{M}$ is known and derive the GLRT based on primary data. Then, an adaptive detector is obtained by substituting $\boldsymbol{M}$ by an appropriate estimate based on the secondary data. Following this rationale in the section we design an adaptive detector for the problem at hand.

As preliminary step towards the receiver derivation, let us define the following quantities. Specifically, denote by $\boldsymbol{Z} = [\boldsymbol{z}_1 \ \boldsymbol{z}_2]$ the primary data matrix and $\boldsymbol{Z}_S = [\boldsymbol{z}_{11} \ \ldots \ \boldsymbol{z}_{1K} \ \boldsymbol{z}_{21} \ \ldots \ \boldsymbol{z}_{2K}]$ the overall matrix of the training samples. Under the assumption that $\boldsymbol{M}$ is known, the GLRT is given by

$$\frac{\max\limits_{\alpha_1,\alpha_2} f_1(\boldsymbol{Z}; \boldsymbol{M}, \alpha_1, \alpha_2)}{f_0(\boldsymbol{Z}; \boldsymbol{M})} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{8}$$

where $f_j(\boldsymbol{Z}, \cdot)$ is the probability density functions (PDF) of primary data under H_j, $j = 0, 1$, namely

$$\begin{aligned} f_0(\boldsymbol{Z}; \boldsymbol{M}) &= \frac{1}{(2\pi)^N \det(\boldsymbol{M})} \\ &\quad \times \exp\left\{-\frac{1}{2}\mathrm{Tr}[\boldsymbol{M}^{-1}\boldsymbol{Z}\boldsymbol{Z}^T]\right\}, \\ f_1(\boldsymbol{Z}; \boldsymbol{M}, \alpha_1, \alpha_2) &= \frac{1}{(2\pi)^N \det(\boldsymbol{M})} \\ &\quad \times \exp\Big\{-\frac{1}{2}\mathrm{Tr}\left[\boldsymbol{M}^{-1}\left(\boldsymbol{u}_1\boldsymbol{u}_1^T + \boldsymbol{u}_2\boldsymbol{u}_2^T\right)\right]\Big\}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{u}_1 &= \boldsymbol{z}_1 - \alpha_1\boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2, \\ \boldsymbol{u}_2 &= \boldsymbol{z}_2 - \alpha_1\boldsymbol{v}_2 - \alpha_2\boldsymbol{v}_1. \end{aligned} \tag{10}$$

Substituting (9) in (8), after some algebraic manipulations, the natural logarithm of (8) can be recast as

$$\boldsymbol{z}_1^T\boldsymbol{M}^{-1}\boldsymbol{z}_1 + \boldsymbol{z}_2^T\boldsymbol{M}^{-1}\boldsymbol{z}_2 - \min_{\alpha_1,\alpha_2} f(\alpha_1, \alpha_2) \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{11}$$

where η is the suitable modification of the threshold in (8), and

$$f(\alpha_1, \alpha_2) = \boldsymbol{u}_1^T\boldsymbol{M}^{-1}\boldsymbol{u}_1 + \boldsymbol{u}_2^T\boldsymbol{M}^{-1}\boldsymbol{u}_2. \tag{12}$$

In the next step, our objective is to minimize $f(\alpha_1, \alpha_2)$ with respect to α_1 and α_2. To this end, we evaluate the derivatives with respect to α_1 and α_2, which are given by

$$\begin{aligned} \frac{\partial f(\alpha_1, \alpha_2)}{\partial \alpha_1} &= -2\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{u}_1 - 2\boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{u}_2, \\ \frac{\partial f(\alpha_1, \alpha_2)}{\partial \alpha_2} &= 2\boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{u}_1 - 2\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{u}_2. \end{aligned} \tag{13}$$

Setting to zero the two derivatives of (13), yields

$$\begin{aligned} \hat{\alpha}_1 &= \frac{\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{z}_1 + \boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{z}_2}{\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{v}_1 + \boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{v}_2}, \\ \hat{\alpha}_2 &= \frac{\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{z}_2 - \boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{z}_1}{\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{v}_1 + \boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{v}_2}. \end{aligned} \tag{14}$$

Substituting (14) in (11), the GLRT can be recast as

$$\left(\hat{\alpha}_1^2 + \hat{\alpha}_2^2\right)\left(\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{v}_1 + \boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{v}_2\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{15}$$

The most natural estimator of $\boldsymbol{M}$ in Gaussian clutter is the Sample Covariance Matrix (SCM) based on the secondary data, namely, $\boldsymbol{S} = \boldsymbol{Z}_S\boldsymbol{Z}_S^T$. Plugging $\boldsymbol{S}$ in place of $\boldsymbol{M}$ into (15), the Two-Step GLRT (TS-GLRT) is finally given by

$$\frac{\left(\boldsymbol{v}_1^T\boldsymbol{S}^{-1}\boldsymbol{z}_1 + \boldsymbol{v}_2^T\boldsymbol{S}^{-1}\boldsymbol{z}_2\right)^2 + \left(\boldsymbol{v}_1^T\boldsymbol{S}^{-1}\boldsymbol{z}_2 - \boldsymbol{v}_2^T\boldsymbol{S}^{-1}\boldsymbol{z}_1\right)^2}{\boldsymbol{v}_1^T\boldsymbol{S}^{-1}\boldsymbol{v}_1 + \boldsymbol{v}_2^T\boldsymbol{S}^{-1}\boldsymbol{v}_2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{16}$$

As a final remark, transferring problem (1) from complex domain to real domain, is equivalent to doubling the number of secondary data and, hence the TS-GLRT can work when $K \geqslant N/2$ instead of $K \geqslant N$ which is required by the traditional detectors in [1–3]. Moreover, we expect that the TS-GLRT exhibits superior detection performance with respect to their counterparts which do not capitalize on the real and even PSD of the clutter.

4. PERFORMANCE ASSESSMENT

This section is devoted to the performance assessment of the newly proposed detector in terms of Probability of Detection (P_d). To this end, we compare the TS-GLRT with several traditional detectors, including the GLRT [1], the Adaptive Matched Filter (AMF) [2], and the Rao test [3]. We make use of standard Monte Carlo counting techniques and evaluate the thresholds necessary to ensure a preassigned value of P_{fa} resorting to $100/P_{fa}$ independent trials. Moreover, the P_d values are estimated over 10^4 independent trials, and $P_{fa} = 10^{-4}$.

Clutter model: We assume a clutter-dominated environment with the covariance matrix given by

$$\boldsymbol{M}_0 = \sigma_n^2\boldsymbol{I}_N + \sigma_c^2\boldsymbol{M}_c, \tag{17}$$

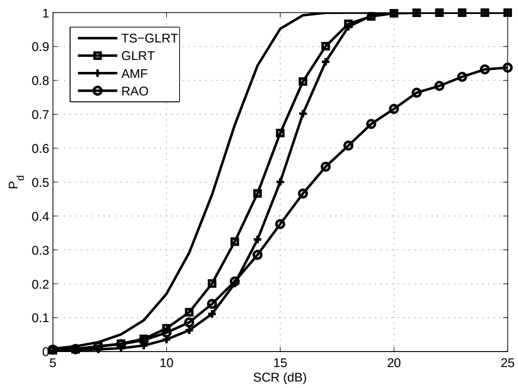

Fig. 1. $P_{\rm d}$ versus SNR for the TS-GLRT, the GLRT, the AMF and the Rao test; $N = 8$, $K = 16$, and $P_{\rm fa} = 10^{-4}$.

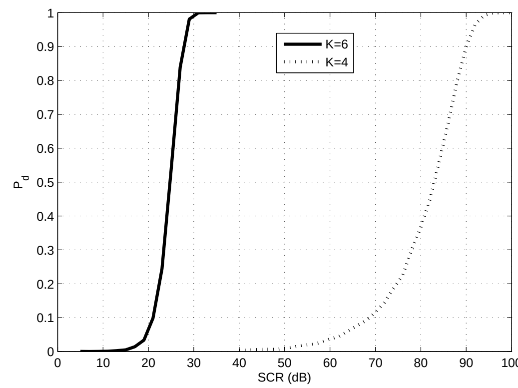

Fig. 3. $P_{\rm d}$ versus for the TS-GLRT; $N = 8$, $K < N$, and $P_{\rm fa} = 10^{-4}$.

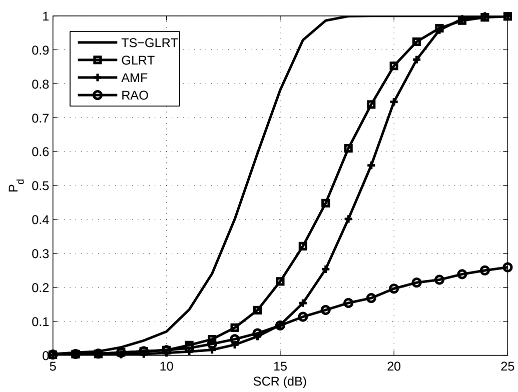

Fig. 2. $P_{\rm d}$ versus SNR for the TS-GLRT, the GLRT, the AMF and the Rao test; $N = 8$, $K = 12$, and $P_{\rm fa} = 10^{-4}$.

where $\sigma_n^2 = 1$, $\sigma_c^2 > 0$ is evaluated assuming a Clutter-to-Noise Ratio (CNR) of 30 dB, and the (i, j)th element of $\boldsymbol{M}_c$ is given by $\rho^{|i-j|}$ with $\rho = 0.9$. Finally, the Signal-to-Clutter Ratio (SCR) is defined as

$$\mathrm{SCR} = |\alpha|^2 \boldsymbol{v}^\dagger \boldsymbol{M}_0^{-1} \boldsymbol{v}. \tag{18}$$

In Fig. 1, we study the performance of four different detectors assuming $N = 8$ and $K = 16$. As it can be seen from the figure, the TS-GLRT guarantees a P_d gain with an order of 2.5 dB with respect to the GLRT at $P_d = 0.9$. Moreover, the above-mentioned P_d gain can be increased by decreasing K, as shown in Fig. 2, where we plot the P_d of the considered detectors for the same system parameters as in Fig. 1, but for $K = 12$. Precisely, in this case the P_d gain at $P_d = 0.9$ between the TS-GLRT and the GLRT increases to 5.5 dB. Thus, for the problem of a binary hypothesis test with real-valued clutter covariance, transferring it from complex domain to real domain is a very effective means to improve performance of detection, especially in the presence of a small number of secondary data.

Finally, in Fig. 3 we plot P_d against SCR assuming $K < N$. In particular, we set $N = 8$ and two cases of K, i.e., $K = 6$ and $K = 4$. We only consider the TS-GLRT, due to the fact that the other three traditional detectors can not work under this condition. As we expected, the larger K, the better P_d the TS-GLRT achieves.

5. CONCLUSIONS

In this paper, we have proposed a decision scheme for adaptive detection in Gaussian clutter with the symmetric PSD constraint. In order to derive the new detector, we assumed that a set of secondary data, free of signal components and sharing the same spectral properties of the clutter as the primary data, is available. Moreover, we transfer the binary hypothesis test problem from complex domain to real domain, and resort to the two-step GLRT-based design procedure. The performance assessment has demonstrated that the proposed receiver can significantly outperform its natural competitors which do not capitalize on the real and even PSD of the clutter, especially in a severely heterogeneous scenario where a very small number of secondary data is available.

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