A NORMALIZED SPATIAL SPECTRUM FOR DOA ESTIMATION WITH UNIFORM LINEAR ARRAYS IN THE PRESENCE OF UNKNOWN MUTUAL COUPLING

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ABSTRACT

Eigenstructure based self-calibration methods usually employ MUSIC algorithm or its variations to estimate DOAs. But ambiguous DOA estimates may be obtained since the spatial spectra of these methods might be seriously disturbed by unknown mutual coupling. In this work, we try to mitigate the perturbations caused by unknown mutual coupling in eigenstructure based self-calibration. We analyse the property of mutual coupling matrix and find that part of the mutual coupling inducing false peaks in the spatial spectrum can be predicted. Thereby, a normalized spatial spectrum is proposed to automatically remove these false peaks. The proposed spectrum is applicable to some types of existing algorithms with a low cost of computation. Since most of the false peaks in the original spatial spectrum are removed, a more robust DOA estimate can be expected. The effectiveness of the normalized spatial spectrum is validated via numerical experiments.

***Index Terms*—** Direction-of-arrival (DOA) estimation, uniform linear array (ULA), mutual coupling, array self-calibration

1. INTRODUCTION

DOA estimation using an array of sensors is a classical problem that arises in radar, sonar, wireless communications, and radio astronomy. The super-resolution DOA estimation methods, such as the eigenstructure methods and the maximum likelihood (ML) algorithms [1], can estimate directions of closely spaced spatial sources which cannot be distinguished by traditional techniques. However, this advantage is based on exact knowledge of array manifold, which is often not available in practice due to unavoidable array modeling errors. These errors, *e.g.*, induced by unknown mutual coupling or array gain/phase perturbations, can cause substantial performance degradation for super-resolution algorithms [2–5]. Thereby, array calibration and robust direction finding (DF) techniques are essential in all practical systems for DOA estimation.

When unknown mutual coupling exists, DOA estimation with arbitrary arrays remains difficult. However, this problem may be solvable for centrosymmetric arrays, such as uniform linear arrays (ULAs) and uniform circular arrays (UCAs), whose mutual coupling matrices (MCM) are banded (or circulant) and symmetric Toeplitz and can be represented by a small number of unknown coefficients. Friedlander *et al*. initially employed this property of centrosymmetric arrays and proposed an iterative self-calibration method [6]. Since then, plenty of self-calibration methods were proposed based on this property. For example, Ye *et al.* [7, 8] proved that if a group of auxiliary sensors is available, then MUSIC algorithm can achieve accurate direction finding without mutual coupling compensation. This result, which is named as auxiliary method, is then extended to different scenarios [9–11]. The applications of these methods are limited by requiring a large amount of auxiliary sensors. Another branch of eigenstructure methods, named as rank-reduction (RARE) estimator [12–14], can make use of the whole array. However, the spectra of RARE methods are susceptible to mutual coupling effects, which may lead to ambiguous DOA estimation. Efforts are made to distinguish [13, 14] and suppress [15, 16] the false peaks in those spectra. The method in [15] requires a multidimensional search of DOAs and is computationally expensive. The recursive RARE (R-RARE) [16] transforms the multidimensional search problem to a sequence of one-dimensional search problems to reduce computational cost. But it still suffers from false peaks when the mutual coupling is strong. It should be mentioned that self-calibration based on sparse representation has attracted wide attention recently [17–21]. These methods usually requires strong assumptions such that there applications are restricted.

In this paper, we consider eigenstructure based DOA estimation in a ULA with unknown mutual coupling. By analysing the property of MCM, we find that part of the mutual coupling inducing false peaks in the RARE spectrum can be predicted. Thereby, we propose a new spatial spectrum which can filter out these predictable peaks automatically. The new spectrum shares two advantages. First, it can be directly used in existing algorithms with little computational

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cost. Second, since many false peaks are filtered out, it is more robust to unknown mutual coupling than the original spatial spectrum.

This paper is organized as follows. The signal model and problem statement are given in Section II. Section III analyses the properties of MCM and proposes the normalized spatial spectrum. Simulations are performed in Section IV to apply the proposed spectrum in existing algorithms and compare it with the original spatial spectrum. Section V concludes the paper.

Our notations are as follows. Lowercase boldface and uppercase boldface are used for vectors and matrices, respectively. For a given matrix $\boldsymbol{X}$, we use $\text{rank}[\boldsymbol{X}]$, $|\boldsymbol{X}|$, and $[X]_{p,q}$ to denote rank, determinant, and the q-th elements of the p-th row of the matrix $\boldsymbol{X}$. For any vector and matrix, $\{\cdot\}^H$ and $\|\cdot\|$ denote the Hermitian (conjugate) transpose and 2-norm, respectively. $\boldsymbol{0}$ is a vector of zeros. $\lceil x \rceil$ rounds x to the nearest integer towards infinity.

2. PROBLEM FORMULATION

2.1. Signal Model

Consider M narrow-band far field signals impinging on an ULA of N sensors. The standard baseband model for the array output is given by

$$\boldsymbol{x}(k) = \boldsymbol{C}\boldsymbol{A}\boldsymbol{s}(k) + \boldsymbol{n}(k), \tag{1}$$

where $\boldsymbol{C}$ and $\boldsymbol{A}$ denote the MCM and array manifold, $\boldsymbol{s}(k)$ is an $M \times 1$ vector of impinging signal waveforms, and $\boldsymbol{n}(k)$ is a vector of additional Gaussian noise with zeros mean and variance σ^2. The array manifold $\boldsymbol{A} = \left[\boldsymbol{a}(\tilde{\theta}_1), \boldsymbol{a}(\tilde{\theta}_2), \ldots, \boldsymbol{a}(\tilde{\theta}_M)\right]$ is an $N \times M$ matrix, the steering vector $\boldsymbol{a}(\tilde{\theta}_m)$ is a function of DOA $\tilde{\theta}_m$, and $S_r = \{\tilde{\theta}_m\}_{m=1}^M$ is the set of real DOAs. The MCM $\boldsymbol{C}$ is modeled as an N-dimensional banded and symmetric Toeplitz matrix to approximate the real-world mutual coupling effect [6]. The first column of $\boldsymbol{C}$ is denoted by $[\boldsymbol{c}^T, \boldsymbol{0}^T]^T$, where $\boldsymbol{c} = [c_1, c_2, \cdots, c_L]$ is the vector of mutual coupling effects and $1 = c_1 > |c_2|, \cdots, |c_L| > 0$.

We make the standard assumptions underlying the eigenstructure based methods for direction finding.
1) $M < N$ and the columns of $\boldsymbol{C}\boldsymbol{A}$ are linear independent.
2) The signals and noise are stationary and ergodic over the observation period.
3) The signals are uncorrelated and the noise is uncorrelated with the signals.

2.2. Self-calibration Based on MUSIC Spectrum

Many eigenstructure methods are variations of the MUSIC algorithm, which can be summarized in three main steps. First, the noise subspace is constructed based on the eigenvalue decomposition of the covariance matrix of array output

$$\boldsymbol{R}_x = \mathbb{E}\left[\boldsymbol{x}(k)\boldsymbol{x}^H(k)\right] = \boldsymbol{U}_s\boldsymbol{\Lambda}_s\boldsymbol{U}_s^H + \sigma_n^2\boldsymbol{U}_n\boldsymbol{U}_n^H, \tag{2}$$

where $\boldsymbol{\Lambda}_s$ is a diagonal matrix of the M principle eigenvalues, $\boldsymbol{U}_s$ is a matrix consisting of the M corresponding eigenvectors, and U_n is a matrix consisting of the remaining $N - M$ eigenvectors. The noise subspace is then spanned by the vectors of $\boldsymbol{U}_n$. Based on the above decomposition, the second step makes use of the subspace property

$$\boldsymbol{U}_n^H\boldsymbol{C}\boldsymbol{A} = \boldsymbol{0}, \tag{3}$$

and constructs MUSIC spectrum

$$P(\theta) = \frac{1}{\|\boldsymbol{U}_n^H\boldsymbol{C}\boldsymbol{a}(\theta)\|^2}. \tag{4}$$

Finally, if $\boldsymbol{C}$ is given, the DOA estimation can be obtained at the peaks of the MUSIC spectrum.

This algorithm can provide accurate DOA estimation [2, 6]. However, when $\boldsymbol{C}$ is unknown, the spectrum (4) does not work any more. In this case, self-calibration methods in [12–14] utilize the banded and symmetric Toeplitz structure of the MCM, *i.e.*,

$$\boldsymbol{C}\boldsymbol{a}(\theta) = \boldsymbol{Q}(\theta)\boldsymbol{c}, \tag{5}$$

and modify the MUSIC spectrum (4) to

$$P_m(\theta) = \frac{1}{\min_{\boldsymbol{c}}\{\left\|\boldsymbol{U}_n^H\boldsymbol{Q}(\theta)\boldsymbol{c}\right\|^2\}}, \tag{6}$$

where $\boldsymbol{Q}(\theta) = \boldsymbol{Q}_1(\theta) + \boldsymbol{Q}_2(\theta)$ and

$$[\boldsymbol{Q}_1(\theta)]_{p,q} = \begin{cases} [\boldsymbol{a}(\theta)]_{p+q-1}, & \forall\, p+q \leq N+1 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

$$[\boldsymbol{Q}_2(\theta)]_{p,q} = \begin{cases} [\boldsymbol{a}(\theta)]_{p-q+1}, & \forall\, p \geq q \geq 2 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

Spectrum (6) is based on the observation that $\|\boldsymbol{U}_n^H\boldsymbol{Q}(\theta)\boldsymbol{c}\| \geq 0$ and equality holds if $\theta \in S_r$ and $\boldsymbol{c} = \boldsymbol{c}_0$, where $\boldsymbol{c}_0$ is the vector of real mutual coupling coefficients. This spectrum, named as RARE spectrum, is proposed for self-calibration of UCAs. The simulations in [14] show that it may yield false peaks, especially when $L > \lceil \frac{N}{2} \rceil$. The authors of [15, 16] extended RARE to the multi-dimensional form

$$P_{mm}(\theta) = \frac{1}{\min_{\boldsymbol{c}}\{\sum_{k=1}^K \|\boldsymbol{U}_n^H\boldsymbol{Q}(\theta_k)\boldsymbol{c}\|^2\}}, \tag{9}$$

and applied it in ULAs, where K is an integer and $1 \leq K \leq M$. However, the proposed methods are still at risk of false peaks if mutual coupling is strong.

3. A NORMALIZED SPATIAL SPECTRUM FOR SELF-CALIBRATION

In this section, we construct a spectrum that is more robust to unknown mutual coupling and can be applied in the above mentioned methods with a low cost of computaion. To achieve this goal, we first point out some properties of $\boldsymbol{Q}(\theta)$ by the following proposition.

Proposition 3.1 *In a ULA where the distance between neighboring sensors is half-wavelength, the $N \times L$ matrix $\boldsymbol{Q}(\theta)$ has the following properties:*
1. If $L \leq \lceil \frac{N}{2} \rceil$, $\boldsymbol{Q}(\theta)$ is full column rank.
2. If $\lceil \frac{N}{2} \rceil < L \leq N$, then $\lceil \frac{N}{2} \rceil \leq \operatorname{rank}[\boldsymbol{Q}(\theta)] \leq L$ and $\operatorname{rank}[\boldsymbol{Q}(\theta)] < L$ only if $\theta \in S_n$, where $S_n = \{\theta | \theta = \arcsin(\frac{k}{N}), k = -N, -N+1, \cdots, N\}$.

The proposition can be proved by induction, which is omitted here due to the page limit. By proposition 3.1, when $L > \lceil \frac{N}{2} \rceil$ and $\exists \theta_f \in S_n$ such that $\boldsymbol{Q}(\theta_f)$ is rank deficient, we can select $\boldsymbol{c}_f$ from the null space of $\boldsymbol{Q}^H(\theta_f)\boldsymbol{Q}(\theta_f)$ such that $\boldsymbol{U}_n^H \boldsymbol{Q}(\theta_f)\boldsymbol{c}_f = \boldsymbol{0}$, $\forall \boldsymbol{U}_n$. In this case, a false peak appears in the spectrum (6) if θ_f is not a real DOA. Notice that the number of these false peaks might be as large as $2N+1$, which will cause serious ambiguous DOA estimate. Therefore, using a small L will suppress false peaks caused by rank deficiency of $\boldsymbol{Q}(\theta)$ and yield a more robust DOA estimate. However, choosing a smaller L also introduces more modeling errors and hence DOA estimation errors in real systems, especially when the mutual coupling effect is strong. When $L > \lceil \frac{N}{2} \rceil$ is selected, one can just throw the peaks at θ_f if $\boldsymbol{Q}(\theta_f)$ is rank deficient. But this approach may miss the real sources around θ_f.

To eliminate the false peaks caused by rank deficiency, we normalize the spectrum (6) by $\|\boldsymbol{Q}(\theta)\boldsymbol{c}\|$ and obtain the following spatial spectrum

$$P_n(\theta) = \frac{1}{f(\theta)}, \text{ with } f(\theta) = \min_{\{\boldsymbol{c}, c_1 = 1\}} \frac{\|\boldsymbol{U}_n^H \boldsymbol{Q}(\theta)\boldsymbol{c}\|^2}{\|\boldsymbol{Q}(\theta)\boldsymbol{c}\|^2}. \quad (10)$$

The optimal solution of (10) is $\boldsymbol{c}_\theta = \boldsymbol{v}_1/[\boldsymbol{v}_1]_1$, where $\boldsymbol{v}_1$ denotes the eigenvector corresponding to the smallest generalized eigenvalue of matrices $\boldsymbol{Q}(\theta)^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{Q}(\theta)$ and $\boldsymbol{Q}(\theta)^H \boldsymbol{Q}(\theta)$. By denoting $\boldsymbol{q}_\theta = \frac{\boldsymbol{c}_\theta}{\|\boldsymbol{Q}(\theta)\boldsymbol{c}_\theta\|}$, $P_n(\theta)$ in (10) can be rewritten as

$$P_n(\theta) = \frac{1}{\|\boldsymbol{U}_n^H \boldsymbol{Q}(\theta)\boldsymbol{q}_\theta\|^2}, \text{ with } \|\boldsymbol{Q}(\theta)\boldsymbol{q}_\theta\| = 1. \quad (11)$$

Hence, we call $P_n(\theta)$ the normalized spatial spectrum.

The performance of this new spectrum can be illustrated in three aspects. First, it will not miss peaks at real DOAs. It is seen that $f(\theta) \geq 0$ and equality holds when $\theta \in S_r$ and $\boldsymbol{c}_\theta = \boldsymbol{c}_0$, where $\boldsymbol{c}_0$ denotes the vector of real mutual coupling coefficients. Therefore, peaks will appear in $P_n(\theta)$ at real DOA locations unless $\boldsymbol{Q}(\theta)\boldsymbol{c}_0 = 0$. Second, $P_n(\theta)$ can eliminate false peaks caused by rank deficiency of $\boldsymbol{Q}(\theta)$, which is obvious according to (11). Last, but not least, it will not introduce new false peaks comparing with the RARE spectrum (6). According to (10), a false peak appears in $P_n(\theta)$ if one of the following two conditions is satisfied, $\|\boldsymbol{U}_n^H \boldsymbol{Q}(\theta)\boldsymbol{c}\|^2 = 0$ or $\|\boldsymbol{Q}(\theta)\boldsymbol{c}\|^2$ goes to infinity. The latter cannot happen since $\|\boldsymbol{Q}(\theta)\|$ and $\|\boldsymbol{c}\|$ are upper bounded. And the former also leads to a false peak in the RARE spectrum.

By defining

$$\boldsymbol{W}_n(\boldsymbol{\theta}) = \sum_{k=1}^{K} \boldsymbol{Q}(\theta_k)^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{Q}(\theta_k),$$
$$\boldsymbol{W}_q(\boldsymbol{\theta}) = \sum_{k=1}^{K} \boldsymbol{Q}(\theta_k)^H \boldsymbol{Q}(\theta_k), \quad (12)$$

the multi-dimensional spectrum (9) can also be normalized and obtained by

$$P_{mn}(\boldsymbol{\theta}) = \frac{\|\boldsymbol{W}_q^{\frac{1}{2}}(\boldsymbol{\theta})\boldsymbol{c}_{\boldsymbol{\theta}}\|^2}{\|\boldsymbol{W}_n^{\frac{1}{2}}(\boldsymbol{\theta})\boldsymbol{c}_{\boldsymbol{\theta}}\|^2}, \quad (13)$$

where $\boldsymbol{c}_{\boldsymbol{\theta}} = \boldsymbol{v}_1/[\boldsymbol{v}_1]_1$, and $\boldsymbol{v}_1$ denotes the eigenvector corresponding to the smallest generalized eigenvalue of matrices $\boldsymbol{W}_n(\boldsymbol{\theta})$ and $\boldsymbol{W}_q(\boldsymbol{\theta})$.

4. SIMULATION RESULTS

In this section, we demonstrate the performance of the normalized spatial spectrum by applying it in existing algorithms. Consider a 9-element ULA whose elements are monopole and vertically polarized with 10m long and 20m spacing and central frequency is 8MHz. The vector of mutual coupling coefficients is derived based on the electromagnetic theory with its value $\bar{\boldsymbol{c}} = [1, 0.046 - 0.428i, -0.116 + 0.213i, 0.117 - 0.119i, -0.108 + 0.066i, 0.096 - 0.031i, -0.084 + 0.007i, 0.072 + 0.010i]$. The MCM $\boldsymbol{C}$ is formed as a banded and symmetric Toeplitz matrix by a truncated version of $\bar{\boldsymbol{c}}$, *i.e.*, $\boldsymbol{c} = [\bar{c}_1, \bar{c}_2, \cdots, \bar{c}_L]$. The additional Gaussian noise is temporally and spatially white. The number of snapshots is 100.

In the first experiment, we examine proposition 3.1 and verify whether the proposed spectra (10) and (13) can suppress false peak caused by rank deficiency. We apply (10) and (13) in two methods, the reduced rank (RARE) method and the recursive-RARE (R-RARE) method [16], respectively. Consider $M = 3$ equal power sources impinging form $[-10°, 8°, 23°]$. The signal-to-noise ration (SNR) is 20dB. Since $L \leq N - M$ is a necessary condition for successful self-calibration in the two methods, we set $L = 6$.

Fig. 1(a) plots the curve of minimum eigenvalue of $\boldsymbol{Q}(\theta)$ varies with θ, *i.e.*, "$\lambda_1(Q)$", and compares the spectra RARE (4) and RARE based on (10). In the figure, S_r and S_n denote the set of real DOAs and the set defined in proposition 3.1,

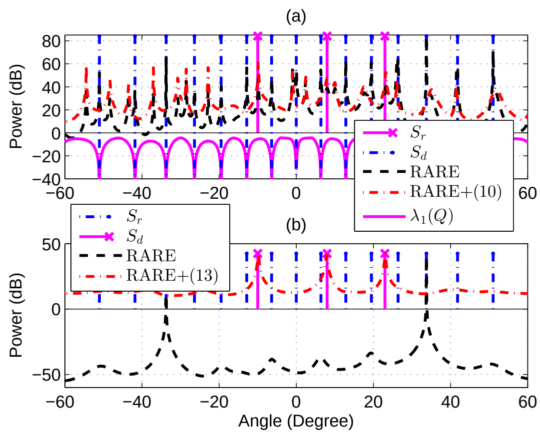

Fig. 1. Comparison of spatial spectra for a 9-element ULA using 100 snapshots. (a) Minimum eigenvalue of $\boldsymbol{Q}(\theta)$, spectra of RARE and RARE based on (10) (b) Spectra of R-RARE [16] and R-RARE based on (13)

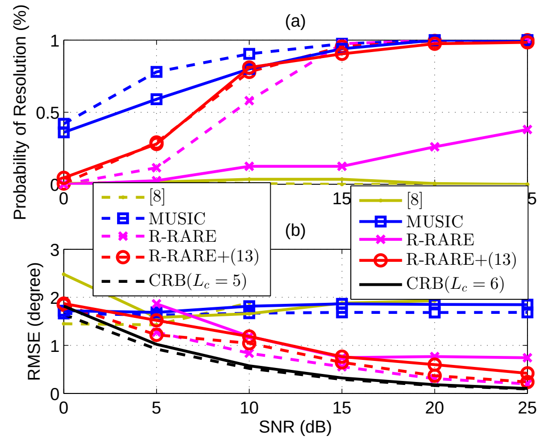

Fig. 2. Performances of four methods at different SNRs with 100 snapshots. (a) Probability of resolution against SNR (b) RMSE of DOA estimation against SNR

respectively. We can observe that the minimum eigenvalue of $\boldsymbol{Q}(\theta)$ equals zero only when $\theta \in S_n$, which coincides with proposition 3.1. It is also seen that peaks exist at directions of signal sources in both spectra. Moreover, the RARE based on (10) can erase the false peaks caused by rank deficiency of $\boldsymbol{Q}(\theta)$ in original RARE and does not yield any new false peak. To remove the remaining false peaks, we apply the R-RARE method.

Fig. 1(b) compares the R-RARE spectrum [16] with R-RARE spectrum based on (13), which are tagged as "R-RARE" and "R-RARE+(13)", respectively. One can observe that R-RARE spectrum fails and the R-RARE spectrum based on (13) can give correct DOA estimate. This is because the initialization step in the R-RARE method is based on the RARE spectrum in Fig. 1(a) which has been ruined by the false peaks at S_n. The results of simulation 1 illustrate that the normalized spectrum can provide more robust DOA estimation when $L \geq \lceil \frac{N}{2} \rceil$.

In the second simulation, we evaluate the self-calibration performance of the proposed spectrum. Specifically, four different subspace algorithms are considered: MUSIC without array calibration, the auxiliary method in [8], the R-RARE method in [16], and the R-RARE method based on (13). We test these algorithms with $M = 3$ equal power sources locates at $[-8°, 10°, 15°]$ and consider $L = 5 \leq \lceil \frac{N}{2} \rceil$ and $L = 6 > \lceil \frac{N}{2} \rceil$, respectively. When $L > \lceil \frac{N}{2} \rceil$, the R-RARE algorithms apply a pre-estimating step to mitigate the impact of false peaks, which pre-estimate the DOAs by using MUSIC without array calibration and consider the true directions lie with in a half-power beam width centered at the pre-estimates. In the auxiliary method, the length of mutual coupling is assumed to be 3 and the rest mutual coupling coefficients are taken as modeling errors, such that the $M + 2L - 1 \leq N$ constraint is satisfied [8]. All the results to be shown were averages over 200 trials.

Fig. 2 illustrates the probability of resolution and the root mean square error (RMSE) against SNR, respectively. Here, the signals are assumed to be successfully resolved if the biases of the DOA estimates are less than 2°, and the RMSE is calculated based on estimates of successful resolutions. In the figure, dash lines and solid lines correspond to $L = 5$ and $L = 6$, respectively, and "CRB" denotes Cramer-Rao bound of DOA estimation.

We can observe in Fig. 2(a) that the probabilities of resolution of different methods increase with SNR except that of the auxiliary method [8]. The reason is the resolution ability of auxiliary method is weakened by a reduced array aperture and modeling errors. It is also seen that the probability of resolution of "R-RARE+(13)" is stable and larger than that of the R-RARE, which decreases drastically when $L > \lceil \frac{N}{2} \rceil$. In Fig. 2(b), the RMSE of the MUSIC method and auxiliary method is lower bounded for modeling errors, while that of the R-RARE methods decrease with the increasing of SNR. There is a gap between the RMSE curves of R-RARE methods and the CRB. This gap can be reduced by embedding the estimated mutual coupling coefficients in the eigenstructure based algorithms.

5. CONCLUSIONS

Based on the analysis of MCM, we proposed a normalized spatial spectrum and applied it in the RARE and R-RARE methods. Theoretical analysis and simulation results show that new spectrum can suppress the false peaks caused by rank deficiency and increase the probability of resolution.

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