DIRECTION-OF-ARRIVAL ESTIMATION WITH ESPAR ANTENNAS USING BAYESIAN COMPRESSIVE SENSING

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ABSTRACT

This paper presents a novel approach of direction-of-arrival (DoA) estimation for the electronically steerable parasitic array radiator (ESPAR) antennas, using only a single radio-frequency (RF) chain. Starting from the problem formulation in the Bayesian compressive sensing (BCS) framework, the CS measurements are projected onto the beamspace of the unique configuration of the ESPAR antenna. In this work, measurements collected at multiple snapshots are considered. First, we propose to solve the sparse recovery problem by the multi-task BCS [1]. Then, the DoAs are estimated by employing a noise filter on the recovered sparse signal. In this method, the number of sources need not be known *a priori*, and computation complexity is reduced by avoiding computing the correlation matrix of measurements unlike the traditional DoA estimation techniques. Simulations show that the proposed method can recover closely spaced sources using a small number of noisy snapshots, and it performs better with more sources than other state-of-the-art algorithms.

Index Terms— ESPAR antenna, DoA estimation, Array signal processing, Bayesian compressive sensing

1. INTRODUCTION

Recently, there has been increasing interests in DoA estimation with the ESPAR antennas [2]. The ESPAR antenna, a kind of smart antenna, uses only a single RF chain to transmit/receive data, thereby reducing hardware complexity, cost and power consumption. Thus, it is well suited for various applications including small radio terminals. In the ESPAR array, directional beamforming is achievable by tuning reactance values loaded to the parasitic elements, which are mutually coupled with the sole active element.

However, the use of a single RF chain in the ESPAR antennas may pose some difficulties in applying the DoA estimation algorithms, derived with the traditional antenna arrays. In the ESPAR array, only the single-port output is attainable, while signals impinging on the parasitic elements cannot be observed. However, currents induced on the parasitic elements are mutually coupled with that on the active element. Moreover, there is a non-linear relationship between the single-port output and reactance loads. To solve these problems, signal processing in the ESPAR array is generally performed in reactance domain (beamspace) instead of element domain. In [3], authors developed a reactance-domain (RD) multiple signal classification (MUSIC) algorithm for the ESPAR antenna, to provide a high resolution method. Taillefer et al. [4] demonstrated that it is suited to exploit the invariances of the regular structure of a hexagonal ESPAR array; therefore, the ESPRIT algorithm was modified in the reactance domain. Indeed, the subspace algorithms require the evaluation of the covariance matrix estimated from the measurements and assume a large number of snapshots for data measurements. This implies an unavoidable increase of the receiver complexity and a delay in the DoA estimation. Since the signals impinging on an antenna array is sparse in the spatial domain, it is possible to employ the emerging compressive sensing theory [5] for DoA estimation. A DoA estimation approach based on CS has been studied for ESPAR antennas in [6], where the DoA estimation problem was cast to an multiple measurement vector (MMV) problem, and then the l_1-SVD (singular value decomposition) algorithm [7] was exploited as the numerical solution. The main drawback of this approach as well as the RD subspace algorithms is that it require a prior knowledge of the number of sources, that is not usually achievable in practice.

Alternatively, the Bayesian compressive sensing (BCS) [8] has been proposed, where the original deterministic CS problem is reformulated in its probabilistic counterpart and then efficiently solved with the relevance vector machine (RVM). In [9], the BCS has been studied for the DoA estimation with a traditional uniform linear array. A novel DoA estimation method based on BCS is studied for the ESPAR antenna for the first time. This method avoids the computation of the correlation matrix of the measurements by directly linking the measurements to the parameters being estimated. Therefore, the computational complexity is reduced. More-

Thanks to the EPSRC Project EP/M014126/1, Large Scale Antenna Systems Made Practical: Advanced Signal Processing for Compact Deployments [LSAS-SP], for funding.

over, the BCS-based method does not require the knowledge of the number of source signals, and thus it is more practical compared to the l_1-SVD and the RD-MUSIC algorithms.

The rest of the paper is organized as follows. The ESPAR antenna and the problem formulation are given in Section 2. The proposed DoA estimation approach is presented in Section 3. The simulations are given in Section 4. Finally, Section 5 concludes this paper.

2. PROBLEM FORMULATION

2.1. ESPAR Antenna

Consider an ESPAR antenna with $M+1$ elements, whose structure example can be found in [2]. One active element (#0) is located in the center of a circle with radius d, where d is usually set to smaller than a half of wavelength for strong mutual coupling between elements. M parasitic elements sit at the equal angular separations on the circle, i.e., the m-th parasitic element is located at angle $\phi_m = (m-1)\frac{2\pi}{M}, m \in \{1, \cdots, M\}$. The active element is connected to the single RF chain and fed to a low noise amplifier (LNA) with a loading impedance Z_s. It is noted that, throughout this work, the loading impedance Z_s is assumed to be perfectly matched with the input impedance (Z_{in}) seen by the active element, that leads to 100% radiation efficiency of the antenna system. Parasitic elements are connected to variable reactors (varactors), which control the reactance loads of parasitic elements, denoted by a vector $\mathbf{x} = [x_1, \cdots, x_M]^T$. $(\cdot)^T$ defines transpose operator.

In this work, we focus analysis on the 2-dimensional DoA estimation, i.e., only the azimuth angle θ is considered. Assume that, at an instant sensing period, the k-th set of reactance loads $\mathbf{x}_k$ is used. The corresponding k-th beampattern voltage response of the ESPAR antenna is represented as [10]:

$$B_k(\theta) = \mathbf{w}_k^T \mathbf{a}(\theta), \tag{1}$$

where $\mathbf{a}(\theta) = [1, e^{-jd\frac{2\pi}{\lambda}\cos(\theta-\phi_1)}, \cdots, e^{-jd\frac{2\pi}{\lambda}\cos(\theta-\phi_M)}]^T$ is the steering vector determined from the antenna geometry, where λ is the carrier wavelength. In (1), $\mathbf{w}_k \in \mathbb{C}^{(M+1)\times 1}$ is an equivalent weight vector defining the k-th beampattern, given by

$$\mathbf{w}_k = (\mathbf{Z} + \mathbf{X}_k)^{-1}\mathbf{u}_0, \tag{2}$$

where entries of the matrix $\mathbf{Z} \in \mathbb{C}^{(M+1)\times(M+1)}$ represent the mutual impedance between antenna elements, and $\mathbf{u}_0 = [1, 0, \cdots, 0]^T$ is a selection vector with $(M+1)$ dimensions. The matrix $\mathbf{X}_k$ is an $(M+1) \times (M+1)$ diagonal loading matrix used to form the k-th beampattern, i.e.,

$$\mathbf{X}_k = diag\left(\begin{bmatrix} Z_s & j\mathbf{x}_k^T \end{bmatrix}\right), \tag{3}$$

where j is the complex unit.

2.2. DoA Estimation Problem

Let us consider L signals $s_l(t)$ from unknown directions $\theta_l, l \in \{1, \cdots, L\}$ impinging on the ESPAR array. The incident signals are assumed to be narrowband, far-field and characterized by the same frequency content. The voltage responses of all elements for an impinging signal are combined due to strong mutual coupling and collected from the single-port. At time t, when the k-th beampattern is formed, the single-port output of the ESPAR array is a linear combination of L signals, which is written as

$$\begin{aligned} y_k(t) &= \sum_{l=1}^{L} \mathbf{w}_k^T \mathbf{a}(\theta_l) s_l(t) + e_k(t), \\ &= \mathbf{w}_k^T \mathbf{A}(\boldsymbol{\theta}) \mathbf{s}(t) + e_k(t), \end{aligned} \tag{4}$$

where $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_L)]$ is the matrix of steering vectors corresponding to unknown directions $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_L]$, $\mathbf{s}(t) = [s_1(t), \cdots, s_L(t)]^T$ is the signal vector, and $e_k(t)$ is measurement noise represented as zero-mean Gaussian distributions with variance σ^2. According to (4), the DoA estimation problem is to find the unknown directions $\theta_l, \forall l$ and the number of sources L, given the knowledge of $y_k(t)$ and the mapping $\boldsymbol{\theta} \rightarrow \mathbf{A}(\boldsymbol{\theta})$. It is worth emphasizing that this problem is non-linear, since the targeting parameters $\boldsymbol{\theta}$ are present in the exponential terms of the elements of $\mathbf{A}(\boldsymbol{\theta})$.

To apply the CS theory, the DoA estimation problem has to be reformulated as a sparse signal representation problem. The whole azimuth plane is discretized as a sampling grid, denoted as $\tilde{\boldsymbol{\theta}} = [\tilde{\theta}_1, \cdots, \tilde{\theta}_{N_\theta}]$, where $N_\theta \gg L$. It is noted that, in this work, the true signal directions $\theta_l, \forall l$ are assumed to belong to the fine sampling grid $\tilde{\boldsymbol{\theta}}$. In other words, we do not consider the case where the true signal directions are off-grid, that may be solved in the future work. An overcomplete dictionary is constructed as $\mathbf{A}(\tilde{\boldsymbol{\theta}}) = [\mathbf{a}(\tilde{\theta}_1), \cdots, \mathbf{a}(\tilde{\theta}_{N_\theta})]$. The use of the overcomplete dictionary $\mathbf{A}(\tilde{\boldsymbol{\theta}})$ allows one to transform the problem of parameter estimation of $\boldsymbol{\theta}$ to the problem of sparse spectrum estimation of $\tilde{\mathbf{s}}(t)$ (i.e., only a few entries of $\tilde{\mathbf{s}}$ are non-zero for $\tilde{\theta}_n = \theta_l$). Equation (4) is rewritten as

$$y_k(t) = \mathbf{w}_k^T \mathbf{A}(\tilde{\boldsymbol{\theta}}) \tilde{\mathbf{s}}(t) + e_k(t). \tag{5}$$

The sparse representation problem in (5) is linear, since $\mathbf{A}(\tilde{\boldsymbol{\theta}})$ is known and does not depend on the actual source directions $\boldsymbol{\theta}$. In the framework of CS, $\mathbf{w}_k^T$ can be considered as a projection vector, which corresponds to one CS measurement, and $K (K < N_\theta)$ such measurements constitute the overall CS measurement vector $\mathbf{y} = [y_1(t), \cdots, y_K(t)]^T \in \mathbb{C}^{K\times 1}$. It is noted that the ESPAR antenna sequentially forms K beampatterns, and thus K sensing periods are required to achieve one measurement vector. For notation simplicity, here we drop off the time index, so that it can be considered as a "single snapshot" measurement as that in a conventional antenna array. Moreover, the signal vector $\tilde{\mathbf{s}}(t)$ is not required to be the same during the block of K sensing periods, which is assumed in the reactance domain MUSIC and ESPRIT algorithms. Instead, we just assume that the positions of the non-zero elements of $\tilde{\mathbf{s}}(t)$ are kept the same through this duration.

In practice, one may perform multiple sets of CS measurements (i.e., multi-snapshot model), expressed as

$$\mathbf{y}_i = \mathbf{W}\mathbf{A}(\tilde{\boldsymbol{\theta}})\tilde{\mathbf{s}}_i + \mathbf{e}_i, \quad i = 1, \cdots, T \tag{6}$$

where $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_K]^T$ is the projection matrix determined by K sets of reactance loads, and $\mathbf{e}_i = [e_1, \cdots, e_K]_i^T$ is the noise vector. T is the number of snapshots. In this model, the directions of signals $\boldsymbol{\theta}$ are assumed to be time invariant through out the T "snapshots".

3. BAYESIAN COMPRESSIVE SENSING APPROACH

Due to the linearity of the sparse signal representation problem, we can consider the real-valued expression of (6) [1]

$$\hat{\mathbf{y}}_i = \hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}})\hat{\mathbf{s}}_i + \hat{\mathbf{e}}_i, \quad i = 1, \cdots, \hat{T}, \tag{7}$$

which is suitable for the BCS algorithm. Although vectors and matrices in (7) have double dimensions compared to those in (6), the sparsity of the problem is not destroyed. Generally, the T measurements, $\{\hat{\mathbf{y}}_i\}_{i=1}^T$ are statistically correlated. The multi-task (MT) CS algorithm [1] is developed for simultaneous inversion of the multiple related signals, based on a hierarchical Bayesian model. In this model, given a common prior shared among multiple snapshots, individual tasks (i.e., inverse CS mapping $\hat{\mathbf{y}}_i \rightarrow \hat{\mathbf{s}}_i$) are performed independently.

Let $\hat{\boldsymbol{\alpha}} = \{\hat{\alpha}_n\}_{n=1}^{\hat{N}_\theta}$ be hyperparameters, shared among all T snapshots and controlling the sparseness of the signal vectors $\hat{\mathbf{s}}_i$. To enforce sparsity over the parameters $\hat{\mathbf{s}}_i$, Gamma priors are assumed on the hyperparameters $\hat{\boldsymbol{\alpha}}$ and noise precision $\beta = \sigma^{-2}$. A zero-mean Gaussian prior is defined for each component of $\hat{\mathbf{s}}_i$. By integrating out the noise precision, the likelihood function of $\hat{\mathbf{s}}_i$ is expressed as [1]

$$\begin{aligned} &Pr(\hat{\mathbf{s}}_i|[\hat{\mathbf{y}}_i, \hat{\boldsymbol{\alpha}}]) \\ &= \frac{\Gamma(a + \hat{N}_\theta/2)\left[1 + \frac{1}{2b}(\hat{\mathbf{s}}_i - \hat{\boldsymbol{\mu}}_i)^T \hat{\boldsymbol{\Sigma}}_i^{-1}(\hat{\mathbf{s}}_i - \hat{\boldsymbol{\mu}}_i)\right]^{-(a+\hat{N}_\theta/2)}}{\Gamma(a)(2\pi b)^{\hat{N}_\theta/2}|\hat{\boldsymbol{\Sigma}}_i|^{1/2}} \end{aligned} \tag{8}$$

where

$$\hat{\boldsymbol{\mu}}_i = \hat{\boldsymbol{\Sigma}}_i(\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}}))^T \hat{\mathbf{y}}_i, \tag{9}$$

$$\hat{\boldsymbol{\Sigma}}_i = \left((\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}}))^T(\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}})) + diag(\hat{\boldsymbol{\alpha}})\right)^{-1}. \tag{10}$$

In (8), a, b are parameters determining the Gamma distribution of the noise precision β. This is a multivariate Student-t distribution.

[1] $\hat{\mathbf{x}} = [\mathcal{R}\{\mathbf{x}\}, \mathcal{I}\{\mathbf{x}\}]^T \in \mathbb{R}^{2N\times 1}$ is the real-valued vector of a complex vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$, and $\hat{\mathbf{X}} = \begin{bmatrix} \mathcal{R}\{\mathbf{X}\} & -\mathcal{I}\{\mathbf{X}\} \\ \mathcal{I}\{\mathbf{X}\} & \mathcal{R}\{\mathbf{X}\} \end{bmatrix} \in \mathbb{R}^{2N\times 2M}$ is the real-valued matrix of a complex matrix $\mathbf{X} \in \mathbb{C}^{N\times M}$, where $\mathcal{R}\{\cdot\}$ and $\mathcal{I}\{\cdot\}$ are the real and imaginary parts of a complex number, respectively.

The most probable hyperparameters $\hat{\boldsymbol{\alpha}}_{MP}$ are estimated by maximizing logarithm of the marginal likelihood: [9]

$$\begin{aligned} \mathcal{L}(\boldsymbol{\alpha}) &= \sum_{i=1}^{T} \log Pr(\hat{\mathbf{y}}_i|\hat{\boldsymbol{\alpha}}) \\ &= -\frac{1}{2}\sum_{i=1}^{T}\left[\log(|\hat{\mathbf{C}}_i|) + (\hat{N}_\theta + 2a)\log(\hat{\mathbf{y}}_i^T \hat{\mathbf{C}}_i \hat{\mathbf{y}}_i + 2b)\right] \\ &\quad + const, \end{aligned} \tag{11}$$

where

$$\hat{\mathbf{C}}_i = \mathbf{I} + (\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}}))^T diag(\hat{\boldsymbol{\alpha}})^{-1}(\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}})). \tag{12}$$

The solution to $\boldsymbol{\alpha}_{MP}$ is obtained by iterative algorithm, such as the fast RVM [11, 1]. With $\boldsymbol{\alpha}_{MP}$, the sparse signal is recovered as the equation shown at the bottom of the next page.

With the recovered $\mathbf{s}_{MT-BCS}$, a noise filtering with a threshold parameter $\eta \in [0, 1]$ [9] is used to estimate the DoAs. The main idea is to select a number (i.e., the estimated source numbers L_e) of spikes, which constitute to the η portion of the total energy of $\mathbf{s}_{MT-BCS}$. Using the minimum energy among the selected spikes as the energy threshold ξ_{L_e}, the estimate DoAs are decided as the positions corresponding to the spikes with energy higher than ξ_{L_e}.

4. NUMERICAL RESULTS

This section evaluates the performance of the proposed DoA estimation approach based on the MT-BCS for the ESPAR antenna, which is compared to that of the RD-MUSIC algorithm [3] as well as the l_1-SVD algorithm [6]. It is noted that the RD-MUSIC and l_1-SVD algorithms require the prior knowledge of the number of source signals.

The simulated ESPAR antenna has $M + 1 = 7$ elements assumed to be thin electrical dipoles with the length of $\lambda/2$ each. The spacing between two adjacent elements is set to $d = \lambda/4$. The mutual impedance matrix $\mathbf{Z}$ is determined by the configuration of the antenna array, which can be calculated by the analytical formulas given in [12]. In the ESPAR array, the projection matrix $\mathbf{W}$ determines K directional beampatterns. We design the $K = 6$ directional beampatterns dividing the angular space of the ESPAR antenna into $K = 6$ sectors, each of which is accessed by the corresponding beampattern. The set of reactance loads for the first sector beampattern maximizing the beam gain to the look direction $0°$ is $\mathbf{x}_1 = [31.2, 17.03, -37.55, -37.55, 17.03, 31.2]$. The remaining 5 beampatterns are formed by circularly permuting elements of $\mathbf{x}_1$. Unlike a uniform linear array, the circular ESPAR antenna array is able to detect the DoA in the full azimuth plane. The angular sampling grid $\tilde{\boldsymbol{\theta}}$ is set to be samples uniform with $\Delta\tilde{\theta} = 1°$ (i.e., $N_\theta = 360$).

First, consider two closely spaced source signals, where the angular displacement is $15°$. The transmitted signals are

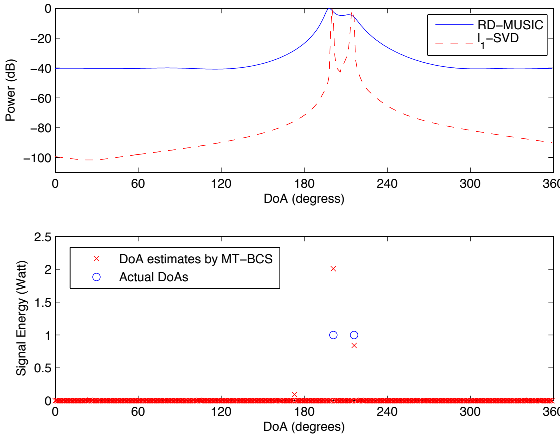

Fig. 1. DoA estimation of two closely spaced signals, $\theta_1 = 200°$, $\theta_2 = 215°$, SNR= 10 dB, $T = 100$.

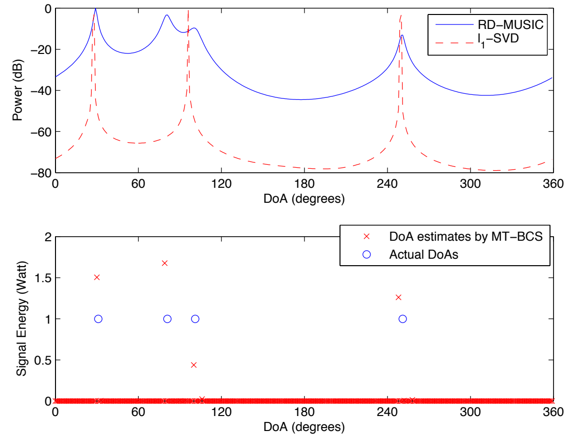

Fig. 2. DoA estimation of four signals, $\theta_1 = 30°$, $\theta_2 = 80°$, $\theta_3 = 100°$,$\theta_4 = 250°$, SNR= 10 dB, $T = 100$.

assumed to be binary phase-shift (BPSK) signals. The number of snapshots is set to $T = 100$, and the two signals are with the same SNR value, $SNR = 10$ dB. An energy threshold parameter $\eta = 0.95$ is assumed in the MT-BCS algorithm. It is noted that the results from the proposed MT-BCS algorithm are not plotted in their logarithm version, since the use of the noise filtering process sets the element energies to zeros when they are smaller than the threshold (i.e., the logarithm value of a zero is minus infinity). From Fig. 1 we can observe that the RD-MUSIC algorithm is unable to exactly recover the two source signals. On the contrary, the two source signals can be exactly recovered by the l_1-SVD algorithm and MT-BCS algorithm. However, there is no knowledge of the number of source signals in the MT-BCS algorithm. In Fig. 2, $L = 4$ incident signals are considered. It shows that the MT-BCS algorithm outperforms both the RD-MUSIC and l_1-SVD algorithms in the case with four signals impinging on the ESPAR antenna. Specifically, only the MT-BCS algorithm is able to recover two closely placed signals, when the number of source signals increases to 4.

5. CONCLUSIONS

In this paper, an new DoA estimation method based on the MT-BCS is studied for the first time within the framework of DoA estimation using ESPAR antenna. Starting from a sparse signal representation of the DoA estimation problem with an ESPAR antenna, the problem is cast in the framework of multi-task Bayesian learning. The main advantage of this method is avoiding the requirement of the knowledge of the number of incident signal, and the computational complexity is reduced by avoiding the computation of correlation matrix from measurements. The simulations show superiority of the MT-BCS approach to the RD-MUSIC as well as the l_1-SVD algorithms, in the recovery of two closely spaced signals, using a small number of snapshots.

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$$\hat{\boldsymbol{s}}_{MT-BCS} = \frac{\sum_{i=1}^{T} \left[(\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}}))^T (\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}})) + diag(\boldsymbol{\alpha}_{MP}) \right]^{-1} (\hat{\mathbf{W}}\hat{\mathbf{A}}(\tilde{\boldsymbol{\theta}}))^T \hat{\mathbf{y}}_i}{T} \tag{13}$$

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