SUPER-RESOLUTION DOA ESTIMATION VIA CONTINUOUS GROUP SPARSITY IN THE COVARIANCE DOMAIN

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ABSTRACT

Estimation of directions-of-arrival (DoA) in the spatial covariance model is studied. Unlike the compressed sensing methods which discretize the search domain into possible directions on a grid, the theory of super resolution is applied to estimate DoAs in the continuous domain. We reformulate the spatial spectral covariance model into a Multiple Measurement Vector (MMV)-like model, and propose a block total variation norm minimization approach, which is the analog of Group Lasso in the super-resolution framework and that promotes the group-sparsity. The DoAs can be estimated by solving its dual problem via semidefinite programming. This gridless recovery approach is verified by simulation results for both uncorrelated and correlated source signals.

Index Terms— Directions of Arrival, Super Resolution, Continuous Sparse Recovery, MMV, Group Lasso

1. INTRODUCTION

Directions-of-arrival (DoA) estimation is a common objective in array signal processing. Locating, with high resolution, closely-spaced DoAs with few snapshots is the main design consideration. The multiple signal classification (MUSIC) [1] is one widely used example of high-resolution DoA estimatior. Inspired by compressed sensing (CS), sparse model formulations are also suggested for DoA estimation as well, usually by discretizing the variable space. Such approaches are vulnerable to grid basis mismatch [2] which leads to failure in recovering signals or degrading DoA estimation performamnce. Thus, to accommodate this situation in the covariance domain, the Sparse Spectral Fitting with Modeling Uncertainty (SSFMU) method was proposed [3]. In [4], by linearizing the grid basis mismatch in a spatial covariance model, an alternating Lasso is proposed to jointly estimate DoA and its mismatch (more references of other approaches dealing with grid basis mismatch in [4]).

The super-resolution (SR) approach presented by Candès and Fernandes-Granda [5, 6] aims to provide a continuous parameter recovery by solving a total variation (TV) norm minimization of a complex measure, which is not the TV norm used in image processing. In [7], atomic norm minimization (ANM) is proposed to estimate continuous frequency spectrum with a subset of sensors. However, the above SR methods are only studied in single-measurement-vector (SMV) model. In [8], an exact joint sparse frequency recovery method is proposed by using ANM in multiple-measurement-vector (MMV) system, and a theoretical analysis on the continuous dictionary setting is provided. In [9], the TV norm minimization employed in MMV is studied to improve performance of DoA estimation, but the source signal is assumed zero-mean positive-valued random variables, which is not a general case. In [10], a sublinear time randomized algorithm is designed to recover sparse Fourier sampling signals with continuous-valued frequencies.

In this work, we formulate the DoA estimation problem in the spatial covariance model, and reformulate it into a MMV-like model. Use of the covariance model in the formulation of a DoA estimator is desirable for a number of reasons, including computational savings for large number of snapshots, and exploitation of the methods that extrapolate array appertures through their co-arrays, such as for the case of minimum-redundancey [11] or co-prime arrays [12]. We extend the theory of super-resolution from SMV to MMV by defining a block total variation (BTV) norm for a complex measure with same locations but different amplitudes at multiple snapshots. Then, we propose a BTV norm minimization approach for the MMV-like model. The performance of the proposed method is demonstrated by simulations for cases of uncorrelated and correlated source signals and compared with MUSIC, ANM-MMV [8], and the Cramer-Rao Lower Bound (CRLB).

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. The DoA Estimation Problem

We consider a planar or linear array with M sensors. The sampled vector $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T \in \mathbb{C}^{M\times 1}$ of the signal model at time t is

$$\mathbf{y}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{n}(t), t = 1, \ldots, T, \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_K(t)]^T \in \mathbb{C}^{K\times 1}$ denotes K far-field narrowband zero-mean sources with power $\sigma_1^2, \ldots, \sigma_K^2$ and covariance matrix $\mathbf{C_x}$ which impinge on the array from angles $\theta_1, \ldots, \theta_K$, and $\mathbf{g}(\theta_k) \in \mathbb{C}^{M\times 1}$ denotes the steering vector for the kth source located at θ_k with m-th entry $e^{-j2\pi\frac{d_m}{\lambda}sin\theta_k}$ in which λ is the wavelength and d_m is the distance of the m-th sensor from a reference one. For a uniform linear array (ULA), $d_m = md$ where d is the inter-element spacing. The steering matrix $\mathbf{G} = [\mathbf{g}(\theta_1), \cdots, \mathbf{g}(\theta_K)] \in \mathbb{C}^{M\times K}$ is formed by the steering vectors $\{\mathbf{g}(\theta_k)\}_{k=1}^{K}$. Denote $\mathbf{n}$ as Additive White Gaussian Noise (AWGN) with $\mathbb{C}\mathcal{N}(0, \sigma^2\mathbf{I})$. T is the number of measurement snapshot. The DoA support set is denoted as $\mathcal{T}_\theta = \{sin(\theta_k)\}_{k=1}^{K} \subset \mathbb{T} = [-1, 1]$ for arriving signals. The MMV system is defined as

$$\begin{aligned}\mathbf{Y} &= [\mathbf{y}(1), \ldots, \mathbf{y}(T)] = \mathbf{G}\mathbf{X} + \mathbf{N} \\ &= [\mathbf{G}\mathbf{x}(1), \ldots, \mathbf{G}\mathbf{x}(T)] + \mathbf{N},\end{aligned} \tag{2}$$

where $\mathbf{Y} \in \mathbb{C}^{M\times T}$, $\mathbf{X} = [\mathbf{x}(1), \ldots, \mathbf{x}(T)] \in \mathbb{C}^{K\times T}$, and $\mathbf{N} = [\mathbf{n}(1), \ldots, \mathbf{n}(T)] \in \mathbb{C}^{M\times T}$. The covariance matrix of observed vectors for K uncorrelated sources is expressed as

$$\tilde{\mathbf{R}} = E[\mathbf{y}\mathbf{y}^H] = \mathbf{G}\mathbf{C}_{\mathbf{x}}\mathbf{G}^H + \sigma^2\mathbf{I}.$$

In practice, by averaging the measured snapshots, $\mathbf{R} = \sum_{t=1}^{T} \mathbf{y}(t)\mathbf{y}(t)^H/T$ is used as the estimate of covariance matrix. Then the measurement based on spatial covariancel model can be rewritten as

$$\mathbf{R} = \sum_{k=1}^{K} \sigma_k^2 \mathbf{g}(\theta_k)\mathbf{g}(\theta_k)^H + \mathbf{V}, \tag{3}$$

where $\mathbf{V}$ represents the contributions of AWGN and the approximation error due to the sample averaging. Based on the above models, the goal of DoA estimation problem is to estimate the support set $\mathcal{T}_\theta$. In the next subsection, the super-resolution theory is introduced how to fit into the DoA estimation problem in the scenario of SMV.

2.2. Preliminary Method of Continuous Signal Recovery

In the theory of super-resolution, consider a continuous signal $s(\tau)$ which has sparse representations in the domain $[-1, 1]$ and a weighted linear combinations of spikes [5]:

$$s(\tau) = \sum_{k=1}^{K} a_k \delta_{\tau_k}, \tag{4}$$

where a_k may be real or complex valued, $\tau_k \in [-1, 1], \forall k$ and δ_{τ_k} is a Dirac measure at location τ_k. Denote $\mathbf{s} = [a_1, \ldots, a_K]^T$ as the data vector. The Fourier transform of $s(\tau)$ is written as

$$r(n) = \int_{-1}^{1} e^{-j2\pi n\tau} s(d\tau) = \sum_{k=1}^{K} a_k e^{-j2\pi n\tau_k}, n = -f_c, \ldots, f_c$$

where f_c is an integer and $2f_c + 1$ is the number of Fourier transform frequency coefficients. With arbitrary noise $\mathbf{e}$ cosidered in this model, we simplify the above equation as

$$\mathbf{r} = \mathcal{F}s + \mathbf{e}, \tag{5}$$

where $\mathbf{r} = [r(-f_c), \ldots, r(f_c)]^T \in \mathbb{C}^{M\times 1}$, and $\mathcal{F}$ denotes the linear operator to measure the $2f_c + 1$ lowest frequency coefficients.

In order to estimate τ_k, the total variation (TV) norm for a complex meaure [13] on a Borel set $B \in$ Borel σ-algebra $\mathcal{B}(\mathbb{T})$ is introduced and defined as $\|s\|_{TV} = \sup \sum_{k=1}^{\infty} |s(B_k)|$, where the supremum is taken over all partitions of B into countable and disjoint measurable subsets B_k. The minimization of $\|s\|_{TV}$ in the continuous domain is used to promote the sparsity of continuous signal s, which is the analog of the l_1-norm minimization of $\|\mathbf{s}\|_1 = \sum_k |a_k|$ in the discrete domain. In [6], convex optimization problem is suggested as

$$\min_{s} \|s\|_{TV} \quad \text{s.t. } \|\mathcal{F}s - \mathbf{r}\|_2 \leq \epsilon. \tag{6}$$

When the signal-measurement-vector system (1) is considered, by letting $\tau_k = sin(\theta_k), \forall k$ and $f_c = (M-1)/2$, the DoA estimation problem can be cast in the super-resolution framework as follows

$$\mathbf{r} = \mathcal{F}s + \mathbf{e} = \mathbf{G}\mathbf{x}(t) + \mathbf{n}(t) = \mathbf{y}(t), \tag{7}$$

and then solved by the TV norm minimization (6). For the spatial covariance model (3), we can also vectorize the covariance matrix into a SMV system, and solve it by TV norm minimization [14].

3. THE PROPOSED METHOD

3.1. Reformulation of the Spatial Covariance Model

Instead of vectorizing the spatial covariance model (3), we recast it into a MMV-like model by the following:

$$\begin{aligned}\mathbf{R} = [\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_{M-1}] &= \sum_{k=1}^{K} \sigma_k^2 \mathbf{g}(\theta_k)\mathbf{g}(\theta_k)^H + \mathbf{V}, \\ &= \sigma_1^2\bar{\mathbf{G}}(\theta_1) + \cdots + \sigma_K^2\bar{\mathbf{G}}(\theta_K) + \mathbf{V},\end{aligned} \tag{8}$$

where $\mathbf{g}(\theta_k)\mathbf{g}(\theta_k)^H = \bar{\mathbf{G}}(\theta_k)$ is a Toeplitz matrix expressed by $\bar{\mathbf{G}}(\theta_k) = [\mathbf{a}_0(\theta_k), \mathbf{a}_1(\theta_k), \ldots, \mathbf{a}_{M-1}(\theta_k)] \in \mathbb{C}^{M\times M}$. For ULA, the lth column of $\bar{\mathbf{G}}(\theta_k)$ is represented as $\mathbf{a}_l(\theta_k)$= $[e^{-j(-l)\xi_k}, \ldots, e^{-j(M-1-l)\xi_k}]^T \in \mathbb{C}^{M\times 1}$, $\forall l = 0, \ldots, M-1$, in which $\xi_k = \frac{d}{\lambda}2\pi sin\theta_k$. Then, the lth column $\mathbf{r}_l \in \mathbb{C}^{M\times 1}$ can be expressed as

$$\begin{aligned}\mathbf{r}_l &= \sigma_1^2\mathbf{a}_l(\theta_1) + \cdots + \sigma_K^2\mathbf{a}_l(\theta_K) + \mathbf{v}_l = \sum_k \sigma_k^2\mathbf{a}_l(\theta_k) + \mathbf{v}_l, \\ &= \mathbf{A}_l\mathbf{p} + \mathbf{v}_l, \forall l = 0, \ldots, M-1\end{aligned} \tag{9}$$

where $\mathbf{A}_l = [\mathbf{a}_l(\theta_1), \ldots, \mathbf{a}_l(\theta_K)] \in \mathbb{C}^{M\times K}$, $\mathbf{p} = [\sigma_1^2, \ldots, \sigma_K^2]^T \in \mathbb{R}^{K\times 1}$, and $\mathbf{v}_l$ is the lth column of $\mathbf{V}$. The matrix $\mathbf{A}_l$ is composed of every lth column from matrices $\bar{\mathbf{G}}(\theta_1), \ldots, \bar{\mathbf{G}}(\theta_K)$. Therefore, $\mathbf{R}$ can be rewritten as

$$\begin{aligned} \mathbf{R} &= [\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_{M-1}] \\ &= [\mathbf{A}_0\mathbf{p}, \mathbf{A}_1\mathbf{p}, \ldots, \mathbf{A}_{M-1}\mathbf{p}] + \mathbf{V}, \end{aligned} \tag{10}$$

which is a similar form to an MMV system in Equation (2). In Equation (10), we have M vectors, $\mathbf{r}_0, \ldots, \mathbf{r}_{M-1}$ with the same power vector $\mathbf{p}$. Unlike the MMV system, each matrix $\mathbf{A}_l$ is different, and each column of $\mathbf{A}_i$ is a rotational steering vector to the corresponding column of $\mathbf{A}_j$, i.e., $\mathbf{a}_i(\theta_k) = \mathbf{a}_{i+1}(\theta_k)e^{j\xi_k}$. Equation (10) will be used to estimate the DoA support set by the proposed method in the next subsections. We will show how to extend the SR theory from SRV to MMV-like system before introducing the proposed method.

3.2. Continuous Group-Sparsity Recovery Method

Based on the theory of super-resolution, we extend a continuous signal into the MMV space by defining $s(\tau; t), \tau \in [-1, 1], t = 1, \ldots, T$ as

$$s(\tau; t) = \sum_{k=1}^{K} b_{kt}\delta_{\tau_k}, \tag{11}$$

where b_{kt} is a real or complex-valued amplitude of measurement at time t, and $\tau_k \in [-1, 1], \forall k$ is a location of kth spike. Denote $\mathcal{T} = \{\tau_k\}_{k=1}^{K}$ as the support set and $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_T]$ as the data matrix where $\mathbf{s}_t = [b_{1t}, \ldots, b_{Kt}]^T$. Similarly in [5], the Fourier transform of $s(\tau; t)$ with respect to τ is

$$r(n; t) = \int_{-1}^{1} e^{-j2\pi n\tau} s(d\tau) = \sum_{k=1}^{K} b_{k,t} e^{-j2\pi n\tau_k}, \tag{12}$$
$$n = -f_c, \ldots, f_c, \;\; t = 1, \ldots, T.$$

When Gaussian noise is considered in this model, Equation (12) can be simplified as

$$\mathbf{r}_{sr}^t = \mathcal{F}s(\tau; t) + \mathbf{e}^t, \forall t = 1, \ldots, T \tag{13}$$

where $\mathbf{r}_{sr}^t = [r(-f_c; t), \ldots, r(f_c; t)]^T \in \mathbb{C}^{M\times 1}$, and $\mathbf{e}^t$ denotes i.i.d. Gaussian noise vector with $\mathbb{C}\mathcal{N}(0, \sigma^2\mathbf{I})$. Let $\mathbf{R}_{sr} = [\mathbf{r}_{sr}^1, \ldots, \mathbf{r}_{sr}^T]$.

By using multiple measurements to estimate τ_k, a block total variation (BTV) norm for a complex meaure with multiple measurements on a set $B \in \mathcal{B}(\mathbb{T})$ is defined as

$$\|s\|_{TV,p} = \sup \sum_{k=1}^{\infty} \|s(B_k; :)\|_p, \tag{14}$$

where $\|s(B_k; :)\|_p = (\sum_{t=1}^{T} |s(B_k; t)|^p)^{1/p}$ and $s(B_k; t) = b_{k,t}$ if the supremum is taken over all partitions $\{B_k\}$ of B belonging to Borel σ-algebra [13] to optimally have a finite and disjoint measurable subsets $\{B_k\}$ at time t. Since at different time t, multiple continuous signals $s(\tau; t)$ share the same spike locations, the group sparsity of $s(\tau; t)$ can be promoted by using the minimization of $\|s\|_{TV,p}$. This is equivalent to the minimization of $\|\mathbf{S}\|_{1,p} = \sum_k \|\mathbf{S}_{k,:}\|_p$ where $\mathbf{S}_{k,:}$ is the kth row of data matrix $\mathbf{S}$ by the notion of Group Lasso [15]. Similarly in [6], based on Equation (11) and (13), the block total variation (BTV) minimization problem is proposed as

$$\min_s \|s\|_{TV,p} \;\; \text{s.t.} \; \sum_{t=1}^{T} \|\mathcal{F}s - \mathbf{r}_{sr}^t\|_2 \le \epsilon, \tag{15}$$

where $1 \le p \le +\infty$.

When considering the MMV-like model (10) and letting $\tau_k = sin(\theta_k)$, $t = l$, $T = M - 1$, $\mathbf{R}_{sr} = \mathbf{R}$ and $f_c = (M-1)/2$, the DoA estimation problem can be formulated in the new super-resolution framework as the following

$$\mathbf{r}_{sr}^l = \mathcal{F}_l s(\tau; l) + \mathbf{e}^l = \mathbf{A}_l\mathbf{p} + \mathbf{v}_l = \mathbf{r}_l, \; l = 0, \ldots, M-1,$$

and then solved by the proposed BTV norm minimization

$$\min_s \|s\|_{TV,1} \;\; \text{s.t.} \; \sum_{l=0}^{M-1} \|\mathcal{F}_l s - \mathbf{r}_l\|_2 \le \epsilon. \tag{16}$$

Note that the minimization of $\|s\|_{TV,1}$ is analog to $\|\mathbf{S}\|_{1,1}$ in this case. A theorem about DoA resolution for MMV system can be claimed similarly by using Theorem 1.2 in [5].

Theorem 1 *Let $\mathcal{T} = \{\tau_k\}_{k=1}^{K}$ as the support set. If the minimum distance $\Delta(\boldsymbol{\theta})$ obeys*

$$\Delta(\boldsymbol{\theta}) = \inf_{\tau_i, \tau_j \in \mathbb{T}} |\tau_i - \tau_j| \ge \frac{4}{f_c}\frac{\lambda}{d},$$

then the high resolution detail of continuous signal s can be recovered with high probability by solving block total variation norm minimization problem (16).

In order to estimate the support set, the dual form of (16) is derived as

$$\begin{aligned} &\max_{\mathbf{U}} \; \text{Re}\{\langle \mathbf{R}, \mathbf{U}\rangle\} - \epsilon\|\mathbf{U}\|_F \\ &\text{s.t.} \; \|\mathcal{F}_l^* \mathbf{u}_l(\tau)\|_\infty \le 1, \forall l = 0, \ldots, M-1, \end{aligned} \tag{17}$$

where $\mathbf{U} = [\mathbf{u}_0, \ldots, \mathbf{u}_{M-1}] \in \mathbb{C}^{M\times M}$ and $\mathcal{F}_l^*\mathbf{u}_l(\tau) = \sum_{|k|\le f_c} u_{l,k} e^{j\frac{d}{\lambda}2\pi(k-l)\tau}$ where $\mathbf{u}_l = [u_{l,-f_c}, \ldots, u_{l,f_c}]^T \in \mathbb{C}^{M\times 1}$. $\text{Re}\{\langle \mathbf{R}, \mathbf{U}\rangle\}$ takes the real part of $tr(\mathbf{U}^H\mathbf{R})$ where $tr(\cdot)$ takes the sum of diagonal entries of matrix. By a generalized Slater condition [16], strong duality holds since $\mathbf{u}_l = \mathbf{0}, \forall l$, which satisfies the constraint, is contained in the feasible set. Although this problem is still with infinite constraints, it can be reformulated as a semidefinite matrix and an affine hyperplane. Thus, the dual problem is rewritten as

$$\begin{aligned} &\max_{\mathbf{U}} \; \text{Re}\{< \mathbf{R}, \mathbf{U} >\} - \epsilon\|\mathbf{U}\|_F \\ &\text{s.t.} \; \begin{bmatrix} \mathbf{Q}^l & \mathbf{u}_l \\ \mathbf{u}_l^H & 1 \end{bmatrix} \succeq 0, \forall l = 0, \ldots, M-1 \\ &\quad \sum_{i=1}^{M-j} \mathbf{Q}_{i,i+j}^l = \begin{cases} 1, & j = 0, \\ 0, & j = 1, 2, \ldots, M-1 \end{cases}, \end{aligned} \tag{18}$$

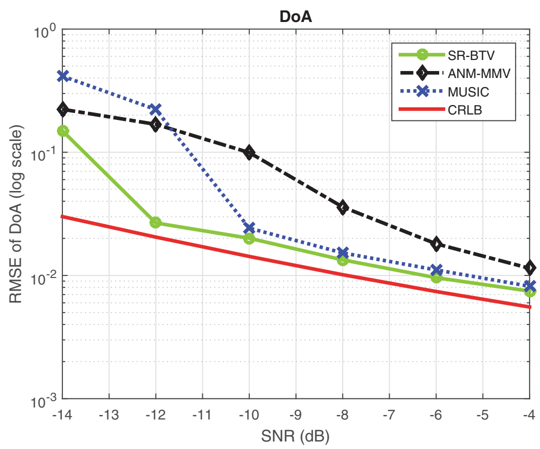

Fig. 1. RMSE of DoA estimation versus SNR for the case of uncorrelated sources.

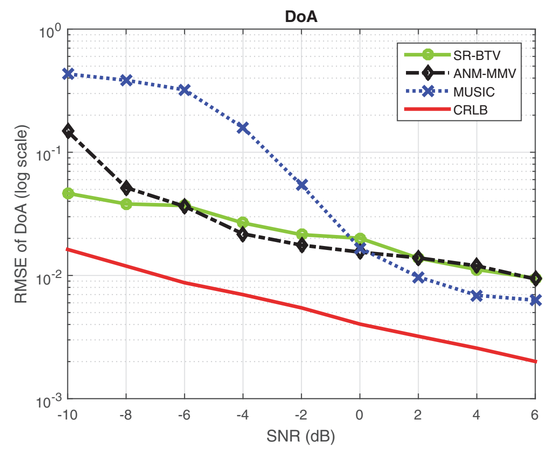

Fig. 2. RMSE of DoA estimation versus SNR for the case of correlated sources.

where $\mathbf{Q}^l \in \mathbb{C}^{M\times M}$ is a Hermitian matrix, $\forall l$. The following lemma is modified from [5] and used to estimate the support set by linking a primal solution with a dual solution.

Lemma 2 *Let s_{est} and $\mathbf{u}_{l,est}$ be a pair of primal-dual solutions to (15) and (18). Then*

$$(\mathcal{F}_l^* \mathbf{u}_{l,est})(\tau) = \mathit{sign}(s_{est}(\tau;l)), \forall \tau \in \mathbb{T} \text{ s.t. } s_{est}(\tau;l) \neq 0.$$

$sign(\cdot)$ takes the sign of any number. By performing the root finding procedure on the $|(\mathcal{F}_l^* \mathbf{u}_{l,est})(\tau)|^2 = 1, \forall l$, we can obtain the estimated support sets $\mathcal{T}_{est}^l = \{\tau_{k,est}^l\}_{k=1}^K$ and its union set $\mathcal{T}_{est} = \bigcup_l \mathcal{T}_{est}^l$. Then, the measurement matrix $\mathbf{G}_{est}$ is reconstructed based on $\mathcal{T}_{est}$. Finally in terms of Equation (2), Group Lasso [15] can be used to determine the true source locations as the following

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \frac{1}{2}||\mathbf{Y} - \mathbf{G}_{est}\mathbf{X}||_F^2 + \gamma||\mathbf{X}||_{2,1}, \tag{19}$$

where $||\mathbf{X}||_{2,1} = \sum_{k=1}^{|\mathcal{T}_{est}|} \|\mathbf{X}_{k,:}\|_2$, and $\mathbf{X}_{k,:}$ denotes the k^{th} row of $\mathbf{X}$ and $|\mathcal{T}_{est}|$ is the cardinality of $\mathcal{T}_{est}$.

4. NUMERICAL RESULTS

The proposed method (SR-BTV) is applied to the DoA estimation problem and compared with MUSIC, ANM-MMV [8] and the CRLB. An uniform linear array (ULA) of $M = 9$ sensors with half-wavelength interelement spacing is considered. The minimum distance $\Delta(\boldsymbol{\theta})$ is set to 1. Suppose $K = 2$ narrowband plane waves impinging on ULA from DoAs with $sin(\boldsymbol{\theta}) = [0.2165251, 0.4665251]$. The distance of two sources is 0.25, which is $\frac{\Delta(\boldsymbol{\theta})}{4}$. Two cases of uncorrelated and correlated sources are considered. In the uncorrelated case, two source signals are zero-mean complex-valued Gaussian random variables with equal power. In the correlated case, the correlation coefficient of two sources is set to 0.9. For MUSIC, the search grid of $[-1, 1]$ is uniformly separated with step size 0.0001. We performed one hundred realizations for each SNR. The number of snapshots is $T = 100$.

The RMSE of DoA estimation for the case of uncorrelated source signals is presented in Figure 1. At high SNR, the performance of SR-BTV and MUSIC are almost the same and approach the CRLB. However, at low SNR, the SR-BTV method shows a lower resolution threshold than MUSIC. For instance, when RMSE $\approx 10^{-1}$, the SR-BTV method outperforms MUSIC about 2 dB. ANM-MMV has slight improvement over MUSIC at low SNR, but does not have good performance at high SNR. In Figure 2, the performance for the correlated case is presented. Since the covariance matrix of source signals is not diagonal anymore, the performance of SR-BTV, ANM-MMV and MUSIC degrade and all are more far away from the CRLB compared with the uncorrelated case. However, the SR-BTV and ANM-MMV are more robust to source correlations and achieves better performance than MUSIC at low SNR. At SNR$= -10$ dB, the RMSE of SR-BTV is approxmately 0.0464 while the RMSE of MUSIC is about 0.4318.

5. CONCLUSION

By reformulating the spatial covariance model into an MMV-like system, group sparsity is exploited in the super-resolution framework. An BTV norm minimization approach is proposed for the reformulated model. The DoAs are estimated by solving its dual. Numerical results demonstrate the robust performance of SR-BTV compared wtih MUSIC and ANM-MMV in cases of uncorrelated and correlated sources.

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