# ITERATIVE QUADRATIC RELAXATION METHOD FOR OPTIMIZATION OF MULTIPLE RADAR WAVEFORMS

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**ABSTRACT**

MIMO radars use multiple waveforms in order to resolve more targets and achieve gains in target detection, parameter estimation and recognition, for example. In this paper, we propose a method for optimizing multiple waveforms with low peak sidelobe and peak cross-correlation levels for MIMO radar. The optimization method relaxes the original quartic problem into a quadratic one and iterates the relaxed problem to improve the solution. The numerical examples demonstrate that good waveforms are obtained with the proposed method.

***Index Terms***— MIMO radar, waveform design, optimization, constant-modulus waveforms, relaxation methods

## 1. INTRODUCTION

Many MIMO radar detection and estimation methods rely on the assumption that the waveforms transmitted simultaneously from different transmitters can be separated from each other at the receiver end. Typically, it is therefore assumed that the used waveforms are orthogonal [1, 2], in which case a bank of matched filters can be used at receiver to identify the waveforms. However, waveforms that would be orthogonal for all delays and Doppler shifts do not exist. It is essential then to optimize the waveforms as well as possible.

When designing a set of waveforms for a MIMO radar, the cross-correlation of the waveforms needs to be low as well in addition to each waveform having small sidelobes. Synthesis of the a desired ambiguity function was proposed in [3]. Simulated annealing and a greedy algorithm were used for optimizing polyphase signals in [4]. Design of waveforms with a peak to average power ratio constraint was studied in [5], whereas Unimodular code design has been proposed in [6] and [7].

In this paper, we propose an iterative quadratic relaxation (IQR) method for radar waveform optimization. This method relaxes the original nonconvex problem into a convex one and solves it in a iterative fashion in order to improve the obtained waveforms. The IQR method is compared with simulated annealing, greedy algorithm, manifold optimization, and the Maximum Block Improvement (MBI) method whose use in waveform optimization was proposed in [11].

We formulate the problem of designing waveforms with minimal peak cross-correlation and peak sidelobe level as a quartic minimax problem on an oblique manifold in Section 2. In Section 4, we describe the proposed IQR method and briefly summarize the other methods that are used for comparison. Numerical examples of the optimization results are given in Section 4, before the final conclusions in Section 5.

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## 2. PROBLEM FORMULATION

Multiple waveforms sets intended for radar use need to have as flat cross-ambiguity function as possible. The cross-ambiguity function is defined for narrowband waveforms as [8]

$$\chi_{ij}(\tau, F_D) = \left| \int s_i(t) s_j^*(t+\tau) e^{j2\pi F_D t} dt \right|^2, \quad (1)$$

where $\tau$ is the time delay and $F_D$ is the Doppler frequency of the target. The narrowband condition may be stated as [9]

$$\frac{2v\mathrm{TB}}{c} \ll 1, \quad (2)$$

where $v$ is the velocity of the target, $\mathrm{TB}$ is the time–bandwidth product of the waveform, and $c$ is the propagation speed of the waveform.

If the narrowband assumption is not valid, the cross-ambiguity function has to be written as [9]

$$\chi_{ij}(\tau, F_D) = \left| \sqrt{\gamma} \int s_i(t) s_j^*(\gamma t+\tau) e^{j2\pi F_D t} dt \right|^2. \quad (3)$$

The temporal compression factor $\gamma$, which describes how much the waveform compresses or stretched in time because of the Doppler shift, is given by

$$\gamma = 1 + \frac{F_D}{F_c}, \quad (4)$$

where $F_c$ is the carrier frequency.

As most radars use digital signal processing, we are mainly interested in sampled discrete-time signals, so we define the *discrete-time cross-ambiguity function* as

$$\chi_{ij}(\tau, F_D, T_s) = \left| \sqrt{\gamma} T_s \sum_k s_i(kT_s) s_j^*(\gamma k T_s + \tau) e^{j2\pi F_D t} \right|^2, \quad (5)$$

where $T_s$ is the sampling interval. Using the definition of Riemann integral, we see that as $T_s \to 0$, the sample cross-ambiguity function converges to the continuous one if the continuous-time cross-ambiguity function is integrable.

In order to correctly separate the waveforms at the receivers, the cross-ambiguity function of the waveforms should be as low as possible. Also the self-ambiguity function should ideally be flat beyond the main lobe, and there should not be significant sidelobes.

The peak cross-correlation (PCC) can be used as a measure of the maximum of the cross-ambiguity function. The normalized PCC is defined as

$$\mathrm{PCC}_k(T_s) = \sup_{i,\tau,F} \frac{\chi_{ki}(\tau, F, T_s)}{\chi_{kk}(0,0,T_s)}. \tag{6}$$

For the self-ambiguity function, we define the normalized peak sidelobe (PSL) as

$$\mathrm{PSL}_k(T_s) = \sup_{(\tau,F)\notin\mathcal{M}_k} \frac{\chi_{kk}(\tau, F, T_s)}{\chi_{kk}(0,0,T_s)}, \tag{7}$$

where the set $\mathcal{M}_k$ is defined as the set of delay $\tau$ and Doppler frequency $F$ values that encompass the main peak of the waveform $k$. An example of this set could be an ellipsoid $\mathcal{M}_k = \{(\tau, F)|(\tau/\tau_0)^2 + (F/F_0)^2 \le 1\}$, where $\tau_0$ and $F_0$ are the main lobe half-widths in delay and Doppler frequency, respectively. Typically, $F$ and $\tau$ are continuous but can be discretized in suitable conditions discussed below.

In the following, we will consider waveforms that are linearly modulated trains of rectangular pulses. Many commonly used waveforms employed in pulse compression in radars and in the digital modulation of communication systems are this type of signals, including binary, QAM, and polyphase signals. However, frequency modulation does not fall into this category.

Let the vector $\boldsymbol{s}_k$ contain the samples of the $k$th baseband waveform $s_k(t)$, i.e. $(\boldsymbol{s}_k)_n = s_k(nT_s)$, where $T_s$ is the sampling interval. The dimensions of $\boldsymbol{s}_k$ depend on the number of symbols and sampling rate, and at critical sampling rate, the $s_k$ would be a $N_p \times 1$ vector, where $N_p$ is the number of pulses. We define the delay and Doppler matrix $\boldsymbol{D}(\tau, F_D, T_s)$ as

$$(\boldsymbol{D}(\tau, F_D, T_s))_{n,m} = \gamma T_s^2 e^{j2\pi F_D n T_s} \delta_{h(n),m}, \tag{8}$$

where

$$h(n) = \left\lfloor \frac{nT_s + \tau}{\gamma T_s} \right\rfloor \tag{9}$$

and $\gamma$ is given by (4).

Using the delay and Doppler matrix, the discrete-time cross-ambiguity function can then be written as

$$\chi_{ij}(\tau, F_D, T_s) = \left| \boldsymbol{s}_i^H \boldsymbol{D}(\tau, F_D, T_s) \boldsymbol{s}_j \right|^2. \tag{10}$$

In order to simplify the waveform optimization problem, it is assumed next that the narrowband assumption holds, and that the waveform is sampled so that the symbol duration is an integer multiple of sampling interval $T_s$. Consequently, for two values of delay $\tau_1$ and $\tau_2$ such that $|\tau_1 - \tau_2| \le T_s$, obviously $\chi_{ij}(\tau_1, f) = \chi_{ij}(\tau_2, f)$ as the sampled signals differ only by a phase shift. Therefore, we need only to consider delays that are integer multiples of the sampling interval.

With these assumptions and critical sampling, the delay and Doppler matrix in (11) becomes $N_p \times N_p$ matrix

$$\begin{aligned}(\boldsymbol{D}_{k,f})_{n,m} &= (\boldsymbol{D}(kT_s, F_D, T_s))_{n,m} \\ &= T_s e^{j2\pi F_D n T_s} \delta_{n+k,m} \\ &= T_s e^{j2\pi f n} \delta_{n+k,m},\end{aligned} \tag{11}$$

where $f = F_D T_s$ is the normalized Doppler frequency.

Furthermore, we assume without loss of generality that each waveform is normalized such that

$$\chi_{kk}(0,0,T_s) = \|s_k\|^2 = 1. \tag{12}$$

The PSL and PCC of the $i$th waveform can now be expressed as

$$\mathrm{PSL}_i = \sup_{k,f} \left| \boldsymbol{s}_i^H \boldsymbol{D}_{k,f} \boldsymbol{s}_i \right|^2, |f| \ge \delta_{k0} f_0 \tag{13}$$

and

$$\mathrm{PCC}_i = \sup_{j,k,f} \left| \boldsymbol{s}_i^H \boldsymbol{D}_{k,f} \boldsymbol{s}_j \right|^2, \tag{14}$$

where $f_0$ is the half-width of the main peak in normalized Doppler frequency.

The goal of minimizing the maximum PSL and PCC of the waveform set can now be formulated as an optimization problem

$$\text{minimize} \max_{i,j,k,f} \left| \boldsymbol{s}_i^H \boldsymbol{D}_{k,f} \boldsymbol{s}_j \right|^2 \qquad |f| \ge \delta_{ij}\delta_{k0} f_0 \tag{15a}$$

$$\text{s.t. } \|\boldsymbol{s}_i\| = 1, \qquad \forall i. \tag{15b}$$

The function $\left| \boldsymbol{s}_i^H \boldsymbol{D}_{k,f} \boldsymbol{s}_j \right|^2$ is a continuous, multivariate polynomial and $k \in \mathbb{Z}$ whereas $f \in [-1/2, 1/2]$ due to aliasing. Consequently, the maximum exists. By discretizing the normalized Doppler frequency, one obtains a minimax optimization problem of multivariate quartic polynomials on oblique manifold.

It is often necessary to constrain the peak-to-average ratio (PAR) of power for each waveform in order to avoid amplifier non-idealities in the front-end. For the linearly modulated sequences of rectangular pulses, the PAR of the $i$th waveform is equal to

$$\mathrm{PAR}_i = \frac{\max_k |(\boldsymbol{s}_i)_k|^2}{\frac{1}{N_p}\|\boldsymbol{s}_i\|^2}, \tag{16}$$

where $N_p$ is the number of pulses. If the maximum allowed PAR is equal to $\mathrm{PAR}_{\max}$, the constraint on the modulation symbols $(\boldsymbol{s}_i)_k$ can be written as

$$|(\boldsymbol{s}_i)_k|^2 \le \frac{\mathrm{PAR}_{\max}}{N_p} \; \forall k, \tag{17}$$

where the normalization $\|\boldsymbol{s}_i\| = 1$ is assumed.

## 3. OPTIMIZATION METHODS

It was shown in the previous section that the waveform optimization for MIMO radars can be written as a quartic minimization on an oblique manifold. Given the minimax nature of the problem, directly applying manifold optimization methods to this problem is unlikely to yield satisfactory solutions due to possibly a large number of local minima. We propose iterative quadratic relaxation (IQR) method for the waveform optimization and compare it with some other suggested methods in the literature.

### 3.1. Iterative Quadratic Relaxation

The iterative quadratic relaxation (IQR) algorithm is based on approximating the quartic problem with a quadratic one and solving the approximation iteratively. At each iteration, a new waveform set is obtained from the previous set by solving a quadratic problem. The new waveforms are obtained from the current waveforms $\boldsymbol{s}_i^{(m)}$, where $m$ denotes the iteration, by solving an optimization problem

$$\min \max_{j,k,f} \left| \boldsymbol{x}_i^H \boldsymbol{D}_{k,f} \boldsymbol{s}_j^{(m)} \right|^2, \qquad |f| \geq \delta_{k0}\delta_{ij} f_0 \tag{18a}$$

$$\text{s.t. } \boldsymbol{x}_i^H \boldsymbol{s}_i^{(m)} = 1 \tag{18b}$$

for each $\boldsymbol{x}_i$. This problem is quadratically constrained problem (QCP) that is convex and solvable in polynomial time. The updated waveform $\boldsymbol{s}_i^{(m+1)}$ is obtained from the solution by normalization, i.e.

$$\boldsymbol{s}_i^{(m+1)} = \frac{\boldsymbol{x}_i}{\|\boldsymbol{x}_i\|}. \tag{19}$$

The optimization decouples into separate problem for each waveform. However, in order to avoid unwanted increase in the cross-correlation, the waveforms need to be updated one by one in a serial fashion. When no significant improvement is achieved with further iterations, it is possible to improve the solution further by switching back to the original problem (15) and using a manifold optimization method or a greedy algorithm.

The optimization problem (18) can be geometrically interpreted as minimizing the objective on a hyperplane that is a tangent plane to the surface of the unit complex hypersphere $\|\boldsymbol{s}_i\| = 1$ at the point $\boldsymbol{s}_i^{(m)}$. After the optimal point on this tangent hyperplane has been found, it has to be projected on the unit complex hypersphere to satisfy the constraint (18b) for the next iteration.

For optimization with a PAR constraint, an additional constraints are needed. These constraint is given by

$$\mathrm{Re}\Big[(\boldsymbol{x}_i)_k^* (\boldsymbol{s}_i^{(m)})_k\Big] = \frac{\mathrm{PAR}_{\max}}{N_p} \; \forall k. \tag{20}$$

Naturally, this constraint does not guarantee that the PAR of the solution would be less or equal to $\mathrm{PAR}_{\max}$. In order to obtain a solution satisfying the PAR constraint, we first define a vector-valued function $\boldsymbol{y}_i(p)$ whose elements are given by

$$(\boldsymbol{y}_i(p))_k = \frac{(\boldsymbol{x}_i)_k}{|(\boldsymbol{x}_i)_k|^p}. \tag{21}$$

It is clear then that the PAR of $\boldsymbol{y}_i(p)$ is decreasing and equal to one for $p = 1$, but equal to the PAR of $\boldsymbol{x}_i$ for $p = 0$. Thus, suitable value for $p$ is easily obtained from the following minimization

$$\min p \tag{22a}$$

$$\text{s.t. } \frac{\max_k |(\boldsymbol{y}_i(p))_k|^2}{\frac{1}{N_p}\|\boldsymbol{y}_i(p)\|^2} \leq \mathrm{PAR}_{\max} \tag{22b}$$

$$0 \leq p \leq 1. \tag{22c}$$

The updated waveform is then given by

$$\boldsymbol{s}_j^{(m+1)} = \frac{\boldsymbol{y}_i(p)}{\|\boldsymbol{y}_i(p)\|} \tag{23}$$

as scaling does not change the PAR value.

### 3.2. Maximum Block Improvement

Maximum Block Improvement (MBI) is an algorithm for real-valued multivariate polynomial optimization. It works by increasing the number of variables so that a linear relaxation of the multivariate polynomial is achieved [10]. In many cases, this linearized problem can be easily solved. At each iteration, the set of variables that improves the objective function most is updated.

Solving the waveform optimization problem using Maximum Block Improvement (MBI) algorithm was proposed in [11]. The linearization of the waveform optimization problem can be reformulated as

$$\begin{aligned} \min \max_{i,j,k,f} \; & \tilde{\boldsymbol{s}}_{i,1}^T \boldsymbol{A}_{k,f} \tilde{\boldsymbol{s}}_{j,2} \tilde{\boldsymbol{s}}_{i,3}^T \boldsymbol{A}_{k,f} \tilde{\boldsymbol{s}}_{j,4} \\ & + \tilde{\boldsymbol{s}}_{i,1}^T \boldsymbol{B}_{k,f} \tilde{\boldsymbol{s}}_{j,2} \tilde{\boldsymbol{s}}_{i,3}^T \boldsymbol{B}_{k,f} \tilde{\boldsymbol{s}}_{j,4} \\ \text{s.t. } & |f| \geq \delta_{k0}\delta_{ij} f_0, \; \|\boldsymbol{s}_i, j\| = 1 \; \forall i, j \end{aligned} \tag{24}$$

At each iteration, one set of variables $\boldsymbol{s}_{i,j}$ is optimized while the others are kept constant. However, due to the minimax nature of the optimization, this is not a convex problem.

### 3.3. Simulated Annealing

Simulated annealing (SA) may be used for waveform optimization. It is a heuristic search method for finding an approximation of the global optimum of a multivariate or combinatorial function [12]. It was used for multiple waveform optimization in [4]. Similar heuristic approaches include genetic algorithm used in [13], tabu search used in [14], and the cross-entropy method in used [15], for example. All of these methods are stochastic search algorithms in which the generation of solution candidates is randomized and worse solutions can be accepted in order to avoid local optima.

An essential part of the simulated annealing is the method forming the solution candidates. Here, we add a small complex Gaussian perturbation to the waveforms at each update, i.e. $\boldsymbol{x}_i = \boldsymbol{s}_i^{(m)} + \boldsymbol{\epsilon}_i$, where $\boldsymbol{\epsilon}_i \sim \mathcal{CN}(\boldsymbol{0}, \sigma_\epsilon^2 \boldsymbol{I})$. Equations (21), (22), and (23) can then be used to obtain a candidate solution that satisfies the PAR constraint and the norm constraint.

In order to achieve the best approximate solution, we apply a modest reheating at the best point of the initial optimization for possibility to improve the solution further to a better local optimum nearby. This approach differs from the greedy optimization approach used in [4].

### 3.4. Greedy Optimization

In greedy waveform optimization, a single symbol in the waveform set is modified until a minimum of the objective function, in this case (15a), has been found. Each symbol is modified in turn until no improvement can be made by changing any single symbol. This type of optimization was used in [4] to improve the solution found using the simulated annealing.

The PAR constraint can be easily accommodated into the greedy optimization by enforcing (17) for the symbol being modified. The other symbols can then be scaled to meet the norm constraint in (15b).

## 4. NUMERICAL EXAMPLES

Numerical examples comparing the waveform optimization methods described in the previous section are provided next. In order to

**Table 1**. PSL and PCC of the optimized waveform sets

| Method | Point 1 | Point 2 | Point 3 | Point 4 | Point 5 |
|---|---|---|---|---|---|
| SA | -9.90 | -9.94 | -9.92 | -9.91 | -9.66 |
| SA+Greedy | -9.90 | -9.96 | -9.93 | -9.92 | -9.68 |
| Manifold | -9.06 | -9.38 | -8.88 | -9.19 | -9.31 |
| Manifold+Greedy | -9.08 | -9.43 | -9.00 | -9.24 | -9.31 |
| IQR | -10.11 | -10.00 | -10.08 | -10.18 | -9.97 |
| IQR+Greedy | -10.11 | -10.00 | -10.08 | -10.18 | -9.97 |
| Greedy | -8.05 | -7.29 | -7.31 | -7.58 | -8.24 |
| MBI | -6.26 | -6.92 | -6.16 | -6.35 | -7.20 |

Comparison of PSL and PCC levels of the optimized $4 \times 40$ waveform sets without a PAR constraint. IQR algorithm provides the best results.

compare the waveform optimization methods, we optimized a waveform set consisting of four polyphase waveforms with 40 symbols in each. The objective was to obtain waveforms with lowest possible peak sidelobe and peak cross-correlation levels. The optimization was done starting from five different initial points either with no PAR constraint or PAR constrained to 1.5 or to 1. The last case corresponds constant-modulus waveforms. In addition to the optimization methods of the previous section, we used also a quasi-Newton method on the manifold. Furthermore, the greedy algorithm was used in conjunction with the other algorithms to see if the waveforms can be improved the further.

The results of the waveform optimization without a PAR constraint are shown in Table 1. It is immediately clear that MBI performs the worst in comparison to the other methods considered. The greedy algorithm by itself fares slightly better than the MBI, but it is still behind the other algorithms. The algorithm providing the waveforms with the lowest PSL and PCC is the IQR in this case. However, the computational complexity of the IQR is correspondingly high. The benefit of using the greedy algorithm after the optimization with another method is marginal at best in this case.

Table 2 shows the optimization results with PAR constrained to be 1.5 at most. The results in this case are similar as without the PAR constraint IQR being the best followed by SA and then the quasi-Newton on the manifold. The differences between these methods are small, while the MBI and the greedy algorithm are clearly worse. Furthermore, the improvement achieved by using after the other methods is insignificant. The results for constant-modulus (PAR equal to one) waveforms are shown in Table 3. The results follow the same pattern as in the other cases, but the differences in the achieved PSL and PCC are smaller between the algorithms.

Table 4 summarizes the results by averaging the PSL and PCC level obtained using 20 different initial points for the three aforementioned cases. It can be seen that the impact of the PAR constraint on the PSL and PCC levels is small. Also, using the greedy algorithm to improve the solutions obtained with the other optimization methods does not provide a significant benefit. It should be noted that as the PAR constraint has only a small impact on the achieved PSL and PCC level. As low PAR simplifies the transmitter design, it would thus be sensible to use low-PAR or constant-modulus waveforms.

## 5. CONCLUSIONS

In this paper, we formulated the MIMO radar waveform optimization problem as a quartic minimax problem on an oblique manifold and proposed an iterative quadratic relaxation (IQR) method for solving it. Optimizing waveforms with a constraint on the peak to average ratio of power, including constant-modulus waveforms, can also be done using the proposed method.

In the numerical examples, the IQR method was found out to be superior both compared to maximum block improvement linearizing the problem and quasi-Newton method on the manifold. The IQR method also typically reached achieved lower PSL and PCC levels compared to simulated annealing.

Comparison of the computational complexity of the optimization methods was not included. This should be done in a subsequent study.

**Table 2**. PSL and PCC of the optimized waveform sets with PAR constraint

| Method | Point 1 | Point 2 | Point 3 | Point 4 | Point 5 |
|---|---|---|---|---|---|
| SA | -9.63 | -9.54 | -9.74 | -9.61 | -9.62 |
| SA+Greedy | -9.64 | -9.54 | -9.76 | -9.62 | -9.62 |
| Manifold | -9.06 | -7.84 | -9.34 | -9.12 | -8.97 |
| Manifold+Greedy | -9.08 | -8.80 | -9.36 | -9.23 | -9.03 |
| IQR | -9.70 | -9.72 | -9.76 | -9.70 | -9.67 |
| IQR+Greedy | -9.70 | -9.72 | -9.77 | -9.70 | -9.68 |
| Greedy | -8.23 | -6.92 | -8.46 | -8.29 | -8.09 |
| MBI | -6.88 | -6.49 | -6.07 | -6.82 | -6.59 |

Comparison of PSL and PCC levels of the optimized $4 \times 40$ waveform sets with the PAR constrained to be equal to or less than 1.5. IQR algorithm provides the best results, but the difference to the SA which is small.

**Table 3**. PSL and PCC of the optimized constant-modulus waveform sets

| Method | Point 1 | Point 2 | Point 3 | Point 4 | Point 5 |
|---|---|---|---|---|---|
| SA | -9.73 | -9.82 | -9.63 | -9.65 | -9.78 |
| SA+Greedy | -9.73 | -9.82 | -9.63 | -9.65 | -9.78 |
| Manifold | -8.60 | -8.69 | -8.59 | -8.82 | -8.92 |
| Manifold+Greedy | -8.62 | -8.73 | -8.72 | -8.82 | -8.92 |
| IQR | -9.97 | -9.73 | -9.81 | -10.07 | -9.92 |
| IQR+Greedy | -9.97 | -9.82 | -9.81 | -10.07 | -9.92 |
| Greedy | -7.80 | -7.37 | -7.46 | -7.76 | -7.51 |
| MBI | -6.88 | -7.53 | -6.07 | -7.45 | -7.46 |

Comparison of PSL and PCC levels of the optimized $4 \times 40$ constant-modulus waveform sets with unit PAR. IQR algorithm provides the best results in all but one of the starting points.

**Table 4**. Average PSL and PCC

| Method | Free PAR | PAR 1.5 | PAR 1 |
|---|---|---|---|
| SA | -9.84 | -9.62 | -9.64 |
| SA+Greedy | -9.85 | -9.63 | -9.64 |
| Manifold | -9.13 | -8.91 | -8.61 |
| Manifold+Greedy | -9.21 | -9.08 | -8.71 |
| IQR | -10.07 | -9.72 | -9.83 |
| IQR+Greedy | -10.07 | -9.72 | -9.84 |
| Greedy | -7.78 | -7.69 | -7.60 |
| MBI | -6.61 | -6.71 | -6.99 |

Maximum of PSL and PCC averaged over random 20 initializations for the $4 \times 40$ waveform sets. PAR constraint increases the PSL and PCC only slightly.

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