MIMO RADAR WAVEFORM DESIGN FOR MULTIPLE EXTENDED TARGET ESTIMATION BASED ON GREEDY SINR MAXIMIZATION

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ABSTRACT

The paper studies waveform and receiver design for MIMO radars through a greedy procedure based on maximization of the signal-to-interference-plus-noise ratio (SINR). The procedure enables joint design of estimation waveforms for multiple extended targets and maximizes also the sum of radar mutual information measures. Numerical results obtained from simulations are presented to illustrate the proposed approach. Different scenarios with various levels of interference among targets are considered and characteristics of the waveforms designed in each scenario are discussed. Results indicate that the designed waveforms overlap in fewer frequency bands when interference between target reflections is stronger.

Index Terms— MIMO radar, extended targets, waveform design, interference, SINR.

1. INTRODUCTION

Over the past decade, waveform adaptation has emerged as a meaningful approach to improve the performance of radar and communication systems [1]. With modern radars implemented in software [2], transmitted radar waveforms and corresponding receivers can be easily tailored to specific targets and/or environment features [3].

When the focus of the radar system is to extract information and to determine unknown characteristics of targets known to be present in the environment, estimation waveforms are used to make measurements with the goal of decreasing the uncertainty about specific target parameters. In this case radar waveform design is accomplished using information theoretic criteria that are based on the mutual information between a specific target response and the corresponding reflected signal observed at the radar receiver.

This paper presents a new approach to radar waveform design that is applicable to general bi-static radar systems and multiple extended targets. The proposed approach leverages results applicable to waveform design in mutually interfering wireless communication systems and studies application of the greedy waveform design algorithm outlined in [4] to designing MIMO radar waveforms. In addition, spectral properties of resulting radar waveforms in specific scenarios corresponding to various levels of interference among the reflected radar signals are also discussed.

The paper is organized as follows: the system model is introduced in Section 2 followed by presentation of the radar target vector channel in Section 3 and of the information theoretic measures used for waveform design in Section 4. The proposed approach for radar waveform design is discussed in Section 5 and is illustrated with numerical results obtained from simulations in Section 6. Final remarks and conclusion are given in Section 7.

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2. SYSTEM MODEL

We consider a general MIMO radar system in which multiple antennas are used for the transmission and processing of multiple waveforms and in which multiple targets may be present within the radar scene. Using transmit and receive beamforming, the transmitted waveforms are directed toward the known location of each target and the receiver processes the reflected signal looking in each target direction, and we denote by $\alpha_T^{(\ell)}$ and $\alpha_R^{(\ell)}$ the pathloss coefficients corresponding to the free-space propagation between the radar transmitter and target ℓ and between the target and the radar receiver, respectively.

Let L be the number of extended targets present, with the impulse response of a given target ℓ denoted by $h_\ell(t)$ and modeled as collections of point targets, or "reflection centers", such that their impulse responses are given by [5]:

$$h_\ell(t) = \sum_{r_c=1}^{R_c} \eta_{r_c}^{(\ell)} \delta(t - \tau_{r_c}^{(\ell)}), \tag{1}$$

where r_c is the index of each reflection center, and $\eta_{r_c}^{(\ell)}$ is the magnitude of the response from each reflection center received with a corresponding delay $\tau_{r_c}^{(\ell)}$ determined by its range. Each of the L targets present is assumed to reside in a single transmit/receive beam pair cell. This assumption is not excessively restrictive, as even beams with small radial width will cover a large area relative to the target size when the target is distant from the transmitter and receiver.

We note that, while the actual impulse response of each target is unknown, information about the statistics of their corresponding frequency responses is assumed known as in [5], such that the frequency response $H_\ell(f)$ of target ℓ with impulse response $h_\ell(t)$ is characterized by mean $\mu_{H_\ell}(f)$ and variance $\sigma_\ell^2(f)$ [6]:

$$\mu_{H_\ell(f)} = E[H_\ell(f)], \tag{2}$$

$$\sigma_{H_\ell}^2(f) = E[|H_\ell(f) - \mu_{H_\ell}(f)|^2]. \tag{3}$$

Assuming Swerling-type targets [5] with Gaussian processes as impulse responses, their corresponding frequency responses are also Gaussian and independent, such that

$$E[H_\ell(f_k) \cdot H_m^*(f_j)] = \begin{cases} 0, \text{ for } k \neq j,\ \ell \neq m \\ \sigma_{H_\ell}^2(f_k), \text{ for } k = j,\ \ell = m. \end{cases} \tag{4}$$

The radar transmitter sends multiple waveforms $s_d(t)$ normalized to unit energy with energy levels p_d, $d = 1, \ldots, L$, with each waveform focused in the direction of target d by transmit beamforming vector $\mathbf{u}_d \in \mathbb{C}^{N_T \times 1}$, where N_T is the number of transmit antennas in the transmitter array. Thus, the transmitted MIMO radar signal is a sum of all the beamformed waveforms:

$$\mathbf{s}(t) = \sum_{d=1}^{L} \mathbf{u}_d s_d(t) \sqrt{p_d}. \tag{5}$$

Assuming that target ℓ is at azimuth angle τ_ℓ relative to the transmit antenna array and that the transmit array manifold vector in the direction of the target is represented by $\mathbf{a}_T(\tau_\ell) \in \mathbb{C}^{N_T \times 1}$, the signal reflected by the target ℓ can be written as

$$y_\ell(t) = h_\ell(t) \star [\mathbf{a}_T^H(\tau_\ell)\mathbf{s}(t)\alpha_T^{(\ell)}], \quad \ell = 1, \ldots, L, \tag{6}$$

where $\star$ represents the convolution operation, and $(\cdot)^H$ denotes the complex conjugate transpose operation.

At the MIMO receiver, signal (6) reflected by target ℓ is received from azimuth direction ρ_ℓ through the receive antenna array with manifold vector $\mathbf{a}_R(\rho_\ell) \in \mathbb{C}^{N_R \times 1}$ in the direction of the target, and the corresponding received signal is:

$$\mathbf{z}_\ell(t) = [\alpha_R^{(\ell)}\mathbf{a}_R(\rho_\ell)\mathbf{a}_T^H(\tau_\ell)\alpha_T^{(\ell)}] \sum_{d=1}^{L} \mathbf{u}_d[h_\ell(t) \star s_d(t)]\sqrt{p_d}. \tag{7}$$

Combining signals reflected from all targets and assuming that a vector noise process $\mathbf{w}(t)$ corrupts signals at the receive antenna array, the total received signal expression becomes:

$$\mathbf{z}(t) = \sum_{\ell=1}^{L} \mathbf{z}_\ell(t) + \mathbf{w}(t). \tag{8}$$

The noise processes in $\mathbf{w}(t)$ correspond to bandpass filtered versions of the noise processes that corrupt signals at all receive antennas, which are assumed to be white and Gaussian with power spectral density (PSD) $Q_\epsilon(f) = \sigma^2$ for all frequencies f and all receive antennas $\epsilon = 1, \ldots, N_R$.

The received signal vector $\mathbf{z}(t)$ is processed by the N_R-antenna array of the MIMO radar receiver through beamforming vectors $\mathbf{v}_r \in \mathbb{C}^{N_R \times 1}$ to yield scalar signals $z_r(t) = \mathbf{v}_r^H\mathbf{z}(t)$ for each of the $r = 1, \ldots, L$ receive directions, where

$$z_r(t) = \sum_{\ell=1}^{L}\sum_{d=1}^{L} \lambda_{rd}^{(\ell)}[h_\ell(t) \star s_d(t)]\sqrt{p_d} + \mathbf{v}_r^H\mathbf{w}(t) \tag{9}$$

and $\lambda_{rd}^{(\ell)} = \alpha_R^{(\ell)}\mathbf{v}_r^H\mathbf{a}_R(\rho_\ell)\mathbf{a}_T^H(\tau_\ell)\mathbf{u}_d\alpha_T^{(\ell)}$. The scalar $\lambda_{rd}^{(\ell)}$ combines the pathloss coefficients along with the transmit and receive beamforming and array manifold vectors, and is implied by the position of the target, which is determined by angles ρ_ℓ and τ_ℓ corresponding to transmit waveform/direction d and receive direction r.

3. THE RADAR TARGET VECTOR CHANNEL

Upon uniform sampling with period T, equation (9) is equivalent to the discrete-time vector equation

$$\mathfrak{z}_r = \sum_{\ell=1}^{L}\sum_{d=1}^{L} \lambda_{rd}^{(\ell)}\sqrt{p_d}\boldsymbol{\mathcal{S}}_d\mathfrak{g}_\ell + \mathfrak{n}_r, r = 1, \ldots, L, \tag{10}$$

$$\text{where } \boldsymbol{\mathcal{S}}_d = \begin{bmatrix} s_d(k) & \ldots & s_d(k-K+1) \\ \vdots & \ddots & \vdots \\ s_d(k+K-1) & \ldots & s_d(k) \end{bmatrix} \tag{11}$$

is a $K \times K$ circulant convolution matrix corresponding to the transmitted waveform in direction d, and

$$\mathfrak{g}_\ell = \begin{bmatrix} g_\ell(0) \\ \vdots \\ g_\ell(K-1) \end{bmatrix}, \quad \mathfrak{n}_r = \begin{bmatrix} n_r(k) \\ \vdots \\ n_r(k+K-1) \end{bmatrix}, \tag{12}$$

are $K \times 1$ vectors containing samples of the target impulse response[1] and noise, respectively.

[1] $g_\ell(k) = \eta_{r_c}^{(\ell)}$ when a reflection center r_c is present at the location corresponding to a delay of $\tau_{r_c}^{(\ell)} = kT$ along the target, and $g_\ell(k) = 0$ when no reflection center is present at that location.

With these notations, one can easily observe that the equivalent discrete-time model of the radar target channel appears similar to that of discrete multitone (DMT) or OFDM systems used for information transmission [7, Ch. 6.13], and following the approach outlined there based on the discrete Fourier transform (DFT) and properties of circulant matrices, the equivalent frequency domain model of the radar target channel is obtained:

$$\mathbf{z}_r = \sum_{\ell=1}^{L}\sum_{d=1}^{L} \lambda_{rd}^{(\ell)}\sqrt{p_d}\mathbf{H}_\ell\mathbf{s}_d + \mathbf{n}_r, r = 1, \ldots, L. \tag{13}$$

The elements of $\mathbf{s}_d$ vector are the K-point DFT of the K samples of the d-th transmitted radar waveform $\{s_d(k) \ldots s_d(k-K+1)\}$ that make up the circulant matrix $\boldsymbol{\mathcal{S}}_d$, and the matrix $\mathbf{H}_\ell$ is a $K \times K$ diagonal matrix with the elements equal to the K-point DFT of the target impulse response samples $g_\ell(0), \ldots, g_\ell(K-1)$. We note that, in this case the diagonal channel matrices $\mathbf{H}_\ell$ contain samples of the Fourier transform of the target impulse response $h_\ell(t)$ and are characterized by their means and variances given in (2) and (3), respectively, and calculated for a set of K frequencies of interest.

4. RADAR WAVEFORM DESIGN BASED ON GREEDY SINR MAXIMIZATION

To facilitate joint design of radar waveforms by jointly processing the reflected signals in (13) we assume, similar to [5], that beamforming at the transmitter is defined such that the waveform transmitted in the direction of a specific target ℓ is only reflected by target ℓ, which implies $\lambda_{rd}^{(\ell)} = 0, \;\forall d \neq \ell$. Under this assumption the inner summation over d in (13) reduces to a single term, and (13) can be written compactly as

$$\mathbf{z} = \sum_{\ell=1}^{L} \underbrace{\bar{\mathbf{H}}_\ell\mathbf{s}_\ell\sqrt{p_\ell}}_{\mathbf{y}_\ell} + \mathbf{n} = \sum_{\ell=1}^{L} \mathbf{y}_\ell + \mathbf{n} \tag{14}$$

$$\text{where } \mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_r \\ \vdots \\ \mathbf{z}_L \end{bmatrix}, \; \bar{\mathbf{H}}_\ell = \begin{bmatrix} \lambda_{1\ell}^{(\ell)}\mathbf{H}_\ell \\ \hline \vdots \\ \hline \lambda_{r\ell}^{(\ell)}\mathbf{H}_\ell \\ \hline \vdots \\ \hline \lambda_{L\ell}^{(\ell)}\mathbf{H}_\ell \end{bmatrix}, \; \mathbf{n} = \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_r \\ \vdots \\ \mathbf{n}_L \end{bmatrix}. \tag{15}$$

Note that equation (14) appears similar to that corresponding to the vector channel model used to design transmission waveforms in cooperative multi-link wireless communication systems presented in [4], with input signals $\mathbf{s}_\ell$ having energies p_ℓ, channel matrices $\bar{\mathbf{H}}_\ell$, and noise vector $\mathbf{n}$. The main difference is implied by the fact that, unlike [4], where the channel matrices are deterministic and the transmitted signal vectors are random and characterized by their corresponding transmit covariance matrices, in the radar vector channel the channel matrices corresponding to the radar targets are random, while the transmitted signal vectors are deterministic.

In this setup, we consider that each individual radar waveform $\mathbf{s}_\ell$ is synthesized from an ensemble of N_{wfms} waveforms, that is

$$\mathbf{s}_\ell = \sum_{w=1}^{N_{\text{wfms}}} \mathbf{s}_\ell^{(w)}, \tag{16}$$

which are jointly optimized for the multiple extended targets present over the frequency bands of interest. We note that, using a greater number of synthesis waveforms for each target will result in an increase in signal diversity, which has been shown to improve radar target estimation performance [8]. The waveform design problem in this case is similar to that of designing codewords for optimal interference avoidance in the multibase wireless communication channel scenario considered in [4], and following a similar approach, our goal is to design the synthesis waveforms to maximize their corresponding SINR at the radar receiver when reflections received due to all other waveforms are regarded as interference.

To formalize this approach, we assume that the $(KL \times 1)$ received signal $\mathbf{z}$ in (14) is processed using a linear receiver filter $\tilde{\mathbf{c}}_\ell$, having the same block structure as shown for the received signal in (15). Furthermore, assuming the radar system looks only in the direction of target ℓ to estimate the frequency response of target ℓ, $\tilde{\mathbf{c}}_\ell$ will be composed of zeros except for the ℓ-th block of dimension $K \times 1$ which is denoted by $\mathbf{c}_\ell$. We note that $\tilde{\mathbf{c}}_\ell$ (and implicitly the block $\mathbf{c}_\ell$) can be regarded as a superposition of N_{wfms} linear filters $\tilde{\mathbf{c}}_\ell^{(q)}$, each decode a piece of target information that is embedded in the reflection of the q-th synthesizing waveform by target ℓ. The SINR corresponding to the q-th synthesizing waveform for target ℓ along with the associated matched filter receiver $\mathbf{c}_\ell^{(q)}$ is written as:

$$\gamma_\ell^{(q)} = \frac{(\mathbf{c}_\ell^{(q)})^H \mathbf{Y}_\ell^{(q)} \mathbf{c}_\ell^{(q)}}{(\mathbf{c}_\ell^{(q)})^H \mathbf{R}_i^{(q)} \mathbf{c}_\ell^{(q)}}, \tag{17}$$

where $$\mathbf{Y}_\ell^{(q)} = |\lambda_{\ell\ell}^{(\ell)}|^2 p_\ell E\left[\mathbf{H}_\ell \mathbf{s}_\ell^{(q)} (\mathbf{s}_\ell^{(q)})^H \mathbf{H}_\ell^H\right] \tag{18}$$

is the correlation matrix of the desired component in the reflected signal due to the q-th transmitted waveform in target ℓ direction, and

$$\mathbf{R}_i^{(q)} = \underbrace{\sum_{\kappa=1,\kappa\neq q}^{N_{\text{wfms}}} |\lambda_{\ell\ell}^{(\ell)}|^2 p_\ell E\left[\mathbf{H}_\ell \mathbf{s}_\ell^{(\kappa)} (\mathbf{s}_\ell^{(\kappa)})^H \mathbf{H}_\ell^H\right]}_{\text{self-interference}} + \underbrace{\sigma_n^2 \mathbf{I}_K}_{\text{noise}}$$
$$+ \underbrace{\sum_{\eta=1}^{N_{\text{wfms}}} \sum_{m=1,m\neq\ell}^{L} |\lambda_{\ell m}^{(m)}|^2 p_m E\left[\mathbf{H}_m \mathbf{s}_m^{(\eta)} (\mathbf{s}_m^{(\eta)})^H \mathbf{H}_m^H\right]}_{\text{interference from other targets}} \tag{19}$$

is the correlation matrix of the combined noise and interference seen from the perspective of the q-th waveform in the direction of target ℓ. We note that $\mathbf{R}_i^{(q)}$ contains self-interference terms corresponding to the reflections of the other synthesizing waveforms intended for target ℓ, as well as the interference terms implied by the reflections of the other radar waveforms intended for different targets.

The SINR expression (17) is a ratio of quadratic forms, which is maximized when $\mathbf{c}_\ell^{(q)}$, and consequently $\mathbf{s}_\ell^{(q)}$, correspond to the eigenvector associated to the maximum generalized eigenvalue of the matrix pair $(\mathbf{Y}_\ell^{(q)}, \mathbf{R}_i^{(q)})$ [9, p. 50]:

$$\mathbf{Y}_\ell^{(q)} \mathbf{c}_\ell^{(q)} = \zeta \mathbf{R}_i^{(q)} \mathbf{c}_\ell^{(q)} \quad \ell = 1, \ldots, L, \;\; q = 1, \ldots, N_{\text{wfms}} \tag{20}$$

As discussed in [4], applying this procedure iteratively for all waveforms in the ensemble $\ell = 1, \ldots, L$, $q = 1, \ldots, N_{\text{wfms}}$, with each waveform normalized to have equal energy and under a joint power constraint, a fixed point will be reached beyond which further eigen-updates will not result in waveforms changes anymore. The procedure is formally implemented in Algorithm 1, for which the fixed point is reached when the Euclidian distance between the same signal vector at two consecutive iterations is within a given tolerance ϵ.

Noting that $\mathbf{R}_i^{(q)}$ and $\mathbf{Y}_\ell^{(q)}$ will each have diagonal structure as long as target frequency responses are uncorrelated as implied by (4) and noise is white, their eigenvectors will be canonical vectors with a single nonzero entry at indices corresponding to individual frequency bands. Thus, waveforms for intended for each target are

Algorithm 1 : Radar Waveform & Receiver Filter Design

Input:
– Number of targets L present and of frequencies of interest K
– Target frequency responses $\mathbf{H}_\ell$ (normalized to unit energy, $\forall \ell$)
– Reflection coefficients λ_{rd}^{ℓ}, for all targets and directions $\forall d, \ell$
– Pre-defined fixed tolerance ϵ
Randomly initialize synthesizing waveforms $\mathbf{s}_\ell^{(q)}, \forall \ell, q$
while $\max_{\ell,q} |\mathbf{s}_\ell^{(q)} - \tilde{\mathbf{s}}_\ell^{(q)}| > \epsilon$ **do**
  for $\ell = 1, \ldots, L$ **do**
    for $q = 1, \ldots, N_{\text{wfms}}$ **do**
      Determine $\mathbf{c}_\ell^{(q)}$ as the eigenvector corresponding to the maximum generalized eigenvalue in (20)
      Set $\mathbf{s}_\ell^{(q)} = \mathbf{c}_\ell^{(q)}$
      Normalize $\mathbf{s}_\ell^{(q)} = \frac{p_\ell}{\sqrt{N_{\text{wfms}}}} \frac{\mathbf{s}_\ell^{(q)}}{(|\mathbf{s}_\ell^{(q)}|)}$
    end for
  end for
end while
Output:
– Optimized radar waveforms $\mathbf{s}_\ell^{(q)}, \forall \ell, q$
– Associated receiver filters $\mathbf{c}_\ell^{(q)}$ $\forall \ell, q$

synthesized in frequency by selecting the frequency bands according to the superpositions indicated by the canonical vectors, and the synthesizing waveforms can be regarded as the degrees of freedom in the waveform design process. As the number of degrees of freedom N_{wfms} increases, so does the flexibility in the shape of the synthesized waveform for each target which can be expected to result in an improved ability to estimate the frequency response of each target.

Each waveform designed in this context can be viewed as an incremental addition to the total sum waveform (16) generated for target ℓ, and, as argued in [4], designing waveforms using this strategy is similar to an iterative water filling approach [10] approach. This is intuitive since each individual waveform designed for each target can be thought of as an incremental allocation of the total power available for that target.

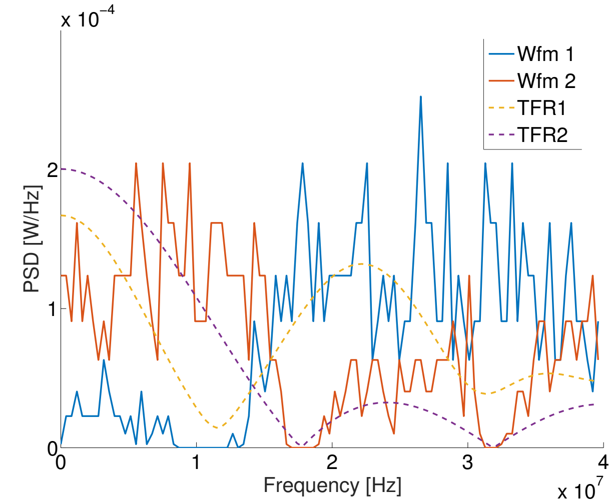
5. SIMULATIONS AND NUMERICAL RESULTS

To illustrate the performance of the presented approach, a scenario with $L = 2$ targets present at known locations was considered, with target frequency responses (TFR) having power spectral densities (PSD) similar to those in [5]. A bistatic MIMO radar system was assumed, with transmitter-receiver separation of 12 km, phased arrays with $N_T = N_R = 25$ elements and $1/2$ wavelength spacing, and classical beamforming was used for transmission and reception.

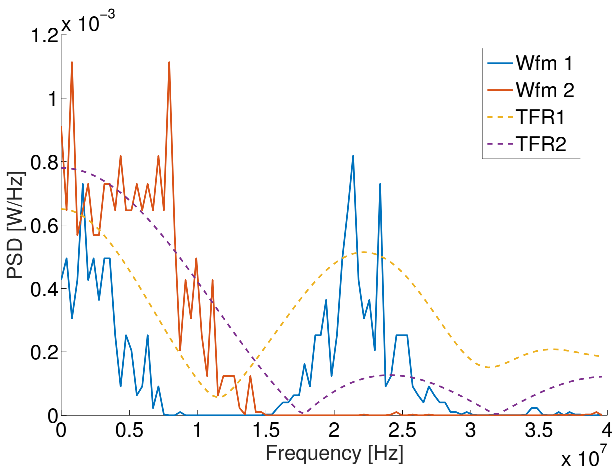
A total power constraint was enforced at the radar transmitter, with radar signals considered over an 80 MHz bandwidth centered at a carrier frequency of 8 GHz, implying a 3.75 m range resolution. Radar waveforms were designed at baseband from 0 to 40 MHz over $K = 101$ frequency bins using $N_{\text{wfms}} = 500$ synthesizing waveforms. In the case of high SNR, a transmit power of 1 kW is assumed to achieve a SNR of the order of $\sim$ 35 dB in the presence of AWGN with variance $\sigma_n^2 = -164$ dBm/Hz, which is similar to [5]. In the case of low SNR, a noise variance of -141dBm/Hz is assumed implying a total received SNR on the order of ~ 18 dB.

5.1. Weak Interference Case

Similar to [5], in this case targets are separated by 3° from the perspective of the receiver and were located at $(75^\circ, 70^\circ)$ and $(48^\circ, 73^\circ)$, respectively, relative to the baseline between the radar transmitter and receiver.

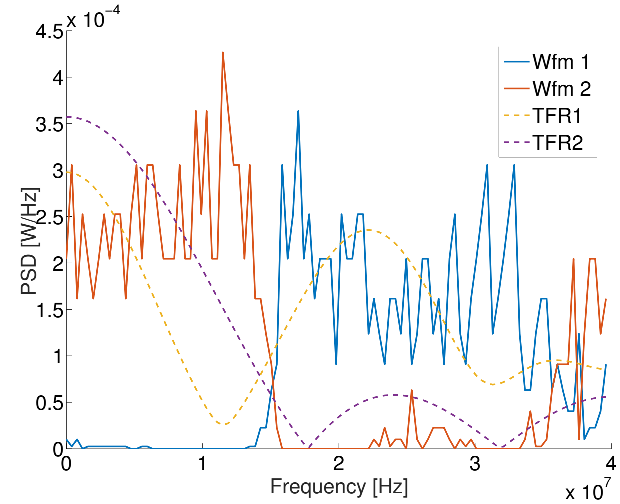


(a) High SNR

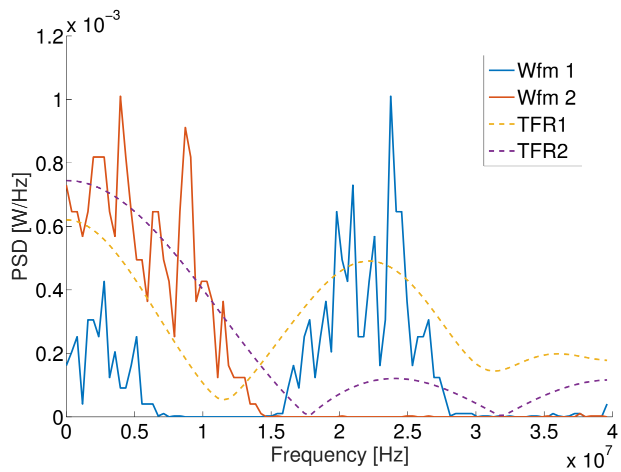


(b) Low SNR

Fig. 1. Waveforms designed for two weakly-interfering targets.



(a) High SNR



(b) Low SNR

Fig. 2. Waveforms designed for two strongly-interfering targets.

The corresponding waveforms designed using Algorithm 1 are illustrated in Figure 1 for both the high and low SNR scenarios, from where it can be observed that, when the target reflections interfere weakly with each other (good target separation from the perspective of the receiver) the resulting radar waveforms overlap in frequency. This agrees with the observations made in [5] where a different waveform design method for a similar two-target estimation problem was used.

5.2. Strong Interference Case

In this case targets which are separated by 0.5° from the perspective of the receiver and were located at $(75^\circ, 70^\circ)$ and $(55^\circ, 70.5^\circ)$, respectively, relative to the transmitter and receiver baseline.

The corresponding waveforms designed using Algorithm 1 are illustrated in Figure 2 for similar high and low SNR scenarios, from where it can be observed that when the target reflections interfere strongly with each other (poor target separation from the perspective of the receiver), the resulting radar waveforms display almost no overlap in frequency at high SNR, which again agrees with the observations made in [5]. At low SNR however, the radar waveforms there is more overlap in frequency between the two waveforms, which should be expected.

6. CONCLUSIONS

In this paper, a new approach to radar waveform design is presented, that is based on leveraging results applicable to waveform design in mutually interfering wireless communication systems. Specifically, the paper presents application of a greedy algorithm based on SINR maximization [4] to jointly designing MIMO radar waveforms for multiple extended target estimation.

Numerical results obtained from simulations are presented to illustrate the proposed approach. These indicate that, when targets are received from well separated arrival angles more of the waveform power may be allocated to overlapping frequency bands from one target to another, while when the separation is smaller, less power is allocated to overlapping frequencies.

Future work includes consideration of the scenario in which the priority for estimating some targets may be higher than others. This is of particular interest in the case where the total radar cross section of some targets are significantly smaller than others so that estimation of that target may be more difficult.

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