ROBUST TRANSMIT PRECODING FOR UNDERLAY MIMO COGNITIVE RADIO WITH INTERFERENCE LEAKAGE RATE LIMIT

Mohannad H. Al-Ali and K. C. Ho

ECE Department, University of Missouri, Columbia, MO 65211, USA
mohannadalalicom@yahoo.com, hod@mail.missouri.edu

ABSTRACT

This paper addresses the problem of optimizing the information rate of a multiple-input multiple-output (MIMO) secondary user (SU) in an underlay cognitive radio (CR) network when channel state information (CSI) from the SU to the primary user (PU) is inaccurate. Rather than applying the commonly used interference temperature metric, the proposed SU transmit precoder limits the interference leakage rate (LR) to maintain the quality of service (QoS) for the PU. We model the uncertainty in CSI as deterministic with the Schatten norm and apply the worst-case principle to derive a robust solution. Two solution methods are proposed to address the design with the LR metric. The first simplifies the LR metric that is valid under the low interference-to-noise ratio (INR) condition and the second uses an iterative linearization technique. We demonstrate the performance of the proposed solutions by numerical simulations.

Index Terms— Convex optimization, CSI uncertainty, leakage rate, MIMO cognitive radio, Schatten norm

1. INTRODUCTION

The emergence of cognitive radio (CR) has brought a promising solution to use efficiently the scarce spectrum for the legacy wireless systems. Among the different proposed paradigms, underlay CR represents a widely accepted technique in the research study and the regulation body [1, 2]. The underlay paradigm allows the coexistence of unlicensed or secondary users (SUs) with licensed or primary users (PUs), provided that their aggregate interference to PUs is below a preassigned limit [3–5].

The use of multiple antennas at the CR network nodes brings the benefits of spatial precoding that can substantially enhance the performance of SUs, reduce their interference to PUs, and meet the quality of service of the end users [3]. Most MIMO CR systems assume accurate knowledge of the channel state information (CSI) for the direct link from SU-Transmit (SU-Tx) to SU-Receive (SU-Rx) and the interference link from SU-Tx to PU-Receive (PU-Rx) (see Fig. 1). In practice SU-Tx only has limited knowledge about the CSI of the interference link [6–9]. Pretending the limited CSI as perfect would lead to the violation of the QoS of PU. It is the objective of this paper to take the inaccuracy of the CSI knowledge into consideration when designing the SU precoder to maximize the performance of SU while maintaining the QoS of PU.

We shall model the uncertainty in CSI as deterministic and within a convex set defined by the Schatten norm, which comprises several commonly used matrix norms in the literature [10, 11], such as the nuclear, Frobenius, and Spectral norms that model the rank, power, and maximum eigenmode of the CSI uncertainty, respectively. The proposed design uses the worst-case principle to obtain a robust solution [11–16].

Unlike the previous studies in underlay CR, this paper adopts the leakage rate (LR) metric as an alternative to the classic interference temperature (IT) metric to assess the impact of interference on PU-Rx. The LR metric was originally appeared in [17]. It has been shown to favor a higher rate for PU than the IT metric, which in some situations is more appropriate [16].

Precoder design using the LR metric is more complicated since the optimization problem is not convex and we propose two solutions for the design. One uses the Lagrange dual formulation and is suitable under low interference-to-noise (INR) condition at PU-Rx. The other employs an iterative linearization approach and is appropriate regardless of the INR value.

The main contribution of our work is that we design a robust SU transmit precoder by maintaining the QoS of PU through the LR metric without restricting the CSI uncertainty defined by a certain matrix norm. The previous studies do not consider the QoS requirement for PU [11, 13], focus on IT metric only [3, 8, 14], or limit to a specific matrix norm for the uncertainty in CSI [12–15]. Indeed, we have not come across similar solutions from literature for this design problem. Although the proposed techniques and numerical results are shown for single pair of PU and SU, they can be extended to handle the existence of multiple PUs in the CR network as elaborated in Sections 3 and 4.

The rest of this paper is organized as follows. Section 2 introduces the system and channel uncertainty models and provides the problem formulation. Section 3 approximates the LR metric at low INR and develops a precoder design through the Lagrange dual formulation. Section 4 proposes an iterative linearization approach to obtain the optimal solution for the precoder. Section 5 provides the simulations and Section 6 concludes the paper.

Notations: Bold upper-case letters denote matrices. $\det(\mathbf{A})$, $\mathrm{Tr}(\mathbf{A})$, $\lambda_{max}(\mathbf{A})$, $[\mathbf{A}]_{ij}$, $\|\mathbf{A}\|_{Sp}$, and $\mathbf{A} \succeq 0$ represent the determinant, the trace, the maximum eigenvalue, the (ith, jth) element, the Schatten norm of order p, and the positive semi-definite (PSD) property of $\mathbf{A}$. $(\cdot)^\dagger$ is the Hermitian transpose, $\log(\cdot)$ is the natural logarithm and $|\cdot|$ is the absolute value of a scalar. $\mathbb{C}^{m\times n}$ is the complex space of $m \times n$ matrices and $\mathbb{H}^n$ is the space of the size n Hermitian matrices. $\mathbf{I}$ is the identity matrix of an appropriate size.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We shall consider a CR system where one PU and one SU, both equipped with multiple transmit and receive antennas, are present as depicted in Fig. 1. The PU has M_p and N_p transmit and receive antennas, while the SU has M_s and N_s. The SU communication

This work was supported by the Higher Committee for Eduacation Develpment in Iraq (HCED).

channel is denoted as $\mathbf{T} \in \mathbb{C}^{N_s \times M_s}$ and the interference channel from SU-Tx to PU-Rx is $\mathbf{S} \in \mathbb{C}^{N_p \times M_s}$. The influence of PU to SU can be reasonably assumed negligible [18–20].

The SU transmitter encodes d data steams using a linear precoder $\mathbf{V} \in \mathbb{C}^{M_s \times d}$ and the corresponding codebook is $\mathbf{Q} = \mathbf{V}\mathbf{V}^\dagger$. We shall model the power characteristics of $\mathbf{Q}$ using a general convex set $\mathcal{Q}$ as [21]

$$\mathcal{Q} = \{\mathbf{Q} \succeq 0 : \mathrm{Tr}\,(\mathbf{Q}) \leq P^{tot}, \lambda_{max}(\mathbf{Q}) \leq P^{max}, \\ [\mathbf{Q}]_{qq} \leq P_q^{ant}, q = 1, \cdots, M_s\} \quad (1)$$

where P^{tot} is the total average transmit power, P^{max} is the maximum average power, and P_q^{ant} is the average power of the qth antenna.

The CSI for $\mathbf{T}$ is perfectly known at SU-Tx, while for $\mathbf{S}$ is partially known. Specifically, we consider the available CSI of $\mathbf{S}$ is in the form of a covariance matrix $\hat{\mathbf{R}}_S \in \mathbb{H}^{M_s}$ and it is related to the actual covariance $\mathbf{R}_S$ by [16]

$$\mathbf{R}_S = \hat{\mathbf{R}}_S + \Delta\mathbf{R}_S\,, \quad (2)$$

where $\Delta\mathbf{R}_S \in \mathbb{H}^{M_s}$ is the uncertainty error matrix. We shall assume that $\Delta\mathbf{R}_S$ belongs to the compact convex set

$$\mathcal{U} = \{\Delta\mathbf{R}_S : \|\Delta\mathbf{R}_S\|_{Sp} \leq \epsilon, \hat{\mathbf{R}}_S + \Delta\mathbf{R}_S \succeq 0\} \quad (3)$$

where ϵ is the uncertainty radius and $\|\cdot\|_{Sp}$ is the Schatten norm of order p defined for any $\mathbf{A} \in \mathbb{C}^{n\times m}$ as [10, Proposition 9.2.3]

$$\|\mathbf{A}\|_{S_p} = \begin{cases} \left(\sum_{i=1}^{\min(n,m)} \sigma_i^p\right)^{1/p}, & 1 \leq p < \infty \\ \sigma_1, & p = \infty \end{cases} \quad (4)$$

where σ_i, $i = 1, \cdots, \min(n, m)$, is the singular values of $\mathbf{A}$ arranged in a decreasing order. It is evident from (2) and (3) that $\mathbf{R}_S$ lies in a deterministic region that is centered at $\hat{\mathbf{R}}_S$.

The objective function for optimization is the mutual information $C(\mathbf{Q})$ of SU defined as [3]

$$C(\mathbf{Q}) = \log\det\left(\mathbf{I} + \frac{1}{\sigma_s^2}\mathbf{T}\mathbf{Q}\mathbf{T}^\dagger\right) \text{ (nats/s/Hz)} \quad (5)$$

where σ_s^2 is the noise power at SU-Rx that is known.

The LR metric [16, 17] is adopted here to provide a measure for the SU interference to PU. This metric quantifies the mutual information of the interference link from SU-Tx to PU-Rx and it was shown in [16] to favor a higher rate for PU than the commonly used IT metric. It is defined as

$$\Gamma(\mathbf{Q}, \mathbf{R}_S) \triangleq \log\det\left(\mathbf{I} + \frac{1}{\sigma_p^2}\mathbf{R}_S^{1/2}\mathbf{Q}\mathbf{R}_S^{1/2}\right) - R_L \quad (6)$$

where R_L is the maximum allowable interference leakage rate at PU-Rx, $\mathbf{R}_S^{1/2}$ is the matrix square root of $\mathbf{R}_S = \mathbf{S}^\dagger\mathbf{S}$, and σ_p^2 is the known noise power at PU-Rx.

Based on the worst-case principle [11–16], the optimization problem to design a robust precoder is

$$\text{(P-I):} \quad \max_{\mathbf{Q}\in\mathcal{Q}}.\ C(\mathbf{Q}) \quad (7)$$

$$\text{s.t.} \quad \max_{\Delta\mathbf{R}_S\in\mathcal{U}}.\ \Gamma(\mathbf{Q}, \mathbf{R}_S) \leq 0\,. \quad (8)$$

P-I optimizes the SU rate under the condition (8) that the interference level in terms of the LR metric is limited to R_L when the interference channel $\mathbf{S}$ is partially known. The $\mathbf{R}_S$ in (8) is dependent on $\Delta\mathbf{R}_S$ through (2). The problem P-I is not convex in the optimization variable $\mathbf{Q}$ due to the LR metric. Two approaches are used to solve P-I, one considers the low INR situation at PU-Rx and the other applies local linearization.

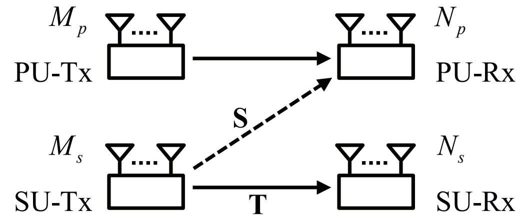

Fig. 1. An SU coexists with a PU in an underlay CR network, where the transmitter and receiver of each user have multiple antennas.

3. LOW INR SOLUTION

We shall first tackle the optimization subproblem of the left side of (8). The LR metric can be expanded as

$$\Gamma(\mathbf{Q}, \mathbf{R}_S) = \frac{1}{\sigma_p^2}\,\mathrm{Tr}\,(\mathbf{R}_S\mathbf{Q}) + o\left(\frac{1}{\sigma_p^2}\|\mathbf{R}_S^{1/2}\mathbf{Q}\mathbf{R}_S^{1/2}\|\right) - R_L \quad (9)$$

where $\|\cdot\|$ is a norm measure [13]. We shall define the interference-to-noise ratio at PU-Rx as $\mathrm{Tr}\,(\mathbf{R}_S\mathbf{Q})/\sigma_p^2$. At low INR regimes where INR $\leq$ 0 dB, the second term on the right side of (9) can be ignored. Hence, for a given precoder $\mathbf{Q}$ and under low INR condition, the left side of the interference constraint (8) can be casted into the following subproblem

$$\text{(P-II):} \quad \max_{\Delta\mathbf{R}_S}.\ \mathrm{Tr}\left(\frac{1}{\sigma_p^2}(\hat{\mathbf{R}}_S + \Delta\mathbf{R}_S)\mathbf{Q}\right) - R_L \quad (10)$$

$$\text{s.t.} \quad \|\Delta\mathbf{R}_S\|_{Sp} \leq \epsilon \quad (11)$$

$$\hat{\mathbf{R}}_S + \Delta\mathbf{R}_S \succeq 0\,. \quad (12)$$

The problem P-II is convex in the uncertainty matrix $\Delta\mathbf{R}_S$ and satisfies the Slater's condition [22]. Therefore, we can represent P-II using its Lagrange dual. The following proposition gives the structure of the optimum objective value for (10) in terms of $\mathbf{Q}$ through the Lagrange dual.

Proposition. The expression of the optimum objective value for the subproblem P-II can be compactly expressed as

$$\frac{1}{\sigma_p^2}\left(\mathrm{Tr}\,(\hat{\mathbf{R}}_S\mathbf{Q}) + \epsilon\|\mathbf{Q}\|_{Sq}\right) - R_L \quad (13)$$

where $1/p + 1/q = 1$ and p is the Schatten norm order.

Proof. The procedure we follow to obtain (13) is to form the Lagrangian of P-II and maximize it over $\Delta\mathbf{R}_S$ to arrive at the dual Lagrangian function. We then minimize the dual function with respect to the dual variables associated with the constraints (11) and (12). For further details, please refer to [16, Appendix D]. ■

(13) provides the insight that the uncertainty in $\mathbf{R}_S$ is being taken care of by introducing the additive factor $\epsilon\|\mathbf{Q}\|_{Sq}$, which ensures a robust solution under the worst-case principle. P-I can now be solved after replacing the left side of (8) by (13) using some optimization package, such as CVX [23]. In the case of multiple PUs, P-I will have additional interference constraints similar to that in (8). In solving P-I each of these constraints will become the form in (13).

From the Schatten norm property, the most conservative robust solution appears by setting $p = \infty$. As $q = 1$ and $\|\mathbf{Q}\|_{S_1} = \mathrm{Tr}(\mathbf{Q})$ then (13) becomes

$$\frac{1}{\sigma_p^2}\,\mathrm{Tr}\,(\tilde{\mathbf{R}}_S\mathbf{Q}) - R_L \quad , \quad \tilde{\mathbf{R}}_S = \hat{\mathbf{R}}_S + \epsilon\mathbf{I}\,. \quad (14)$$

It indicates that the uncertainty in the interference channel can be accounted for simply by increasing the eigenvalues of the available CSI $\hat{\mathbf{R}}_S$ by ϵ.

4. ITERATIVE LINEARIZATION SOLUTION

For a given uncertainty channel matrix, the non-convexity of $\mathbf{Q}$ in (8) is handled by using a local minimization method. Such a technique was previously applied to the log-det function in obtaining a smooth surrogate for a rank function [24].

The first-order Taylor series expansion of (6) around a certain $\mathbf{Q}^{(k)}$, where k is the iteration index, is

$$\Gamma(\mathbf{Q}, \mathbf{R}_S) \approx \log\det\left(\mathbf{I} + \frac{1}{\sigma_p^2}\mathbf{R}_S^{1/2}\mathbf{Q}^{(k)}\mathbf{R}_S^{1/2}\right) + \mathrm{Tr}\left(\mathbf{A}^{(k)}(\mathbf{Q} - \mathbf{Q}^{(k)})\right), \qquad (15)$$

$$\mathbf{A}^{(k)} = \mathbf{R}_S^{1/2}\left(\sigma_p^2\mathbf{I} + \mathbf{R}_S^{1/2}\mathbf{Q}^{(k)}\mathbf{R}_S^{1/2}\right)^{-1}\mathbf{R}_S^{1/2}. \qquad (16)$$

Using the approximation (15), the left side of (8) in P-I can be written in a compact form as

$$\max_{\Delta\mathbf{R}_S \in \mathcal{U}} \mathrm{Tr}\left(\mathbf{A}^{(k)}\mathbf{Q}\right) - R_L^{(k)} \qquad (17)$$

where

$$R_L^{(k)} = R_L - \log\det\left(\mathbf{I} + \frac{1}{\sigma_p^2}\mathbf{R}_S^{1/2}\mathbf{Q}^{(k)}\mathbf{R}_S^{1/2}\right) + \mathrm{Tr}\left(\mathbf{A}^{(k)}\mathbf{Q}^{(k)}\right). \qquad (18)$$

P-I is now tractable with respect to $\mathbf{Q}$. Nevertheless, maximizing over $\Delta\mathbf{R}_S$ for (17) becomes cumbersome.

From the reasoning at the end of Section 3, choosing $\Delta\mathbf{R}_S = \epsilon\mathbf{I}$ would produce the most conservative solution for any p. That is, the correlation matrix

$$\bar{\mathbf{R}}_S \triangleq \hat{\mathbf{R}}_S + \epsilon\mathbf{I} \qquad (19)$$

represents an upper bound for $\mathbf{R}_S$ in its deterministic set defined by $\hat{\mathbf{R}}_S + \mathcal{U}$, i.e., $\bar{\mathbf{R}}_S \succeq \mathbf{R}_S$. Thus, (17) is guaranteed to be non-positive when

$$\Gamma_{\max}^{(k)} \triangleq \mathrm{Tr}(\bar{\mathbf{A}}^{(k)}\mathbf{Q}) - \bar{R}_L^{(k)} \qquad (20)$$

is non-positive, where $\bar{\mathbf{A}}^{(k)}$ and $\bar{R}_L^{(k)}$ are given by (16) and (18) with $\mathbf{R}_S$ replaced by $\bar{\mathbf{R}}_S$ defined in (19).

Algorithm 1 summarizes the steps to solve P-I using the local linearization approach. We shall start with a small value of $\mathbf{Q}$ to begin the iteration.

Based on [10, Proposition 8.6.13] and the fact that $\log(\cdot)$ is a strictly increasing function, the log-det function is strictly increasing with respect to $\mathbf{Q}$. Consequently the proposed local linear approximation will serve as its upper bound and a correct solution is ensured after the iterations.

When there are several PUs, we can modify Algorithm 1 to handle the additional interference constraints through performing steps 2 and 3 for each PU. Furthermore in step 4, these interference constraints in the form of (20) are used to solve P-I. The algorithm terminates when the required rate accuracy for each PU is satisfied.

5. NUMERICAL RESULTS

We shall provide some numerical simulations that illustrate the performance of the proposed designs. The elements of the channel matrices $\mathbf{T}$ and $\mathbf{S}$ are independently drawn from a circularly symmetric complex Gaussian (CSCG) distribution with zero mean and unit variance. We generate the inaccurate CSI matrix $\hat{\mathbf{R}}_S$ by

$$\hat{\mathbf{R}}_S = (\mathbf{S} - \delta\mathbf{S})^\dagger(\mathbf{S} - \delta\mathbf{S}) \qquad (21)$$

$$= \underbrace{\mathbf{S}^\dagger\mathbf{S}}_{\mathbf{R}_S} - \underbrace{(\delta\mathbf{S}^\dagger\mathbf{S} + \mathbf{S}^\dagger\delta\mathbf{S} - \delta\mathbf{S}^\dagger\delta\mathbf{S})}_{\Delta\mathbf{R}_S} \qquad (22)$$

where the elements of the uncertainty $\delta\mathbf{S}$ are also drawn independently from a zero mean CSCG distribution. The variance of $\delta\mathbf{S}$ is adjusted such that the Schatten norm of the second term in (22), $\Delta\mathbf{R}_S$, is not bigger than ϵ. To avoid trivial uncertainty radius we do not allow $\epsilon \geq \|\mathbf{R}_S\|_{Sp}$ to ensure a valid $\hat{\mathbf{R}}_S$. The uncertainty radius is set as $\epsilon = w\|\mathbf{R}_S\|_{S\infty}$ [11], where $w \in (0, 1)$ controls the amount of uncertainty in $\mathbf{S}$. The antenna settings for the SU and PU are $M_s = N_s = 3$ and $M_p = N_p = 6$. The noise power values are $\sigma_s^2 = \sigma_p^2 = 1$. The results shown are the averages over 1000 independent realizations of $\mathbf{T}$ and $\mathbf{S}$.

Algorithm 1 Iterative robust solution for P-I using the local linearization

Requirement: P^{tot}, P^{max}, P_q^{ant} for $q = 1, \cdots, M_s$, R_L, ϵ, and rate accuracy θ

Initialization: Setting $\mathbf{Q}^{(1)} = \gamma\mathbf{I}_{M_s}$ with $\gamma \approx 0$ [24], counter $k = 0$, and obtaining $\Gamma(\mathbf{Q}^{(1)}, \bar{\mathbf{R}}_S)$

repeat

1. $k = k + 1$.
2. At $\mathbf{Q}^{(k)}$, evaluate $\bar{\mathbf{A}}^{(k)}$ using (16) and $\bar{R}_L^{(k)}$ using (18) by replacing $\mathbf{R}_S$ with $\bar{\mathbf{R}}_S$ defined in (19).
3. Obtain $\Gamma_{\max}^{(k)}$ from (20).
4. Solve P-I by replacing the left side of (8) with $\Gamma_{\max}^{(k)}$ and obtain $\mathbf{Q}^{(k+1)}$.
5. Compute $\Gamma(\mathbf{Q}^{(k+1)}, \bar{\mathbf{R}}_S)$ using (6).

until $|\Gamma(\mathbf{Q}^{(k+1)}, \bar{\mathbf{R}}_S) - \Gamma(\mathbf{Q}^{(k)}, \bar{\mathbf{R}}_S)| \leq \theta$.

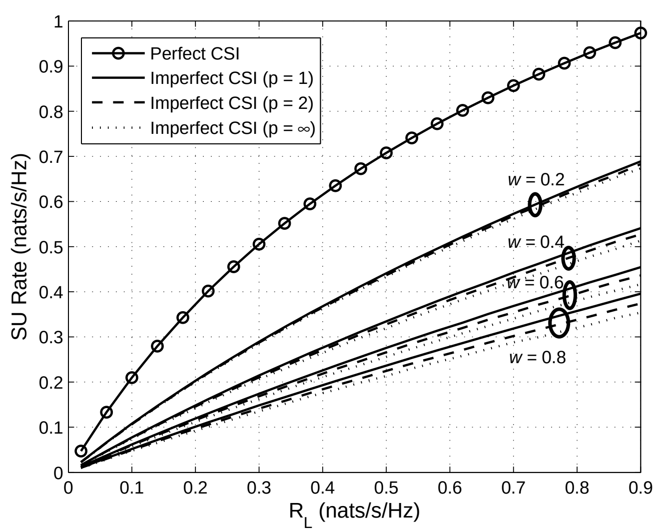

Fig. 2. Rates for SU from the design using low INR approximation versus the leakage rate limit R_L. The results for imperfect interference CSI at $p = 1, 2$, and ∞ are shown. The settings of antennas are $M_s = N_s = 3$ and $M_p = N_p = 6$. The results are generated by the average of 1000 realizations for $\mathbf{T}$ and $\mathbf{S}$ for different values of w.

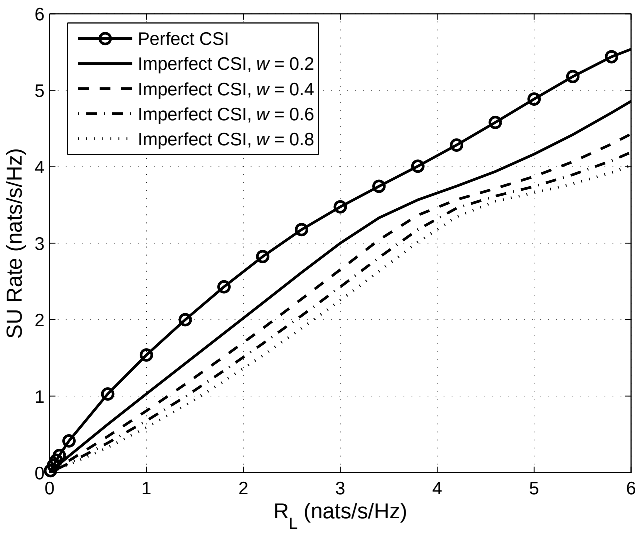

Fig. 3. Rates for SU from the iterative linearization solution for different leakage rate limit R_L and amount of interference CSI uncertainty w. The results are generated by the average of 1000 realizations for $\mathbf{T}$ and $\mathbf{S}$. The settings for antennas are $M_s = N_s = 3$ and $M_p = N_p = 6$.

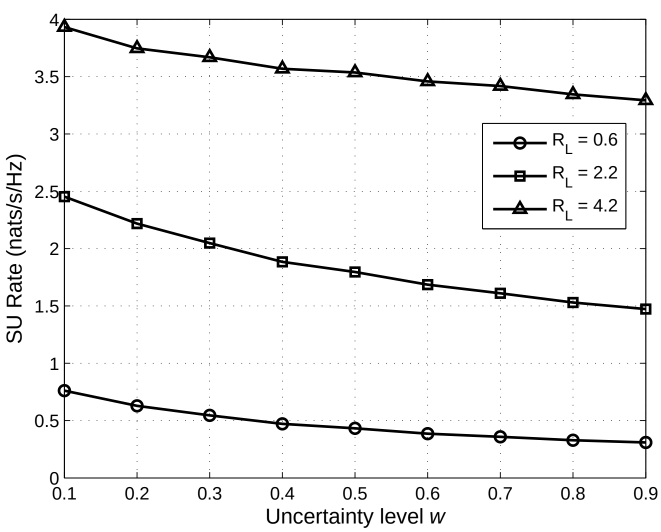

Fig. 4. Rates for SU from the iterative linearization solution for different uncertainty levels $w = \epsilon/\|\mathbf{R}_S\|_{S\infty}$ and leakage rate limit R_L. The results are generated by the average of 1000 realizations for $\mathbf{T}$ and $\mathbf{S}$. The settings for antennas are $M_s = N_s = 3$ and $M_p = N_p = 6$.

We first examine the performance of the precoder design under the low INR condition presented in Section 3. The power settings in the set $\mathcal{Q}$ are $P^{tot} = \sigma_p^2 = 1$, $P^{max} = P_1^{ant} = 0.6$, and $P_q^{ant} = 0.3$ for $q = 2, \cdots, M_s$. Fig. 2 shows the achieved SU rate as the leakage rate limit R_L in nats/s/Hz increases. The result when the interference channel is perfectly known is shown as a reference. When the amount of uncertainty in interference CSI increases (increasing w), the performance is further away from the perfect CSI scenario as expected. The reduction in performance seems to be more sensitive at small uncertainty than large. With respect to different p values for the Schatten norm used in defining the uncertainty set, the difference is more obvious at large amount of uncertainty and high leakage rate limit.

Next, we look at the behavior of the precoder design using the iterative linearization method summarized in Algorithm 1. The SU-Tx power settings are $P^{tot} = 10$, $P^{max} = P_1^{ant} = 6$, and $P_q^{ant} = 5$ for $q = 2, \cdots, M_s$. Fig. 3 illustrates the achieved SU rate at different values of R_L and several levels of the interference CSI uncertainty, with the order of the Schatten norm for CSI uncertainty set fixed to $p = \infty$. We also include the result when the interference CSI is perfectly known for reference purpose. The proposed design yields a solution that follows very well with the ideal solution with perfect CSI. Similar to Fig. 2, higher amount of CSI uncertainty (increasing w) would reduce the performance of SU.

Fig. 4 illustrates the behavior of the SU rate for a given R_L as the amount of CSI uncertainty increases. The simulation setting is similar to that in Fig. 3. The SU rate is more sensitive to the CSI uncertainty level when it is smaller. The SU rate increases with R_L as expected.

The solution using the low INR approximation has similar complexity as the algorithm in [12] for the uncertainty set defined by $p = 2$. At $R_L = 0.9$ nats/s/Hz and $w = 0.2$ the computation time for the proposed algorithm is 1.5 times higher. The required time for the iterative linearization approach is 6.5 times higher for $\theta = 10^{-4}$, where the power settings are similar to that in Fig. 2.

6. CONCLUSION

In this paper, we have proposed two precoder solutions that optimize the performance of SU in a MIMO CR network, where the CSI from SU to PU is partially known and LR metric is used to ensure the QoS of PU. We employed the Schatten norm to characterize the CSI uncertainty. The first solution simplifies the LR metric under low INR condition and applies the Lagrange dual to obtain a tractable and compact interference constraint for the optimization problem to reach a solution. The second exploits linearization through the Taylor series expansion and uses iteration to solve for a solution. Numerical simulations have shown the validity of the proposed techniques and their behaviors as the leakage rate limit and the amount of uncertainty vary.

7. REFERENCES

[1] A. J. Goldsmith, S. A. Jafar, I. Marić, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894-914, May 2009.

[2] Federal Communication Commission Spectrum Policy Task Force, "Report of the Spectrum Efficiency Working Group," *Technical Report 02-135*, no. November, 2002.

[3] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88-102, Feb. 2008.

[4] X. Kang, H. K. Garg, Y.-C. Liang, and R. Zhang, "Optimal power allocation for OFDM-based cognitive radio with new

primary transmission protection criteria," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2066-2075, Jun. 2010.

[5] K. Cumanan, R. Zhang, and S. Lambotharan, "A new design paradigm for MIMO cognitive radio with primary user rate constraint," *IEEE Commun. Lett.*, vol. 16, no. 5, pp. 706-709, May 2012.

[6] F. Gao, R. Zhang, Y.-C. Liang, and X. Wang, "Design of learning-based MIMO cognitive radio systems," *IEEE Veh. Technol.*, vol. 59, no. 4, pp. 1707-1720, Feb. 2010.

[7] H. Yi, "Nullspace-based secondary joint transceiver scheme for cognitive radio MIMO networks using second-order statistics," in *Proc. IEEE ICC*, Cape Town, South Africa, May 2010, pp. 1-5.

[8] Y. Noam, A. Manolakos, and A. J. Goldsmith, "Null space learning with interference feedback for spatial division multiple access," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, pp. 5699-5715, Jul. 2014.

[9] A. Manolakos, Y. Noam, and A. J. Goldsmith, "Interference due to null space mismatch in cooperative multipoint MIMO cellular networks," in *Proc. IEEE ICC*, Sydney, NSW, Jun. 2014, pp. 5148-5153.

[10] D. S. Bernstein, *Matrix Mathematics: Theory, Facts, Formulas*, 2nd ed. Princeton, NJ, USA: Princeton Univ. Press, 2009.

[11] J. Wang, M. Bengtsson, B. Ottersten, and D. P. Palomar, "Robust MIMO precoding for several classes of channel uncertainty," *IEEE Trans. Signal Process.*, vol. 61, no. 12, pp. 3056-3070, Jun. 2013.

[12] K. Cumanan, R. krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for multiuser cognitive radios using worst-case performance optimization," in *Proc. 42nd Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Oct. 2008, pp. 378-382.

[13] J. Wang and D. P. Palomar, "Worst-case robust MIMO transmission with imperfect channel knowledge," *IEEE Trans. Signal Process.*, vol. 57, no. 8, pp. 3086-3100, Aug. 2009.

[14] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4871-4881, Jul. 2009.

[15] I. Wajid, M. Pesavento, Y. C. Eldar, and D. Ciochina, "Robust downlink beamforming with partial channel state information for conventional and cognitive radio networks," *IEEE Trans. Signal Process.*, vol. 61, no. 14, pp. 3656-3670, Jul. 2013.

[16] M. H. Al-Ali and K. C. Ho, "Transmit precoding in underlay MIMO cognitive radio with unavailable or imperfect knowledge of primary interference channel," submitted for publication.

[17] M. Pei, A. Mukherjee, A. L. Swindlehurst, and J. Wei, "Rank minimization designs for underlay MIMO cognitive radio networks with completely unknown primary CSI," in *Proc. IEEE GLOBECOM*, Anaheim, CA, Dec. 2012, pp. 1150-1155.

[18] J. Mao, J. Gao, Y. Liu, G. Xie, and X. Li, "Power allocation over fading cognitive MIMO channels: An ergodic capacity perspective," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1162-1173, Jan. 2012.

[19] S.-M. Cai and Y. Gong, "Cognitive beamforming for throughput maximization with statistical cross channel state information," *IEEE Commun. Lett.*, vol. 18, no. 11, pp. 2031-2034, Aug. 2014.

[20] K.-Y. Wang, N. Jacklin, Z. Ding, and C.-Y. Chi, "Robust MISO transmit optimization under outage-based QoS constraints in two-tier heterogeneous networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 4, pp. 1883-1897, Apr. 2013.

[21] D. P. Palomar, "Unified framework for linear MIMO transceivers with shaping constraints," *IEEE Commun. lett.*, vol. 8, no. 12, pp. 697-699, Dec. 2004.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[23] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," Mar. 2014 [Online]. Available: http://cvxr.com/cvx

[24] M. Fazel, H. Hindi, and S. P. Boyd, "Log-det heuristic for matrix rank minimization with applications to Hankel and Euclidean distance matrices," in *Proc. Amer. Control Conf.*, Denver, CO, Jun. 2003, pp. 2156-2162.