COPRIME ARRAY ADAPTIVE BEAMFORMING BASED ON COMPRESSIVE SENSING VIRTUAL ARRAY SIGNAL

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ABSTRACT

In this paper, we propose a novel adaptive beamforming algorithm for coprime array by compressive sensing the virtual uniform linear array signal. Based on the idea of coprime sampling, a much longer virtual uniform linear array can be generated from a coprime array. With a compressive sensing matrix, a connection can be built between the coprime array with fewer physical sensors and the virtual uniform linear array with much more virtual sensors. Hence, the proposed adaptive beamforming algorithm takes full advantage of the longer virtual array. The performance increment provided by the virtual array is much larger than the performance loss due to the introduced compressive sensing. Hence, the beamformer using the virtual array is expected to obtain much better performance than those using the coprime array directly. Simulation results demonstrate the effectiveness of the proposed adaptive beamforming algorithm.

Index Terms— Adaptive beamforming, compressive sensing, coprime array, virtual array.

1. INTRODUCTION

Coprime sampling utilizes a pair of coprime factors to undersample the signal, and its sparse characteristic is expected to be found broad applications in system identification, target location, urban radar processing and seismic imaging, etc [1–6]. More recently, coprime array was proposed as a typical implementation of coprime sampling, although it is a *de facto* special sparse array. Based on the property of coprime number, a much longer virtual linear array can be obtained by deriving the difference coarray of a coprime array [7]. Generally speaking, longer array means higher resolution and stronger interference suppression capability. For this reason, coprime array has found wide applications and a series of research results have been reported in the past years [7–16].

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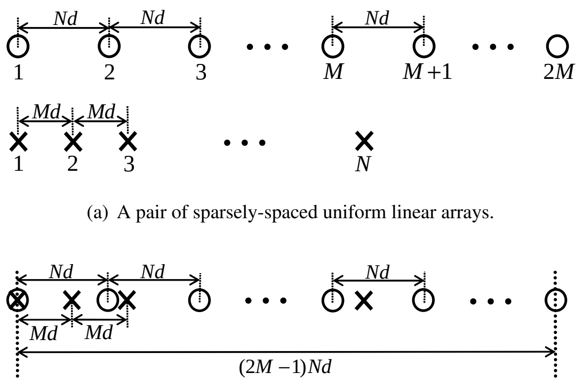
However, most existing researches were focused on taking the advantages of increased degrees-of-freedom (DOF) provided by the generated virtual uniform linear array for direction-of-arrival (DOA) estimation. To the best of our knowledge, few efforts have been put on adaptive beamforming research specially for the coprime array. Since coprime array is a kind of sparse non-uniform linear array, it will suffer performance degradation by directly using the adaptive beamforming algorithms designed for the general array. Therefore, how to design an adaptive beamforming algorithm for the coprime array remains a challenging task.

Motivated by the compressive sensing applications in radar and array signal processing [17–21], in this paper, we propose a novel adaptive beamforming algorithm for the coprime array by compressive sensing virtual array signal. In detail, a compressive sensing kernel, whose dimension is determined by the given coprime array structure, is created to compress the virtual array signal by a random projection. With such a compressive sensing matrix, the connection between the original coprime array and the generated virtual array is built. By calculating the compressed virtual covariance matrix via spatial smoothing, the adaptive beamformer corresponding to the coprime array can be obtained. The performance increment provided by the virtual array is much larger than the performance loss due to the introduced compressive sensing. Hence, the proposed adaptive beamforming algorithm via compressive sensing the virtual array will be better than those directly obtained from the coprime array. Simulation results demonstrate the effectiveness of the proposed adaptive beamforming algorithm specially designed for the coprime array.

2. COPRIME ARRAY SIGNAL MODEL

We consider a pair of sparsely-spaced uniform linear arrays illustrated in Fig. 1(a). The first array has $2M$ sensors spaced Nd apart, and the second array has N sensors spaced Md apart, where M and N are coprime numbers satisfying $M < N$. Without loss of generality, d is chosen to be a half-wavelength. According to the property of coprime number,



(a) A pair of sparsely-spaced uniform linear arrays.

(b) Aligned coprime array.

Fig. 1. The coprime array structure.

the sensors of each array do not overlap except the first one served as the reference when these two arrays are aligned, as shown in Fig. 1(b). Hence, the coprime array actually consists of $2M + N - 1$ physical sensors.

Based on the above $(2M, N)$ coprime array structure, a virtual uniform linear array can be generated by calculating the difference coarray [8]

$$L(m, n) = \pm(Mn - Nm), \tag{1}$$

with $0 \leq m \leq 2M - 1$ and $0 \leq n \leq N - 1$, where a set of consecutive integers from $-MN$ to MN can be obtained in $L(m, n)$. Therefore, a much longer virtual uniform linear array can be generated at locations $\{-MNd, -(MN - 1)d, \cdots, 0, \cdots, (MN - 1)d, MNd\}$. The generated virtual uniform linear array can be used to process the incident signal with far fewer physical sensors, and its computational efficiency is significantly improved as compared to that using actual uniform linear array of the same size.

However, the beamforming weight vector is actually weighted to the physical sensors of coprime array instead of to the virtual sensors of virtual array. That is to say, the beamformer output of coprime array is still given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k), \tag{2}$$

where $\mathbf{w} \in \mathbb{C}^{2M+N-1}$ is the beamforming weight vector, $\mathbf{x}(k) \in \mathbb{C}^{2M+N-1}$ is the coprime array observation vector at time k, and $(\cdot)^H$ is the Hermitian transpose. The array observation vector $\mathbf{x}(k)$ can be modeled as

$$\mathbf{x}(k) = \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k), \tag{3}$$

where $\mathbf{x}_s = \mathbf{a}(\theta_s)s(k)$, $\mathbf{x}_i(k)$ and $\mathbf{x}_n(k)$ are the desired signal, interference and noise, respectively. These components are assumed to be statistically independent to each other. Here, $\mathbf{a}(\theta_s) \in \mathbb{C}^{2M+N-1}$ is the coprime array steering vector of the desired signal waveform $s(k)$ from the direction θ_s.

The coprime array output signal-to-interference-plus-noise ratio (SINR) is defined as

$$\text{SINR} = \frac{\sigma_s^2 \left|\mathbf{w}^H \mathbf{a}(\theta_s)\right|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \tag{4}$$

where $\mathbf{R}_{i+n} = E\{\left(\mathbf{x}_i(k) + \mathbf{x}_n(k)\right)\left(\mathbf{x}_i(k) + \mathbf{x}_n(k)\right)^H\} \in \mathbb{C}^{(2M+N-1)\times(2M+N-1)}$ is the interference-plus-noise covariance matrix, and σ_s^2 is the desired signal power. Maximizing the output SINR (4) is equivalent to minimizing the output variance while keeping the desired signal distortionless pass as

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_s) = 1, \tag{5}$$

which solution

$$\mathbf{w} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_s)}{\mathbf{a}^H(\theta_s) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_s)} \tag{6}$$

is the so-called MVDR beamformer. In the practical applications, the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k) \mathbf{x}^H(k) \tag{7}$$

is usually adopted because the exact $\mathbf{R}_{i+n}$ is not available, where K is the number of snapshots. The resulted adaptive beamformer is called as the sample matrix inversion (SMI) beamformer.

3. THE PROPOSED ALGORITHM

In this section, a novel adaptive beamforming algorithm specially for the coprime array is proposed. The basic idea is to compressively sample the longer virtual uniform linear array by random projection, and then the sample compressed virtual covariance matrix can be calculated by using spatial smoothing. With the compressed virtual steering vector together, the MVDR adaptive beamformer is presented.

In order to obtain the equivalent received signal vector of the virtual array, we first vectorize the sample covariance matrix $\hat{\mathbf{R}}$ in (7) as [8]

$$\mathbf{r} = \text{vec}(\hat{\mathbf{R}}) = \mathbf{V}\mathbf{p} + \sigma_n^2 \text{vec}(\mathbf{I}), \tag{8}$$

where $\mathbf{V} = \left[\mathbf{a}^*(\theta_s) \otimes \mathbf{a}(\theta_s), \mathbf{a}^*(\varphi_1) \otimes \mathbf{a}(\varphi_1), \cdots, \mathbf{a}^*(\varphi_q) \otimes \mathbf{a}(\varphi_q)\right] \in \mathbb{C}^{(2M+N-1)^2 \times (q+1)}$, $\mathbf{p} = [\sigma_s^2, \sigma_1^2, \cdots, \sigma_q^2]^T$ consists of the power of the sources with DOAs $\{\theta_s, \varphi_1, \cdots, \varphi_q\}$, σ_n^2 is the noise power, and $\mathbf{I}$ is the identity matrix. Here, the vectorization operator $\text{vec}(\cdot)$ stacks each column of the matrix one after another. With such a vectorization, the vector $\mathbf{r}$ is equivalent to the received signal of the array which geometry is given by the steering matrix $\mathbf{V}$.

By removing the repeated rows in $\mathbf{V}$ and sorting them, a $(2MN+1) \times (q+1)$ dimensional matrix $\bar{\mathbf{V}}$ is generated corresponding to the virtual uniform linear array generated by coprime array, which the j-th row is corresponding to the sensor located at $(-MN-1+j)d$. In the sequel, we divide these $2MN+1$ sensors into $MN+1$ overlapped subarrays, each of which consists of $MN+1$ sensors. The i-th subarray with $MN+1$ sensors located at $\{(-i+1+g)d,\ 0 \leq g \leq MN\}$ gives a new vector as

$$\bar{\mathbf{z}}_i = \bar{\mathbf{V}}_i \mathbf{p} + \sigma_n^2 \bar{\mathbf{I}}_i, \tag{9}$$

where $\bar{\mathbf{V}}_i \in \mathbb{C}^{(MN+1)\times(q+1)}$ is the steering matrix corresponding to the $(MN+2-i)$-th through $(2MN+2-i)$-th rows of $\bar{\mathbf{V}}_i$, and $\bar{\mathbf{I}}_i$ is a zero vector except an element 1 at i-th position. In such a case, $\bar{\mathbf{z}}_i$ is the received signal vector of the i-th subarray, and now, the signal can be considered to be processed by a virtual array.

However, when using the adaptive beamforming technique, the optimized beamforming weight vector is weighted to the received signal vector of the coprime array instead of that of the virtual array. Hence, we consider to compressively sample the virtual array signal vector $\bar{\mathbf{z}}_i \in \mathbb{C}^{MN+1}$ via a compressive sensing kernel $\mathbf{\Phi} \in \mathbb{C}^{(2M+N-1)\times(MN+1)}$, which connects the coprime array with $2M+N-1$ physical sensors and the virtual array with $MN+1$ virtual sensors. Then, the compressive measurement is given by

$$\bar{\mathbf{z}}_i^{\mathbf{c}} = \mathbf{\Phi} \bar{\mathbf{z}}_i, \tag{10}$$

where the compressive sensing kernel $\mathbf{\Phi}$ can be chosen as a random one, such as Gaussian or Bernoulli kernels. The random compressive sensing kernel meets the incoherence requirement in the compressive sensing theory [22]. With such a random projection, the longer virtual array signal vectors $\{\bar{\mathbf{z}}_i \in \mathbb{C}^{(MN+1)}, i = 1, \cdots, MN+1\}$ are compressed to a set of low-dimensional vectors $\{\bar{\mathbf{z}}_i^{\mathbf{c}} \in \mathbb{C}^{(2M+N-1)}, i = 1, \cdots, MN+1\}$, which now can be used for spatial smoothing.

By averaging over the covariance matrices of the $MN+1$ subarrays, the spatially smoothed compressed covariance matrix yields

$$\mathbf{R_s} = \frac{1}{MN+1} \sum_{i=1}^{MN+1} \bar{\mathbf{z}}_i^{\mathbf{c}} \bar{\mathbf{z}}_i^{\mathbf{c}H}, \tag{11}$$

which can be further expressed as

$$\mathbf{R_s} = \hat{\mathbf{R}}_{\mathbf{c}}^2, \tag{12}$$

where $\hat{\mathbf{R}}_{\mathbf{c}} \in \mathbb{C}^{(2M+N-1)\times(2M+N-1)}$ is the sample compressed virtual covariance matrix. Compared with the sample covariance matrix $\hat{\mathbf{R}}$ in (7) which is directly calculated from the coprime array received signal $\{\mathbf{x}(k), k = 1, \cdots, K\}$, the sample compressed virtual covariance matrix $\hat{\mathbf{R}}_{\mathbf{c}}$ in (12) is calculated from the compressed virtual array received signal $\{\bar{\mathbf{z}}_i^{\mathbf{c}}, i = 1, \cdots, MN+1\}$. The benefit from the longer virtual array is enough to compensate the performance loss due to compressive sensing. Hence, more performance advantage can be obtained by using the sample compressed virtual covariance matrix $\hat{\mathbf{R}}_{\mathbf{c}}$ than the sample covariance matrix $\hat{\mathbf{R}}$.

Similarly, the virtual array steering vector of the desired signal $\bar{\mathbf{v}}(\theta_s) \in \mathbb{C}^{MN+1}$, the corresponding column of $\bar{\mathbf{V}}_i$ in (9), should also be compressed by the same compressive sensing kernel $\mathbf{\Phi}$ as

$$\mathbf{b}(\theta_s) = \mathbf{\Phi} \bar{\mathbf{v}}(\theta_s), \tag{13}$$

where $\mathbf{b}(\theta_s) \in \mathbb{C}^{2M+N-1}$ has the same dimension as the coprime array steering vector. Without loss of generality, a virtual array with the sensors located at $\{0, d, 2d, \cdots, MNd\}$ is adopted to calculate the virtual array steering vector $\bar{\mathbf{v}}(\theta_s)$.

Based on the compressed virtual uniform linear array, the output SINR defined in (4) can be rewritten as

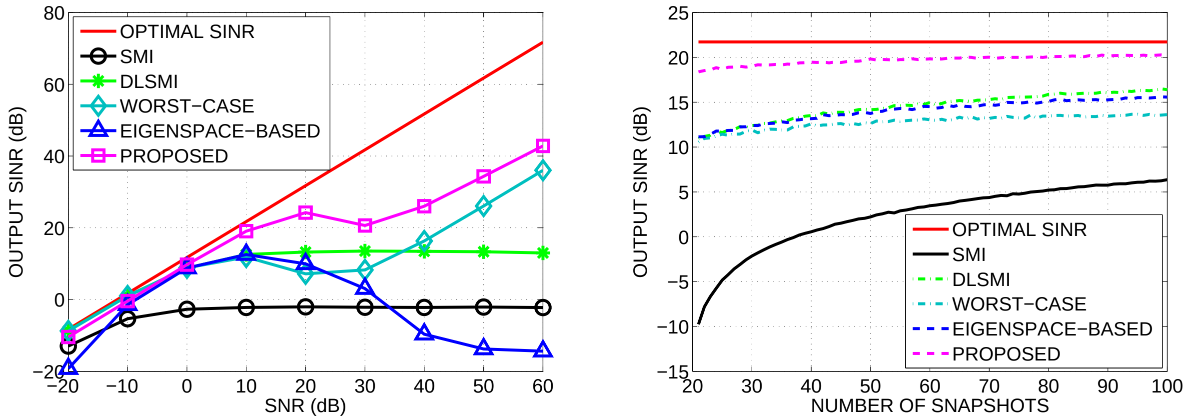
$$\mathrm{SINR}_{\mathbf{c}} = \frac{\sigma_s^2 \left| \mathbf{w}^H \mathbf{\Phi} \bar{\mathbf{v}}(\theta_s) \right|^2}{\mathbf{w}^H \mathbf{R}_{i+n}^{\mathbf{c}} \mathbf{w}}, \tag{14}$$

where $\mathbf{R}_{i+n}^{\mathbf{c}} = \sum_{q=1}^{Q} \sigma_q^2 \mathbf{\Phi} \bar{\mathbf{v}}(\varphi_q) \bar{\mathbf{v}}^H(\varphi_q) \mathbf{\Phi}^H + \sigma_n^2 \mathbf{\Phi} \mathbf{\Phi}^H$ is the theoretical compressed virtual interference-plus-noise covariance matrix. It is usually unavailable, and can be replaced by the sample compressed virtual covariance matrix $\hat{\mathbf{R}}_{\mathbf{c}}$ in (12). Following the MVDR beamforming principle, the proposed adaptive beamforming weight vector is given by

$$\mathbf{w_c} = \frac{\hat{\mathbf{R}}_{\mathbf{c}}^{-1} \mathbf{\Phi} \bar{\mathbf{v}}(\theta_s)}{\bar{\mathbf{v}}^H(\theta_s) \mathbf{\Phi}^H \hat{\mathbf{R}}_{\mathbf{c}}^{-1} \mathbf{\Phi} \bar{\mathbf{v}}(\theta_s)}. \tag{15}$$

Unlike the MVDR beamformer in (6) for the general array, the proposed adaptive beamfomer is specially designed for the coprime array, where extra information can be obtained from a longer virtual uniform linear array. Meanwhile, the designed beamformer weight vector $\mathbf{w_c}$ is weighted to the physical sensors in coprime array instead of the virtual sensors in virtual array. Furthermore, the required compressed virtual interference-plus-noise covariance matrix can be reconstructed to remove the desired signal components in the current sample compressed virtual covariance matrix $\hat{\mathbf{R}}_{\mathbf{c}}$ [23, 24], which can be used to further improve the output SINR performance especially at high SNRs.

In the previous research of coprime array for DOA estimation, the computational complexity is $\mathcal{O}\big((MN)^3\big)$. Benefit from compressive sensing, the computational complexity of the proposed adaptive beamforming algorithm is $\mathcal{O}\big((2M+N)(MN)^2\big)$, which is dominated by spatial smoothing process. It is sightly larger than the general MVDR beamforming algorithm, which computational complexity is $\mathcal{O}\big((2M+N)^3\big)$.



(a) (b)

Fig. 2. Output performance comparison. (a) Output SINR versus SNR; (b) Output SINR versus number of snapshots.

4. NUMERICAL SIMULATION

In our simulations, a coprime array with coprime factors $M = 5$ and $N = 11$ is deployed, which equals to $2M+N-1 = 20$ physical sensors. The desired signal is assumed to be a far-field narrowband waveform from the direction $\theta_s = 5°$, and two interferences are assumed to have DOAs $\varphi_1 = -20°$ and $\varphi_2 = 30°$, respectively. The interference-to-noise ratio (INR) in each sensor is set to be 30dB. The additive noise is modeled as a zero-mean complex white Gaussian process. For each scenario, 1,000 Monte-Carlo runs are performed.

The proposed adaptive beamforming algorithm is compared to the SMI beamformer, the diagonal loading SMI (DLSMI) beamformer, the worst-case performance-based beamformer and the eigenspace-based beamformer. Unlike the proposed one, all other beamformers are optimized from the sample covariance matrix $\hat{\mathbf{R}}$ in (7) directly. In the proposed beamforming algorithm, the complex-valued compressive sensing matrix $\mathbf{\Phi}$ satisfies the orthonormal assumption, namely, $\mathbf{\Phi}\mathbf{\Phi}^H = \mathbf{I}$. The entries of $\mathbf{\Phi}$ are generated from an independent and identically distributed (i.i.d.) zero-mean random Gaussian distribution $\mathcal{CN}(0, \frac{1}{\sqrt{2M+N-1}})$. The diagonal loading factor $\xi = 10\sigma_n^2$ is adopted in the DLSMI beamformer, and the norm upper-bound of the signal steering vector mismatch is chosen to be $\varepsilon = 3$ in the worst-case beamformer. The desired signal direction is assumed to be exactly known by all tested beamforming algorithms. In addition, the optimal SINR (4) is also presented as the benchmark, which is calculated from the exact interference-plus-noise covariance matrix and the desired signal steering vector.

Fig. 2(a) compares the output SINR versus input SNR, where the number of snapshots is fixed to be $K = 30$. Because the proposed beamforming algorithm exploits a much longer virtual array instead of the original coprime array, its output SINR has shown significant improvement compared to the other beamformers especially at high SNRs. In Fig. 2(b), the output SINRs for the tested methods are illustrated against the number of snapshots K, where the SNR in each sensor is fixed to be 10 dB. It is obvious that the proposed beamforming algorithm has much faster convergence rate than others. By comparing with the SMI beamformer only, we can see that the performance loss due to compressive sensing is much less then the performance increment provided by the longer virtual array. Although the proposed beamforming algorithm performs better than others, we note that the desired signal self-nulling phenomenon is still severe especially at high SNRs. In order to avoid or alleviate the self-nulling phenomenon, the interference covariance matrix reconstruction method [23,24] is a good candidate.

5. CONCLUSION

In this paper, we proposed a novel adaptive beamforming algorithm specially designed for the coprime array. By using the property of coprime array, a much longer virtual uniform linear array can be generated from a coprime array. In order to design an adaptive beamformer for the coprime array, a random compressive sensing kernel is adopted to compressively sample the virtual uniform linear array. Hence, a connection is built between the coprime array and the virtual array. By calculating the sample compressed virtual covariance matrix via spatial smoothing, the adaptive beamformer is designed based on the MVDR beamforming principle. Simulation results demonstrate that the proposed beamformer outperforms the others which use the coprime array received signal directly. The main reason is that the performance increment benefited from the much longer virtual uniform linear array is much larger than the performance loss due to compressive sensing.

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