NON-ASYMPTOTIC PERFORMANCE BOUNDS OF EIGENVALUE BASED DETECTION OF SIGNALS IN NON-GAUSSIAN NOISE

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ABSTRACT

The core component of a cognitive radio is its detector. When a device is equipped with multiple antennas, the detection method is usually based on an eigenvalue analysis. This paper explores the performance of the most common largest eigenvalue detector, for the case of a narrowband temporally white signal and calibrated receiver noise. In contrast to popular Gaussian assumption, our performance bounds are valid for any signal and noise that belong to the wide class of sub-Gaussian random processes. Moreover, the results are given in closed-form for any finite number of observations and antennas, in contrary to the widespread asymptotic analysis approach.

Index Terms— Sensor array, cognitive radio, random matrix, Chernoff bound, sub-Gaussian random variables

1. INTRODUCTION

The problem of detecting the presence of an unknown wireless signal using a multi-antenna sensor prompted the emerging field of cognitive radio (CR). CR refers to attempts to improve the current underutilization of the limited radio electromagnetic spectrum [1–3]. To achieve this goal radios need to be equipped with smart sensing capabilities that allow them to decide when and how to transmit so as to minimize interference with other signals.

The detection problem is more difficult when the structure of the transmitted signal cannot be utilized by the receiver. This is the case either when the nature of the signal is unknown at the receiver, or when low signal-to-noise-ratio (SNR) due to shadowing, fading, reflection and/or propagation losses do not allow synchronization with the signal's pilot features [4]. All these scenarios are common in CR applications, which forces the receiver to assume a general unknown signal pattern and apply robust blind detection schemes [5, 6].

The use of multiple receive antennas greatly improves receiver performance. It makes it possible to gather more data samples per time interval, exploit the spatial structure of the received signals and apply schemes that are robust to noise uncertainties. In CR applications, a multi-antenna array is usually employed and yields a spatial diversity gain, especially in a multiapath urban environment.

Many recent papers [7–18] have put forward versions of the multi-antenna detector. The differences have to do with the outcome of different assumptions regarding the signal and noise models. Most of these detectors were developed using a generalized likelihood ratio test (GLRT) approach while assuming a Gaussian noise and signal, which leads to random matrix analysis, and more specifically to an analysis of the eigenvalues of the sample covariance matrix (SCM). The most basic detector is obtained by assuming a temporally white signal of interest and a calibrated white additive Gaussian noise (AWGN) at the receiver, where the obtained GLRT statistic is the largest eigenvalue of the SCM ($\hat{\lambda}_1$) [7, section III-B]. Here, we focus on the performance of this basic detector, which also acts as a constant false-alarm rate (CFAR) detector when a fixed decision threshold is used.

When analyzing the performance of blind multi-antenna detectors, previous papers have assumed Gaussian receiver noise and transmitted signals. Most of them, for example [12, 15, 19–21], turned to large matrix techniques, which are based on earlier asymptotic results by Johnstone [22] and Baik [23]. For finite sample problems these methods yield a Tracy-Widom approximation for the false-alarm probability (P_{FA}) and a Gaussian approximation for the misdetection probability (P_{MD}). The exact P_{FA} of the $\hat{\lambda}_1$ detector in the presence of AWGN can be calculated using the result in Khatri [24], and [25] provides an efficient way to evaluate it. Tropp [26, Theorem 5.1] presented a friendly upper bound on P_{FA} for any noise distribution, as long as the largest eigenvalue of the instantaneous SCM is upper bounded almost surely. Hence, it cannot be applied to unbounded random variables (RV) such as Gaussian and sub-Gaussian (SG) RVs.

In this paper, we investigate the performance of the $\hat{\lambda}_1$ detector for signals and noise that belong to the more general family of SG RVs. The class of SG RVs contains all RVs whose moment generating function (MGF) is bounded by the MGF of a centered Gaussian RV [27]. This is a convenient and fairly wide class that includes, for example, the centered Gaussian and the Gaussian mixture. It also includes all bounded RVs, and in particular all constant envelope signals such as FM radio, a QPSK modulated signal, a polynomial-phase [28] or a chirp [29, 30], which are all commonly implemented by communication applications and active radars.

The main contributions of this paper are new non-asymptotic upper bounds on false-alarm and misdetection probabilities for the $\hat{\lambda}_1$ detector, for a SG transmitted signal and SG noise. These upper bounds can be used to lower bound the receiver operation characteristic (ROC) curve, hence guaranteeing a minimum level of performance for any practical finite sample problem.

This paper is organized as follows. Section 2 formulates the detection problem and the $\hat{\lambda}_1$ detector. The new non-asymptotic error bounds are introduced and proved in Section 3. Section 4 presents a comparison of the bounds against simulation results. Finally, conclusions are drawn in section 5.

2. PROBLEM FORMULATION

Assume that we have a narrowband signal impinging on a set of M antennas. The received signal at the i-th channel at time t can be described by its complex envelope waveform,

$$x_i(t) = h_i s(t) + v_i(t) \qquad \begin{matrix} i = 1, \dots, M \\ t_0 < t < t_1 \end{matrix} \tag{1}$$

where $s(t)$ is the transmitted signal, which is assumed to follow a zero-mean circular symmetric complex σ_s^2-SG distribution, and to be temporally white. h_i is the complex response of the i-th channel, which is unknown at the receiver, and depends on the electromagnetic scattering and path loss. $v_i(t)$ is the additive zero-mean circular symmetric complex white σ_w^2-SG noise of the i-th channel, where noises of different channels are assumed to be calibrated and uncorrelated. Equation (1) can be expressed in vector form by

$$\boldsymbol{x}(t) = \boldsymbol{h}s(t) + \boldsymbol{v}(t) \quad t_0 < t < t_1 \tag{2}$$

where $\boldsymbol{x}(t) = [x_1(t), \ldots, x_M(t)]^T$ is the received data, $\boldsymbol{h} = [h_1, \ldots, h_M]^T$ is the unknown complex response of the different channels, and $\boldsymbol{v}(t) = [v_1(t), \ldots, v_M(t)]^T$ is a spatially uncorrelated noise vector.

The detector has to decide whether the transmitted signal $s(t)$ exists in the measurement or only noise is sensed. Formally, the detector needs to distinguish between the following two hypotheses:

$$\begin{aligned} &\mathcal{H}_0 : \boldsymbol{x}(t) = \boldsymbol{v}(t) \\ &\mathcal{H}_1 : \boldsymbol{x}(t) = \boldsymbol{v}(t) + \boldsymbol{h}s(t) \end{aligned} \tag{3}$$

The sampled versions of (2) and (3) are given by

$$\boldsymbol{x}[n] = \boldsymbol{h}s[n] + \boldsymbol{v}[n] \quad n = 1, 2, \cdots, N \tag{4}$$

$$\begin{aligned} &\mathcal{H}_0 : \boldsymbol{x}[n] = \boldsymbol{v}[n] \\ &\mathcal{H}_1 : \boldsymbol{x}[n] = \boldsymbol{v}[n] + \boldsymbol{h}s[n] \end{aligned} \tag{5}$$

Without loss of generality, we assume a normalized noise power $\mathbb{E}\left\{|v_i[n]|^2\right\} = 1$, and denote the transmitted power by $\Gamma \triangleq \mathbb{E}\left\{|s[n]|^2\right\}$. The received SNR is given by $\|\boldsymbol{h}\|^2\Gamma$. The $M \times M$ covariance matrix of $\boldsymbol{v}[n]$ is the identity matrix $\boldsymbol{R}_v = \boldsymbol{I}$. We can represent the hypothesis test in (5) using the expected covariance matrix $\boldsymbol{R}_x = \mathbb{E}\left\{\boldsymbol{x}[n]\boldsymbol{x}^H[n]\right\}$:

$$\begin{aligned} &\mathcal{H}_0 : \boldsymbol{x}[n] \sim \mathcal{P}_0(\boldsymbol{I}) \\ &\mathcal{H}_1 : \boldsymbol{x}[n] \sim \mathcal{P}_1\left(\boldsymbol{I} + \Gamma\boldsymbol{h}\boldsymbol{h}^H\right) \end{aligned} \tag{6}$$

where $\mathcal{P}_0(\boldsymbol{R})$ and $\mathcal{P}_1(\boldsymbol{R})$ are centered SG random distributions with covariance matrix $\boldsymbol{R}$. Let us also denote the SCM by

$$\hat{\boldsymbol{R}}_x \triangleq \frac{1}{N}\boldsymbol{X}\boldsymbol{X}^H = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}[n]\cdot\boldsymbol{x}^H[n] = \frac{1}{N}\sum_{n=1}^{N}\hat{\boldsymbol{R}}_x[n] \tag{7}$$

where $\boldsymbol{X} = [\boldsymbol{x}[1], \cdots, \boldsymbol{x}[N]]$ is a $M \times N$ matrix that contains all the samples of the received signals. $\hat{\boldsymbol{R}}_x$ is a random matrix, and in the Gaussian case its distribution is known as the Wishart distribution.

In this paper, we investigate the performance of the largest eigenvalue detector. It is defined by

$$T(\boldsymbol{X}) = \lambda_1\left(\hat{\boldsymbol{R}}_x\right) \triangleq \hat{\lambda}_1 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \delta \tag{8}$$

where $\lambda_1\left(\hat{\boldsymbol{R}}_x\right)$ denotes the largest eigenvalue of $\hat{\boldsymbol{R}}_x$, and δ is the decision threshold.

Let the unitary matrix $\boldsymbol{U} = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_M]$ be a basis of eigenvectors of $\boldsymbol{R}_x$ for $\mathcal{H}_1$ where $\boldsymbol{u}_1 = \frac{\boldsymbol{h}}{\|\boldsymbol{h}\|}$. Multiplying the measurement by $\boldsymbol{U}^H$ we obtain a diagonal model, in which the received signal is described by

$$\boldsymbol{y}[n] = \boldsymbol{U}^H\boldsymbol{x}[n] = \boldsymbol{U}^H\boldsymbol{h}s[n] + \boldsymbol{U}^H\boldsymbol{v}[n] = \boldsymbol{e}_1\|\boldsymbol{h}\|s[n] + \boldsymbol{w}[n] \tag{9}$$

where $\boldsymbol{e}_1 = [1, 0, \cdots, 0]^T$ is a unit vector and $\boldsymbol{w}[n]$ is still a spatially and temporally white zero mean normalized noise vector. Moreover, since the SG distribution is rotation invariant, then $w_i[n]$ is also a σ_w^2-SG RV. The diagonal covariance matrix of the received signal for the $\mathcal{H}_1$ hypothesis is given by

$$\boldsymbol{R}_y = \|\boldsymbol{h}\|^2\Gamma\left[\boldsymbol{e}_1, \boldsymbol{0}, \cdots, \boldsymbol{0}\right] + \boldsymbol{I} \tag{10}$$

The hypothesis test of the diagonal model is given by

$$\begin{aligned} &\mathcal{H}_0 : \boldsymbol{y}[n] = \boldsymbol{w}[n] \\ &\mathcal{H}_1 : \boldsymbol{y}[n] = \boldsymbol{w}[n] + \boldsymbol{e}_1\|\boldsymbol{h}\|s[n] \end{aligned} \tag{11}$$

or using a covariance matrix representation,

$$\begin{aligned} &\mathcal{H}_0 : \boldsymbol{y}[n] \sim \mathcal{P}_0(\boldsymbol{I}) \\ &\mathcal{H}_1 : \boldsymbol{y}[n] \sim \mathcal{P}_1\left(\boldsymbol{I} + \|\boldsymbol{h}\|^2\Gamma\left[\boldsymbol{e}_1, \boldsymbol{0}, \cdots, \boldsymbol{0}\right]\right) \end{aligned} \tag{12}$$

Since $\lambda_1\left(\hat{\boldsymbol{R}}_x\right) = \lambda_1\left(\hat{\boldsymbol{R}}_y\right)$, in terms of performance of the largest eigenvalue detector defined by (8), the diagonal problem defined by (11) and (12) is equivalent to the original problem defined by (5) and (6). Therefore, without loss of generality, we work with the equivalent diagonal model described by (11) and (12), where the input samples of the detector are $\boldsymbol{Y} = [\boldsymbol{y}[1], \cdots, \boldsymbol{y}[N]]$. Note that the transformation $\boldsymbol{U}$ is unknown to the receiver, and it is only used to simplify the analysis of the detector's performance. For easier readability, we omit the y notation from the covariance matrix $\boldsymbol{R} \equiv \boldsymbol{R}_y$, and also denote $\boldsymbol{R}_i \triangleq \boldsymbol{R}|\mathcal{H}_i$. Hence, the misdetection and false-alarm probabilities of the largest eigenvalue detector are defined by

$$\begin{aligned} P_{MD} &\triangleq \mathbb{P}\left\{\lambda_1\left(\hat{\boldsymbol{R}}_1\right) < \delta\right\} \\ P_{FA} &\triangleq \mathbb{P}\left\{\lambda_1\left(\hat{\boldsymbol{R}}_0\right) > \delta\right\} \end{aligned} \tag{13}$$

3. NEW PERFORMANCE BOUNDS

Theorem 1 (First diagonal Chernoff P_{MD} upper bound)**.** *For the detection problem defined by* (11)*, the misdetection probability of the largest eigenvalue detector defined by* (13) *is upper bounded by*

$$P_{MD} \leq \exp\left[-2N\theta\left(1 - \frac{\delta}{1 + \|\boldsymbol{h}\|^2\Gamma}\right) + \frac{1}{2}\sigma_1^2\theta^2\right] \tag{14}$$

where δ is the decision threshold, N is the number of observations per channel, $\boldsymbol{h}$ is the channel vector, Γ is the power of the transmitted signal, $\left(\sigma_1^2, B_1\right)$ are the SE parameters of $\frac{2N}{1+\|\boldsymbol{h}\|^2\Gamma}\hat{\boldsymbol{R}}_1[1,1]$ and

$$\theta = \min\left\{\frac{2N}{\sigma_1^2}\left(1 - \frac{\delta}{1 + \|\boldsymbol{h}\|^2\Gamma}\right), \quad B_1\right\} \tag{15}$$

Theorem 2 (Gershgorin-Chernoff P_{FA} upper bound)**.** *For the detection problem defined by* (11)*, the false-alarm probability of the largest eigenvalue detector defined by* (13) *is upper bounded by*

$$P_{FA} \leq \exp\left(-\frac{N\theta\delta}{\sigma_w^2}\right)\cdot M\cdot 16^{M-1}\left[1 - \left(\frac{M-1}{2}\theta^2 + \theta\right)\right]^{-N} \tag{16}$$

where δ is the decision threshold, N is the number of observations per channel, M is the number of channels, σ_w^2 is the SG parameter of the noise and

$$\theta = \frac{1}{M-1} \cdot \min \left\{ \sqrt{2M-1} - 1, \frac{1 - M - \frac{\delta}{\sigma_w^2} + \sqrt{M^2 + 2M\left(\frac{\delta}{\sigma_w^2}\right)^2 - 2M - \left(\frac{\delta}{\sigma_w^2}\right)^2 + 1}}{\delta/\sigma_w^2} \right\} \tag{17}$$

Remark 1. A practical threshold should be chosen in the range:

$$1 < \delta < 1 + \|\boldsymbol{h}\|^2 \Gamma$$

Remark 2. Since $\sigma_1^2 \propto N$, the P_{MD} bound in (14) decays exponentially as N increases.

Remark 3. It can be shown that the P_{FA} bound in (16) decays exponentially as the number of samples N grows. If the number of antennas M also grows, then exponential decay is guaranteed if $N = \Omega\left(M^2\right)$.

Remark 4. If the noise probability distribution function (PDF) is symmetric, then the P_{FA} bound in (16) above can be reduced by a factor of 4^{M-1}.

Before proving Theorems 1 and 2, we note some necessary properties of SG and sub-exponential (SE) RVs (see [27]):

Lemma 1 (sub-Gaussian and sub-exponential properties)**.** *For X centered σ_x^2-SG RV, Y centered σ_y^2-SG RV, $\alpha \in \mathbb{R}$, $\delta > 0$, if X and Y are independent and $M_X(\theta)$ denotes the MGF of X, then:*

1. $M_X(\theta) \leq \exp\left(\frac{1}{2}\sigma_X^2\theta^2\right)$
2. *αX is a centered $\alpha^2\sigma_X^2$-SG RV.*
3. *$(X+Y)$ is a centered $(\sigma_X^2 + \sigma_Y^2)$-SG RV.*
4. *X^2 is a $\left(\sigma_{X^2}^2, B_{X^2}\right)$-SE RV with μ_{X^2} mean.*
5. $$M_{X^2}(\theta) \leq \left(1 - 2\theta\sigma_X^2\right)^{-1/2} \quad \textit{for} \quad 0 < \theta < \left(2\sigma_X^2\right)^{-1}$$ $$\leq \exp\left\{\theta\mu_{X^2} + \frac{1}{2}\sigma_{X^2}^2\theta^2\right\} \quad \textit{for} \quad \theta < B_{X^2}$$
6. *αX^2 is a $\left(\alpha^2\sigma_{X^2}^2, \frac{B_{X^2}}{\alpha}\right)$-SE RV.*
7. *$(X^2 + Y^2)$ is a $\left(\sigma_{X^2}^2 + \sigma_{Y^2}^2, \min\{B_{X^2}, B_{Y^2}\}\right)$-SE RV.*

Proof of Theorem 1. We start by pointing out that the largest sample eigenvalue $\hat{\lambda}_1$ is always equal or greater than any diagonal element of the SCM, and specifically greater than the first diagonal element $\hat{\boldsymbol{R}}_1[1,1]$. This is easily proved by taking

$$\hat{\boldsymbol{R}}[1,1] = \boldsymbol{e}_1^T \hat{\boldsymbol{R}} \boldsymbol{e}_1 \leq \|\boldsymbol{e}_1\|^2 \hat{\lambda}_1 = \hat{\lambda}_1 \tag{18}$$

As a result,

$$P_{MD} = \mathbb{P}\left\{\lambda_1\left(\hat{\boldsymbol{R}}_1\right) < \delta\right\} \leq \mathbb{P}\left\{\hat{\boldsymbol{R}}_1[1,1] < \delta\right\} \tag{19}$$

where

$$\hat{\boldsymbol{R}}_1[1,1] = \frac{1}{N}\sum_{n=1}^{N} \left|\|\boldsymbol{h}\| s[n] + w_1[n]\right|^2 \tag{20}$$

Since both the signal and noise samples follow a SG distribution, $\frac{2N}{1+\|\boldsymbol{h}\|^2\Gamma}\hat{\boldsymbol{R}}_1[1,1]$ is a $\left(\sigma_1^2, B_1\right)$-SE RV. Using property 5 in Lemma 1 it is straightforward to obtain (14) as the Chernoff bound on the CDF of $\hat{\boldsymbol{R}}_1[1,1]$, with the constraint $\theta < B_1$. Finally, by differentiating the bound in (14) with respect to θ, we can find the optimal value of θ which is given in (15). □

Proof of Theorem 2. First, we use Gershgorin's circle theorem. It states that every eigenvalue of a complex matrix A lies within at least one of the Gershgorin discs $D\left(a_{ii}, R_i\right)$, where a_{ii} is the i-th diagonal element, and $R_i = \sum_{j\neq i}|a_{ij}|$ is sum of the absolute values of the non-diagonal entries in the i-th row. The rightmost Gershgorin disc may be used to upper bound the largest eigenvalue

$$\hat{\lambda}_1 \leq \max_{1\leq i\leq M}\left\{\hat{\boldsymbol{R}}_0[i,i] + \sum_{j\neq i}\left|\hat{\boldsymbol{R}}_0[i,j]\right|\right\} \tag{21}$$

where the maximization is over the matrix rows. Second, we apply a union bound over the identically distributed rows of $\hat{\boldsymbol{R}}_0$, and obtain

$$\begin{aligned} P_{FA} &= \mathbb{P}\left\{\lambda_1\left(\hat{\boldsymbol{R}}_0\right) > \delta\right\} \\ &\leq M \cdot \mathbb{P}\left\{\hat{\boldsymbol{R}}_0[1,1] + \sum_{j=2}^{M}\left|\hat{\boldsymbol{R}}_0[1,j]\right| > \delta\right\} \end{aligned} \tag{22}$$

Third, we develop a Chernoff bound on (22). For $\theta > 0$, we get

$$P_{FA} \leq e^{-\theta\delta} \cdot M \cdot \underbrace{\mathbb{E}\left\{e^{\theta\left(\hat{\boldsymbol{R}}_0[1,1] + \sum_{j=2}^{M}\left|\hat{\boldsymbol{R}}_0[1,j]\right|\right)}\right\}}_{\triangleq E_1} \tag{23}$$

In order to get rid of the dependence between $\hat{\boldsymbol{R}}_0[1,1]$ and $\hat{\boldsymbol{R}}_0[1,j]$ in E_1, we use the law of total expectation:

$$\begin{aligned} E_1 &= \mathbb{E}\left\{e^{\theta\hat{\boldsymbol{R}}_0[1,1]} \cdot \mathbb{E}\left(e^{\theta\sum_{j=2}^{M}\left|\hat{\boldsymbol{R}}_0[1,j]\right|}\,\middle|\,\{w_1[n]\}_{n=1}^N\right)\right\} \\ &= \mathbb{E}\left\{e^{\theta\hat{\boldsymbol{R}}_0[1,1]} \cdot \underbrace{\left[\mathbb{E}\left(e^{\theta\left|\hat{\boldsymbol{R}}_0[1,2]\right|}\,\middle|\,\{w_1[n]\}_{n=1}^N\right)\right]}_{\triangleq \mathbb{E}\left(e^{\theta|Z|}\right) \equiv M_{|Z|}(\theta)}^{M-1}\right\} \end{aligned} \tag{24}$$

Since $\hat{\boldsymbol{R}}_0[1,2] \equiv \frac{1}{N}\sum_{n=1}^{N} w_1[n]w_2^*[n]$, using properties 2 and 3 in Lemma 1, Z is a zero mean complex $2\sigma_Z^2\sigma_w^2$-SG RV where $2\sigma_Z^2 \triangleq \frac{1}{N^2}\sum_{n=1}^{N}|w_1[n]|^2 = \frac{1}{N}\hat{\boldsymbol{R}}_0[1,1]$. We can express Z as the sum of its real part Z_r and its imaginary part jZ_i, where each of Z_r, Z_i is a zero mean real $\sigma_Z^2\sigma_w^2$-SG RV. Then, using the triangle inequality we obtain

$$M_{|Z|}(\theta) \leq \mathbb{E}\left[e^{\theta(|Z_r| + |Z_i|)}\right] = \mathbb{E}^2\left[e^{\theta|Z_r|}\right] = M_{|Z_r|}^2(\theta) \tag{25}$$

Using the MGF definition it is easy to show that

$$M_{|Z_r|}(\theta) \leq 2M_{Z_r}(\theta) + 2M_{Z_r}(-\theta) \tag{26}$$

and together with property 1 in Lemma 1 we get

$$M_{|Z_r|}(\theta) \leq 4\exp\left(\frac{1}{2}\theta^2\sigma_Z^2\sigma_w^2\right) = 4\exp\left(\frac{1}{4}\theta^2\frac{\sigma_w^2}{N}\hat{\boldsymbol{R}}_0[1,1]\right) \tag{27}$$

Substituting (27) with (25) back into (24) we obtain

$$\begin{aligned} E_1 &\le \mathbb{E}\left\{ e^{\theta \hat{\boldsymbol{R}}_0[1,1]} \cdot \left[4\exp\left(\frac{1}{4}\theta^2 \frac{\sigma_w^2}{N} \hat{\boldsymbol{R}}_0[1,1] \right) \right]^{2(M-1)} \right\} \\ &= 16^{M-1}\mathbb{E}\left\{ \exp\left[\left(\frac{M-1}{4N^2}\sigma_w^4\theta^2 + \frac{\sigma_w^2}{2N}\theta \right) \frac{2N}{\sigma_w^2}\hat{\boldsymbol{R}}_0[1,1] \right] \right\} \\ &= 16^{M-1}\mathrm{M}_{\frac{2N}{\sigma_w^2}\hat{\boldsymbol{R}}_0[1,1]}\left(\frac{M-1}{4N^2}\sigma_w^4\theta^2 + \frac{1}{2N}\sigma_w^2\theta \right) \end{aligned} \tag{28}$$

Substituting (28) into (23) we get

$$P_{FA} \le \exp\left(-\frac{N\theta\delta}{\sigma_w^2} \right) M \cdot 16^{M-1}\mathrm{M}_{\frac{2N}{\sigma_w^2}\hat{\boldsymbol{R}}_0[1,1]}\left(\frac{M-1}{4}\theta^2 + \frac{\theta}{2} \right) \tag{29}$$

where $\theta > 0$. Since the noise samples follow a SG distribution, $\frac{2N}{\sigma_w^2}\hat{\boldsymbol{R}}_0[1,1]$ is a sum of $2N$ i.i.d. 1-SE RVs. Using property 5 in Lemma 1, (16) is reached with the constraint $0 < \theta < \frac{\sqrt{2M-1}-1}{M-1}$. Finally, by differentiating the bound in (16) with respect to θ, we can find the optimal value of θ which is given in (17). $\square$

4. SIMULATION RESULTS

The bounds given in Theorem 1 and Theorem 2 may be applied to any circular symmetric SG transmitted signal and receiver noise. Here we present simple examples of such signals to compare the bounds against a Monte-Carlo simulation.

To test the bound on P_{MD}, let both the transmitted signal and the noise follow a complex Gaussian distribution. It can be shown that the SE parameters of $\frac{2N}{1+\|\boldsymbol{h}\|^2\Gamma}\hat{\boldsymbol{R}}_1[1,1]$ are $\left(\sigma_1^2 = 8N, B_1 = 1/4\right)$.

To test the bound on P_{FA}, let the noise samples be a combination of a two independent RVs, a complex Gaussian RV and a complex uniform RV:

$$w_i[n] = \nu_i[n] + u_i[n] \tag{30}$$

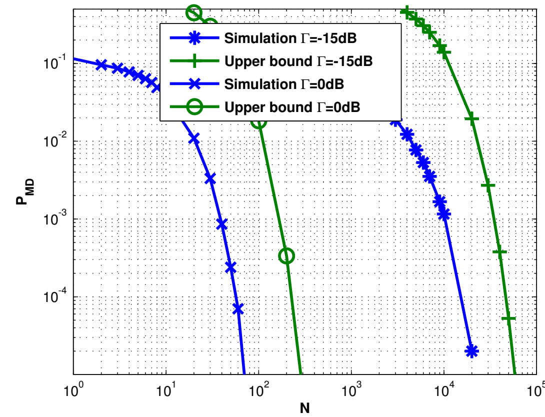
where $\nu_i[n]$ is a centered complex Gaussian RV with a variance equal to $1/2$, and $u_i[n]$ is a centered complex uniform RV with $\Re\{u_i[n]\}, \Im\{u_i[n]\} \in \left[-\sqrt{3/4}, \sqrt{3/4}\right]$ whose variance also equals $1/2$. It is easy to show that $w_i[n]$ is a 1-SG RV.

We simulated SNRs of $\Gamma \in \{0\text{dB}, -15\text{dB}\}$, normalized channel coefficients $\|\boldsymbol{h}\|^2 = M$, an antenna array of size $M = 4$ and a number of samples $N \le 10^5$. For each number of samples, we evaluated 10^5 random experiments.

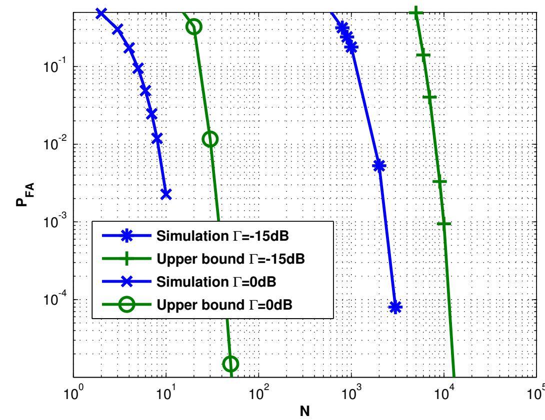
Figure 1 (a) shows the misdetection probability, and Figure 1 (b) shows the false alarm probability as a function of the number of samples, for a fixed decision threshold. Both bounds exhibit a steep exponential slope as the number of samples grows, and for a given low error rate in both cases the gap between the bound and simulation result is less than a factor of 4 in the number of samples.

5. CONCLUSION

The performance of the largest eigenvalue detector was investigated for the case of temporally white signals and calibrated noise power. We derived new closed-form error upper bounds that can be evaluated for any finite number of measurements (samples), and may be used to ensure a minimum performance level. Unlike similar contributions, we did not use the common Gaussian assumption, and our results are valid for the broad class of SG RVs. The bounds were compared to simulation results, and proved to be informative. Our contribution should be useful for cognitive radio, multistatic passive radar and any application that requires detection based on an eigenvalue analysis, especially when the transmitted signal or noise do not obey a strictly Gaussian distribution.



(a) Misdetection probability



(b) False-alarm probability

Fig. 1. Detector performance bounds as function of the number of samples against simulation results for $M = 4$ and $\Gamma \in \{0\text{dB}, -15\text{dB}\}$.

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