# PERFORMANCE ANALYSIS OF A MODIFIED RAO TEST FOR ADAPTIVE SUBSPACE DETECTION

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# ABSTRACT

The problem of detecting a subspace signal is studied in colored Gaussian noise with an unknown covariance matrix. In the subspace model, the target signal belongs to a known subspace, but with unknown coordinates. We propose a modified Rao test (MRT) by introducing a tunable parameter. The MRT is more general, which includes the Rao test and the generalized likelihood ratio test as special cases. Moreover, closed-form expressions for the probabilities of false alarm and detection of the MRT are derived. Numerical results demonstrate that the MRT can offer the flexibility of being adjustable in the mismatched case where the target signal deviates from the presumed signal subspace. In particular, the MRT provides better mismatch rejection capacities as the tunable parameter increases.

*Index Terms*— Adaptive detection, Rao test, subspace signal detection, mismatched signal rejection, constant false alarm rate.

## 1. INTRODUCTION

In recent years, there have been a large number of investigations on the signal detection problem in colored Gaussian noise with an unknown covariance matrix [1–7]. Typically, a set of training (secondary) data is assumed to be available to estimate the unknown noise covariance matrix. Many classic detectors have been proposed, such as the generalized likelihood ratio test (GLRT) detector [8], adaptive matched filter (AMF) [9], and adaptive coherence estimator (ACE) [10]. These GLRT, AMF and ACE are developed for the matched case where the target signal is perfectly matched to the assumed steering vector. Wuhan Radar Academy Wuhan 430019, China

In practice, mismatch of the signal steering vector may exist due to many factors such as wavefront distortions, calibration and pointing errors, and imperfect antenna shape [11]. In [12], Pulsone *et al.* proposed an adaptive beamformer orthogonal rejection test (ABORT) which exhibits better mismatch discrimination capabilities than both the GLRT and AMF. De Maio derived a Rao test in [11], which achieves better rejection capacities of strong mismatched signals than the ABORT. Note that none of the above mentioned detectors can adjust their rejection capabilities of mismatched signals.

In the above detectors, the target steering vector is a rankone signal. In some applications, the signal of interest is naturally multi-rank. For example, the data collected from multiple polarimetric channels in polarization radars can be formulated as a subspace model for target detection [13–16]. The subspace signal model was also employed for multiuser detection [17], and signal estimation and detection in multipath environments [18, 19]. Recently, Liu et al. generalized the Rao test from the rank-1 to rank-r (r > 1) subspace signal model, and obtained a subspace version of the Rao test in [4]. However, the theoretical performance of the subspaceversion Rao test was not examined. Moreover, one common issue in the detectors mentioned above is that the detection performance for matched signals and rejection performance for mismatched signals cannot be adjusted when the target signal has multi-rank. In practice, it is desired to offer a tradeoff between the two performance metrics for matched and, respectively, mismatched signals.

In this paper, we examine the subspace signal detection problem whereby the signal of interest is constrained to a multi-rank subspace with unknown coordinates. A new modified Rao test (MRT) with a tunable parameter is proposed, which includes the GLRT and Rao test as special cases. Moreover, we derive closed-form expressions for the probabilities of false alarm, respectively, detection of the MRT, which are verified by using Monte Carlo (MC) simulations. Numerical results show that the mismatched signal rejection performance of the proposed MRT improves as the tunable parameter increases. Remarkably, the MRT with a large tunable parameter can better reject mismatched signals than existing

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detectors.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts  $(\cdot)^T$ ,  $(\cdot)^*$  and  $(\cdot)^{\dagger}$  denote transpose, complex conjugate and complex conjugate transpose, respectively. The notation ~ means "is distributed as," and  $\mathcal{CN}$  denotes a circularly symmetric, complex Gaussian distribution. E $\{\cdot\}$  denotes the mean of a random argument.  $\stackrel{d}{=}$  means equivalence in distribution.  $\chi^2_n$  denotes the central Chi-squared distribution with *n* degrees of freedom, while  $\chi'^2_n(\zeta)$  denotes the non-central Chi-squared distribution with *n* degrees of freedom and a non-centrality parameter  $\zeta$ .  $|\cdot|$  represents the modulus of a complex number, and  $j = \sqrt{-1}$ .  $\binom{n}{m}$  is the binomial coefficient.  $\mathbf{I}_n$  is the identity matrix

of dimension n, and  $tr(\cdot)$  is the trace of a matrix.

## 2. SIGNAL MODEL

Consider the following model of the test data:

$$\mathbf{x} = \mathbf{S}\mathbf{a} + \mathbf{n},\tag{1}$$

where **S** is a known full-rank matrix of dimension  $Q \times q$ whose columns span the subspace containing target signals; **a** is a deterministic but unknown coordinate vector of dimension q, accounting for the target reflectivity and channel propagation effects; the noise **n** is assumed to have a circularly symmetric, complex Gaussian distribution, i.e.,  $\mathbf{n} \sim C\mathcal{N}(\mathbf{0}, \mathbf{R})$ , where **R** is a positive definite covariance matrix **G** imension  $Q \times Q$ . In practice, the noise covariance matrix **R** is usually unknown. A standard assumption is that there exists a set of homogeneous training data free of target signal components, i.e.,  $\{\mathbf{y}_k | \mathbf{y}_k \sim C\mathcal{N}(\mathbf{0}, \mathbf{R}), k = 1, 2, \dots, K \text{ and } K \geq Q\}$ , which can be used to estimate **R**.

Let the null hypothesis  $(H_0)$  be that the test data are target signal free and the alternative hypothesis  $(H_1)$  be that the test data contain the target signal. Hence, the detection problem is to decide between the null hypothesis and the alternative one:

$$H_0: \begin{cases} \mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}), \end{cases} \quad H_1: \begin{cases} \mathbf{x} \sim \mathcal{CN}(\mathbf{Sa}, \mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}), \end{cases}$$
(2)

where  $k = 1, 2, \dots, K$ .

#### 3. MODIFIED RAO TEST

As shown above, the Rao test has the following form:

$$\Xi_{\text{Rao}} = \frac{T_{\text{AMF}}}{(1 + \mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x})(1 + \mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x} - T_{\text{AMF}})} \overset{H_1}{\underset{H_0}{\gtrless}} \xi_{\text{Rao}},$$
(3)

where  $T_{\text{AMF}} = \mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{S} (\mathbf{S}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{S})^{-1} \mathbf{S}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x}$ . Recall that the GLRT detector proposed in [20] can be written as

$$T_{\text{GLRT}} = \frac{T_{\text{AMF}}}{1 + \mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x}} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\text{GLRT}},\tag{4}$$

where  $t_{\text{GLRT}}$  is the detection threshold. Note that the only difference between the Rao test and the GLRT is the second term  $(1 + \mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x} - T_{\text{AMF}})$  in the denominator of (3). Based on this observation, we propose a modified Rao test (MRT) involving a tunable parameter as follows:

$$\Xi = \frac{T_{\text{AMF}}}{(1 + \mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x}) [1 + \alpha (\mathbf{x}^{\dagger} \hat{\mathbf{R}}^{-1} \mathbf{x} - T_{\text{AMF}})]} \underset{H_0}{\overset{H_1}{\gtrless}} \xi, \quad (5)$$

where  $\xi$  is a detection threshold,  $\alpha \ge 0$  is a tunable parameter. Obviously, the MRT contains the GLRT and Rao test as special cases of  $\alpha = 0$  and  $\alpha = 1$ , respectively.

It should be pointed out that the analytical performance of the Rao test is not examined in [4]. In the sequel, we first investigate the statistical properties of the proposed MRT, and then derive closed-form expressions for its probabilities of false alarm and detection. Apparently, by setting  $\alpha = 1$  we also fill the gap on the analytical performance of the Rao test that is missing in [4].

Similar to [21], it can be shown that

$$\Xi = \frac{T_{\rm AMF}}{(\rho^{-1} + T_{\rm AMF})[1 + \alpha(\rho^{-1} - 1)]} \overset{H_1}{\underset{H_0}{\gtrless}} \xi, \qquad (6)$$

where  $\rho$  is a loss factor whose PDF is

$$f_{\rho}(\rho) = \frac{K!(1-\rho)^{Q-q-1}\rho^{K-Q+q}}{(Q-q-1)!(K-Q+q)!}, \quad 0 < \rho < 1.$$
(7)

After an equivalent transformation, we have

$$\rho T_{\text{AMF}} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)}.$$
(8)

Similar to [22, eq. (B39)], we can derive that

$$\rho T_{\text{AMF}} \stackrel{\text{d}}{=} \begin{cases} \frac{\chi_{2q}^2}{\chi_{2(K-Q+1)}^2}, & \text{under } H_0 \\ \frac{\chi_{2q}^2(2\delta\rho)}{\chi_{2(K-Q+1)}^2}, & \text{under } H_1 \end{cases} \tag{9}$$

with  $\delta = \mathbf{a}^{\dagger} \mathbf{S}^{\dagger} \mathbf{R}^{-1} \mathbf{S} \mathbf{a}$ . Note that to guarantee the positiveness of the right-hand side of (8), the value of the random variable  $\rho$  is now restricted to the range

$$\frac{\xi\alpha}{1-\xi+\xi\alpha} < \rho < 1. \tag{10}$$

Define  $2\tau \stackrel{\mathrm{d}}{=} \chi^2_{2(K-Q+1)}$ , and

$$2t \stackrel{\mathrm{d}}{=} \begin{cases} \chi_{2q}^2, & \text{under } H_0 \\ \chi_{2q}^{\prime 2}(2\delta\rho), & \text{under } H_1 \end{cases}$$
(11)

Then, we obtain

$$\rho T_{\rm AMF} \stackrel{\rm d}{=} \frac{t}{\tau}.$$
 (12)

#### 3.1. Probability of False Alarm

Based on (8), we can obtain the probability of false alarm conditioned on  $\rho$  as

$$P_{\text{FA} \mid \rho} = \int_{0}^{+\infty} \left( \int_{\frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)} \tau}^{+\infty} f_t(t \mid H_0) \, \mathrm{d} t \right) f_\tau(\tau \mid H_0) \, \mathrm{d} \tau$$
$$= \sum_{j=1}^{q} \left( K - Q + q - j \\ q - j \right) \left[ \frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)} \right]^{q-j} \times \left[ \frac{\rho}{\rho - \xi(\rho + \alpha - \rho\alpha)} \right]^{-(K - Q + q - j + 1)}.$$
(13)

Therefore, the probability of false alarm of the Rao test can be obtained by averaging over  $\rho$ , i.e.,

$$P_{\rm FA} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} P_{\rm FA|\rho} f_{\rho}(\rho) \,\mathrm{d}\rho, \tag{14}$$

where  $f_{\rho}(\rho)$  is given in (7). It follows that the MRT exhibits the desirable constant false alarm rate (CFAR) property against the noise covariance matrix, since the probability of false alarm in (14) is irrelevant to the noise covariance matrix.

#### 3.2. Detection Probability

#### 3.2.1. Matched Case

We first consider the matched case. Let

$$\omega = \frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)}\tau,$$
(15)

where the loss factor  $\rho$  is temporarily fixed. The PDF of  $\omega$  conditioned on  $\rho$ , denoted by  $f_{\omega|\rho}(\omega)$ , can be easily obtained by using the PDF of  $\tau$ . As a result, the probability of detection conditioned on  $\rho$  can be obtained as

$$P_{\mathrm{D}\,|\,\rho} = \int_{0}^{+\infty} \left( \int_{\omega}^{+\infty} f_t(t|\,H_1) \mathrm{d}\, t \right) f_{\omega\,|\rho}(\omega) \mathrm{d}\,\omega$$
  
=1 -  $\left[ \frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)} \right]^{q-1}$   
 $\times \left[ \frac{\rho}{\rho - \xi(\rho + \alpha - \rho\alpha)} \right]^{-(K-Q+q)}$   
 $\times \sum_{j=1}^{K-Q+1} {K-Q+q \choose q+j-1} \left[ \frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)} \right]^j$   
 $\times \exp\left\{ -\delta[\rho - \xi(\rho + \alpha - \rho\alpha)] \right\}$   
 $\times \sum_{m=0}^{j-1} \frac{\delta^m}{m!} [\rho - \xi(\rho + \alpha - \rho\alpha)]^m.$  (16)

Furthermore, the detection probability of the MRT is obtained by averaging over  $\rho$ , i.e.,

$$P_{\rm D} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} P_{\rm D|\rho} f_{\rho}(\rho) \,\mathrm{d}\rho, \qquad (17)$$

where  $f_{\rho}(\rho)$  is given in (7).

## 3.2.2. Mismatched Case

Here we consider the mismatched case where the actual target signal subspace deviates from the presumed subspace. To quantify the mismatching, we define the angle  $\phi$  between the actual signal steering vector  $\mathbf{S}_0$  and the nominal subspace  $\mathbf{S}$ as follows [4]

$$\cos^2 \phi = \frac{|\operatorname{tr}(\mathbf{S}^{\dagger} \mathbf{R}^{-1} \mathbf{S}_0)|^2}{\operatorname{tr}(\mathbf{S}_0^{\dagger} \mathbf{R}^{-1} \mathbf{S}_0) \operatorname{tr}(\mathbf{S}^{\dagger} \mathbf{R}^{-1} \mathbf{S})}.$$
(18)

Note that  $\phi = 0$  corresponds to the case where the actual signal belongs to the nominal subspace. For the case of q = 1. the subspace matrix **S** reduces to a steering vector denoted by **s**, and the coordinate vector **a** becomes a scalar denoted by *a*. The angle  $\phi$  between the actual signal steering vector  $\mathbf{s}_0$  and the nominal subspace **s** becomes

$$\cos^2 \phi = \frac{|\mathbf{s}_0^{\dagger} \mathbf{R}^{-1} \mathbf{s}|^2}{(\mathbf{s}^{\dagger} \mathbf{R}^{-1} \mathbf{s})(\mathbf{s}_0 \mathbf{R}^{-1} \mathbf{s}_0)}.$$
 (19)

Next, we derive a closed-form expression for the detection probability of the MRT for the mismatched case with q = 1. It is obtained in [23] that the PDF of  $\rho$  in the mismatched case is

$$f_{\rho}^{\text{mis}}(\rho) = \exp(-\rho\Psi_{\phi}) \sum_{n=0}^{K-Q+2} {\binom{K-Q+2}{n}} \frac{K!}{(K+n)!} \times \Psi_{\phi}^{n} g_{K-Q+2,Q+n-1}(\rho),$$
(20)

where

and

$$\Psi_{\phi} = |a|^2 \mathbf{s}_0^{\dagger} \mathbf{R}^{-1} \mathbf{s}_0 \sin^2 \phi, \qquad (21)$$

$$g_{k,m}(x) = \frac{(k+m-1)!}{(k-1)!(m-1)!} x^{k-1} (1-x)^{m-1}$$
(22)

with 0 < x < 1. According to (16), the detection probability conditioned on  $\rho$  in the case of q = 1 becomes

$$P_{\mathrm{D}|\rho} = 1 - \left[\frac{\rho}{\rho - \xi(\rho + \alpha - \rho\alpha)}\right]^{-(K-Q+1)} \times \sum_{j=1}^{K-Q+1} {\binom{K-Q+1}{j}} \left[\frac{\xi(\rho + \alpha - \rho\alpha)}{\rho - \xi(\rho + \alpha - \rho\alpha)}\right]^{j} \times \exp\left\{-\delta[\rho - \xi(\rho + \alpha - \rho\alpha)]\right\} \times \sum_{m=0}^{j-1} \frac{\delta^{m}}{m!} [\rho - \xi(\rho + \alpha - \rho\alpha)]^{m},$$
(23)



Fig. 1. Probability of false alarm of the MRT versus the detection threshold for q = 2.

where the non-centrality parameter  $\delta$  in the mismatched case becomes

$$\delta = |a|^2 \mathbf{s}_0^{\dagger} \mathbf{R}^{-1} \mathbf{s}_0 \cos^2 \phi. \tag{24}$$

Therefore, the detection probability of the MRT with q = 1 can be expressed as

$$P_{\rm D} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} P_{\rm D|\,\rho} f_{\rho}^{\rm mis}(\rho) \,\mathrm{d}\rho, \tag{25}$$

where  $P_{\mathrm{D}|\rho}$  and  $f_{\rho}^{\mathrm{mis}}(\rho)$  are given in (23) and (20), respectively.

It should be pointed out that the detection probability of the MRT is unavailable for the case of q > 1, since it is difficult to derive the PDF of  $\rho$  for the multi-rank subspace case.

# 4. NUMERICAL RESULTS

In this section, numerical simulations are conducted to confirm the validity of the above theoretical results. A uniform linear array of 5 elements with a half-wavelength spacing is used, i.e., Q = 5. Throughout this section, the (i, j)th element of the noise covariance matrix is chosen to be  $[\mathbf{R}]_{i,j} = \sigma^2 0.95^{|i-j|}$ , where the noise power  $\sigma^2$  is set to be 1. We select  $\mathbf{a} = \sigma_{\mathbf{a}}^2 [1, \dots, 1]^T$ , where  $\sigma_{\mathbf{a}}^2$  is the target power. Define the signal-to-noise ratio (SNR) as

$$SNR = 10 \log_{10} \frac{\sigma_{\mathbf{a}}^2}{\sigma^2}.$$
 (26)

For comparison purposes, we consider the AMF, GLRT, and ACE.

The probability of false alarm of the MRT with q = 2 as a function of the detection threshold is shown in Fig. 1, where the lines denote the results obtained with the finite-sum expression in (14), and the symbols "o" represent the results



Fig. 2. Detection probability versus  $\cos^2 \phi$  in the mismatched case.

obtained with MC simulations. The number of independent trials used in each case is  $10^6$ . It can be seen that the theoretical results are in good accordance with the simulation results.

In Fig. 2, the detection probability curves of the MRT with q = 1 are plotted with respect to  $\cos^2 \phi$  for SNR = 20 dB. For comparison purposes, the ABORT proposed in [12], the GLRT and ACE are also included. Note that we also conducted simulations on the performance of the AMF and observed that the AMF is much more robust than the GLRT. This observation is well-known, and was also made in [12], [9]. For clarity of exposition, the detection probability curve of the AMF is not plotted in Fig. 2.

We can observe that the selectivity of the proposed MRT can be flexibly controlled by adjusting the tunable parameter  $\alpha$ . More specifically, the rejection capabilities of mismatched signals of the MRT increase as the tunable parameter  $\alpha$  increases. Interestingly, the MRT with  $\alpha = 1$  (i.e., the Rao test) has rejection capacities of mismatched signals worse than the ACE in the case of moderate SNR. However, we can select larger  $\alpha$  to improve the selectivity of the proposed MRT. For example, the MRT with  $\alpha = 10$  has mismatched rejection capabilities better than the ABORT, as shown in Fig. 2.

# 5. CONCLUSION

In this paper, we proposed the MRT by introducing a tunable parameter. It subsumes the GLRT and Rao test as particular cases. The performance of the proposed MRT is evaluated in terms of the probabilities of false alarm and detection. It is shown that the MRT has the CFAR property with respect to the noise covariance matrix. Simulation results reveal that the mismatched signal rejection capabilities of the proposed MRT can be flexibly adjusted. Specifically, the mismatched signal rejection capabilities improve as the tunable parameter increases.

## 6. REFERENCES

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