KNOWLEDGE-AIDED HYPERPARAMETER-FREE BAYESIAN DETECTION IN STOCHASTIC HOMOGENEOUS ENVIRONMENTS

Pu Wang[⋆] *Hongbin Li*[†] *Olivier Besson*[‡] *Jun Fang*[⋄]

[⋆] Schlumberger-Doll Research, Cambridge, MA 02139, USA
[†] Stevens Institute of Technology, Hoboken, NJ 07030, USA
[‡] University of Toulouse, ISAE, 31055 Toulouse, France
[⋄] University of Electronic Science and Technology of China, Chengdu 611731, China

ABSTRACT

This paper considers adaptive signal detection in *stochastic homogeneous* environments where the disturbance covariance matrix of both test and training signals, $\mathbf{R}$, is assumed to be a random matrix with *a priori* knowledge of $\bar{\mathbf{R}}$. Unlike existing detectors assuming a known hyperparameter associated with $\bar{\mathbf{R}}$, a knowledge-aided detector with the capability of automatic weighting is considered by accounting for the uncertainty of the prior knowledge. Specifically, the generalized likelihood ratio test (GLRT) is utilized to develop the test statistic, along with the maximum marginal likelihood (MML) estimation of the hyperparameter. The proposed KA-MML-GLRT detector is evaluated by numerical simulations and the results show improved detection performance over conventional and knowledge-aided detectors, especially in the case of limited training signals and inaccurate prior knowledge.

Index Terms— Stochastic homogeneous model, generalized likelihood ratio test, maximum marginal likelihood estimation.

1. INTRODUCTION

For adaptive signal detection, a homogeneous model is usually assumed, where the disturbance in the test signal shares the same covariance matrix with target-free training signals [1, 2]. The classical adaptive matched filter (AMF) and Kelly's generalized likelihood ratio test (GLRT) were proposed for this purpose. Both detectors require the computation of the sample covariance matrix (SCM) from sufficient training signals which may not be available in practice. To mitigate such a demanding requirement of homogeneous training signals, a diagonal loading or colored loading has been found to be an efficient way in [3]. Although the diagonal loading approach appears to be heuristic at first, it turns out to be the solution to the covariance matrix estimation and the subsequent adaptive signal detection in a class of stochastic homogeneous environments [4–9] as well as various stochastic heterogeneous models [10–13].

Mathematically, the adaptive signal detection in the homogeneous environment is formulated as the following binary hypothesis testing:

$$\begin{aligned} H_0: &\quad \mathbf{x}_0 = \mathbf{d}_0, \quad \mathbf{x}_k = \mathbf{d}_k, k = 1, \cdots, K, \\ H_1: &\quad \mathbf{x}_0 = \alpha\mathbf{s} + \mathbf{d}_0, \quad \mathbf{x}_k = \mathbf{d}_k, k = 1, \cdots, K, \end{aligned} \tag{1}$$

where $\mathbf{x}_0 \in \mathbb{C}^{N\times 1}$ is the test signal, $\mathbf{x}_k = \mathbf{d}_k, k = 1, \cdots, K$, are target-free training signals, $\mathbf{s}$ is the *known* array response, α is an *unknown* complex-valued amplitude, $\mathbf{d}_0$ and $\mathbf{d}_k$ are independent, zero-

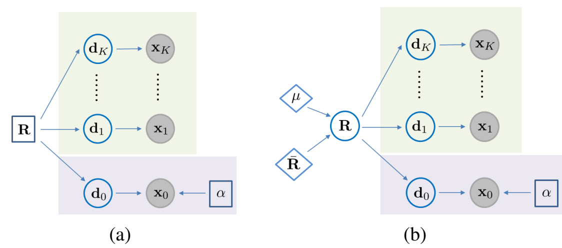

Fig. 1. Directed graphical model representation of (a) *conventional* and (b) *stochastic* homogeneous models; Circles denote random variables, squares denote deterministic model parameters, and diamonds denote user parameters. Shaded circles further represent observed random variables. Note the different (deterministic versus stochastic) ways of treating the interference covariance matrix $\mathbf{R}$.

mean complex-valued Gaussian distributed random vectors with covariance matrices given by

$$E\{\mathbf{d}_0\mathbf{d}_0^H\} = E\{\mathbf{d}_k\mathbf{d}_k^H\} = \mathbf{R}, \tag{2}$$

where the disturbance covariance matrix $\mathbf{R}$ is assumed to be *unknown*.

For the conventional homogeneous model, $\mathbf{R}$ is considered as a *deterministic* hyperparameter, while the stochastic homogeneous model, as considered in this paper, assumes $\mathbf{R}$ to be a random matrix with built-in prior knowledge. A graphical model representation of the two homogeneous models is shown in Fig. 1. Note the different (deterministic versus stochastic) ways of treating the interference covariance matrix $\mathbf{R}$. Specifically, $\mathbf{R}$ is assumed to have a complex inverse Wishart distribution, i.e., $\mathbf{R} \sim \mathcal{CW}^{-1}((\mu - N)\bar{\mathbf{R}}, \mu)$ [4]:

$$p(\mathbf{R}) = \frac{\left|(\mu - N)\bar{\mathbf{R}}\right|^{\mu}}{\tilde{\Gamma}(N, \mu)\,|\mathbf{R}|^{\mu+N}} e^{-(\mu-N)\,\mathrm{tr}(\mathbf{R}^{-1}\bar{\mathbf{R}})}, \tag{3}$$

where

$$\tilde{\Gamma}(N, \mu) = \pi^{N(N-1)/2} \prod_{k=1}^{N} \Gamma(\mu - N + k), \tag{4}$$

with $\Gamma(\cdot)$ denoting the Gamma function. In addition, $\bar{\mathbf{R}}$ denotes the *known* prior covariance matrix obtained from sources such as land-cover/land-use (LCLU) maps, past measurements, etc. [14], and the hyperparameter μ describes the uncertainty of the prior knowledge

$\bar{\mathbf{R}}$ with respect to $\mathbf{R}$. The larger μ is, the more important $\bar{\mathbf{R}}$ is, as we have

$$E\left\{\mathbf{R}|\bar{\mathbf{R}},\mu\right\} = \bar{\mathbf{R}},$$
$$E\left\{(\mathbf{R}-\bar{\mathbf{R}})^2|\bar{\mathbf{R}},\mu\right\} = \frac{\bar{\mathbf{R}}^2 + (\mu-N)\,\mathrm{tr}\{\bar{\mathbf{R}}\}\bar{\mathbf{R}}}{(\mu-N)^2-1}, \quad (5)$$

with $\mu > N+1$.

In this paper, we are interested in developing knowledge-aided Bayesian detection with the capability of combining the available signals and the prior knowledge in a fully automatic, hyperparameter-free way. Our approach is to treat the hyperparameter μ as a *deterministic* but *unknown* parameter. To estimate the hyperparameter, the maximum marginal likelihood (MML) estimation is proposed by finding the marginal likelihood function. The MML estimation of μ utilizes the test and training signals as well as the available prior knowledge by taking into account its uncertainty. Once the MML estimates of μ are obtained in both hypotheses, the test statistic of the proposed KA-MML-GLRT detector is developed in a GLRT-like principle which computes the ratio of the maximum marginal likelihood functions under both hypotheses.

The rest of the paper is organized as follows. Conventional and existing knowledge-aided detectors are briefly reviewed in Section 2. Following that, the KA-MML-GLRT detector and the underlying estimation of unknown parameters are derived in Section 3. Simulation results are provided in Section 4 to show the effectiveness of the proposed KA-MML-GLRT detector. Finally, conclusions are drawn in Section 5.

2. PRIOR ARTS

Assuming the hyperparameter μ is known in advance, knowledge-aided detectors have been proposed in [5, 6, 8]. Particularly, the knowledge-aided GLRT (KA-GLRT) and knowledge-aided AMF (KA-AMF) detectors were developed according to the one-step and two-step GLRT principles [8]:

$$T_{\text{KA-GLRT}} = \frac{|\mathbf{s}^\dagger\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{x}_0|^2}{(\mathbf{x}_0^\dagger\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{x}_0 + K)(\mathbf{s}^\dagger\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{s})}, \quad (6)$$
$$T_{\text{KA-AMF}} = \frac{|\mathbf{s}^\dagger\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{x}_0|^2}{\mathbf{s}^\dagger\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{s}}, \quad (7)$$

where the covariance matrix estimation $\hat{\boldsymbol{\Sigma}}$ linearly combines the sample covariance matrix $\hat{\mathbf{R}}$ and the prior knowledge $\bar{\mathbf{R}}$ with the weighting factors determined by the hyperparameter μ as

$$\hat{\boldsymbol{\Sigma}} = K\hat{\mathbf{R}} + (\mu-N)\bar{\mathbf{R}}, \quad (8)$$

where $\hat{\mathbf{R}}$ is obtained from K training signals:

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^H. \quad (9)$$

Compared with the AMF and Kelly's GLRT, it is easy to see that the knowledge-aided counterparts inherit the same detection statistic but with the sample covariance matrix replaced by the colored loading form in (8).

In practice, the hyperparameter μ may not be known. To address this issue, the simplest way is to take a subjective determination of μ before applying the above knowledge-aided detectors. However, performance loss has been observed and evaluated in [8, Fig.12] if one subjectively chooses an under-estimated or an over-estimated hyperparameter. As a result, it is highly desired that the knowledge-aided detector can *automatically* determine the weighting factors by accounting for the availability of training signals and the uncertainty of the prior knowledge.

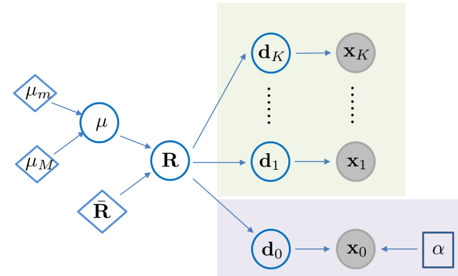

Fig. 2. Directed graphical model representation of the *hierarchical* homogeneous model in [15]; Note the the hyperparameter μ, described by a circle here, is considered to be a random parameter with two hierarchical hyperparameters μ_m and μ_M.

One approach is based on a hierarchical stochastic homogenous model [15], where the hyperparameter μ is further considered as a uniform *discrete* random variable over a pre-specified interval,i.e.,

$$\mu \sim \text{ unif }(\mu_m : \Delta\mu : \mu_M) \quad (10)$$

where μ_m and μ_M are, respectively, the lower and upper bounds of μ, and $\Delta\mu$ is the discretization stepsize. The graphical model representation of the hierarchical stochastic homogenous model is shown in Fig. 2, where the hyperparameter μ is described by a circle, instead of a diamond in Fig. 1. Then the minimum mean squared error (MMSE) estimate of the hyperparameter μ is obtained by first deriving the posterior distribution $p(\mu|\mathbf{x}_1,\cdots,\mathbf{x}_K)$ and then computing the posterior mean of $\mu|\mathbf{x}_1,\cdots,\mathbf{x}_K$:

$$\hat{\mu}_{\text{MMSE}} = \frac{\sum_{\mu=\mu_m}^{\mu_M}\mu h(\mu)}{\sum_{\mu=\mu_m}^{\mu_M}h(\mu)}, \quad (11)$$

where

$$h(\mu) = \frac{|\mu-N|^\mu}{|K\hat{\mathbf{R}} + (\mu-N)\bar{\mathbf{R}}|^{K+\mu}}\frac{\tilde{\Gamma}(N,K+\mu)}{\tilde{\Gamma}(N,\mu)}. \quad (12)$$

From (11), it is seen that the hyperparameter μ can be estimated from the training signals $\{\mathbf{x}_k\}_{k=1}^K$ via $\hat{\mathbf{R}}$, the prior covariance matrix $\bar{\mathbf{R}}$, and the discretized hyperparameter range (μ_m,μ_M). With the MMSE estimate of μ, the KA-AMF detector of (7) and the KA-GLRT detector of (6) can be implemented in a fully automatic hyperparameter-free way.

In the following, we develop a knowledge-aided hyperparameter-free detector in a different way. First, we consider the hyperparameter μ as a *deterministic* but unknown parameter, as opposed to the *stochastic* assumption on μ in [15]. This deterministic approach is useful to skip the selection of a pre-specified interval on the hyperparameter, i.e., $[\mu_m,\mu_M]$, and the discretization step on μ as a grid mismatch effect may be incurred when the true μ falls in between two discretized grids. Second, by treating μ in the deterministic way, it is possible to utilize a GLRT-like principle by computing the ratio of the maximum marginal likelihood functions under both hypotheses.

3. KA-MML-GLRT: KNOWLEDGE-AIDED MAXIMUM MARGINAL LIKELIHOOD GENERALIZED LIKELIHOOD RATIO TEST

In the following, we derive the proposed KA-MML-GLRT detector in detailed steps. The statistic of the KA-MML-GLRT is developed as follows

$$T = \frac{\max_{\mu} \max_{\alpha} \int f_1\left(\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_K \left|\alpha, \mathbf{R}\right.\right) p(\mathbf{R}|\mu; \bar{\mathbf{R}}) d\mathbf{R}}{\max_{\mu} \int f_0\left(\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_K | \mathbf{R}\right) p(\mathbf{R}|\mu; \bar{\mathbf{R}}) d\mathbf{R}}. \tag{13}$$

where $f_{0,1}\{\cdot\}$ denote, respectively, the conditional likelihood function under H_0 and H_1:

$$\begin{aligned} & f_i\left(\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_K \left|\alpha, \mathbf{R}\right.\right), \quad i = 0, 1, \\ =& f_i\left(\mathbf{x}_0 \left|\alpha, \mathbf{R}\right.\right) f\left(\mathbf{x}_1, \cdots, \mathbf{x}_K \left|\mathbf{R}\right.\right) \\ =& \frac{1}{\pi^{(K+1)N} \left|\mathbf{R}\right|^{K+1}} \exp\left\{-\operatorname{tr}\left(\mathbf{R}^{-1}\boldsymbol{\Sigma}_i\right)\right\}, \end{aligned} \tag{14}$$

with

$$\boldsymbol{\Sigma}_i = \mathbf{y}_i \mathbf{y}_i^H + K\hat{\mathbf{R}}, \tag{15}$$

with $\mathbf{y}_i = \mathbf{x}_0 - \beta_i \alpha \mathbf{s}$, $\beta_1 = 1$, $\beta_0 = 0$, and $p(\mathbf{R}|\mu; \bar{\mathbf{R}})$ is the prior distribution of $\mathbf{R}$ in (3). Essentially, the KA-MML-GLRT of (13) computes the marginal likelihood functions of both the test and training signals under H_0 and H_1, and then takes the ratio.

Due to the integral of an inverse complex Wishart distribution, the marginal likelihood functions can be evaluated as

$$\begin{aligned} & \int f_i\left(\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_K \left|\alpha, \mathbf{R}\right.\right) p(\mathbf{R}; \mu, \bar{\mathbf{R}}) d\mathbf{R} \\ =& \frac{\left|(\mu - N)\bar{\mathbf{R}}\right|^{\mu}}{\pi^{(K+1)N}\tilde{\Gamma}(N, \mu)} \int \left|\mathbf{R}\right|^{-(L+N)} e^{-\operatorname{tr}\left(\mathbf{R}^{-1}\bar{\boldsymbol{\Sigma}}_i\right)} d\mathbf{R} \\ =& \frac{\left|(\mu - N)\bar{\mathbf{R}}\right|^{\mu}\tilde{\Gamma}(N, K+\mu+1)}{\pi^{(K+1)N}\tilde{\Gamma}(N, \mu)} \left|\bar{\boldsymbol{\Sigma}}_i\right|^{-L}, \end{aligned} \tag{16}$$

where $L = K + \mu + 1$, and

$$\bar{\boldsymbol{\Sigma}}_i(\alpha, \mu) = \boldsymbol{\Sigma}_i + (\mu - N)\bar{\mathbf{R}} = \mathbf{y}_i\mathbf{y}_i^H + K\hat{\mathbf{R}} + (\mu - N)\bar{\mathbf{R}} \tag{17}$$

with $\alpha = 0$ if $i = 0$.

Defining

$$\eta(\mu) = |(\mu - N)\bar{\mathbf{R}}|^{\mu}\tilde{\Gamma}(N, \mu + K + 1)\tilde{\Gamma}^{-1}(N, \mu), \tag{18}$$

(13) reduces to

$$\begin{aligned} T &= \frac{\max_{\alpha}\max_{\mu>N}\left\{\eta(\mu)\left|\bar{\boldsymbol{\Sigma}}_1(\alpha, \mu)\right|^{-L}\right\}}{\max_{\mu>N}\left\{\eta(\mu)\left|\bar{\boldsymbol{\Sigma}}_0(\mu)\right|^{-L}\right\}} \\ &= \frac{\max_{\mu>N}\left\{\eta(\mu)\max_{\alpha}\left|\bar{\boldsymbol{\Sigma}}_1(\alpha, \mu)\right|^{-L}\right\}}{\max_{\mu>N}\left\{\eta(\mu)\left|\bar{\boldsymbol{\Sigma}}_0(\mu)\right|^{-L}\right\}}. \end{aligned} \tag{19}$$

Given μ, the MML estimate of α can be derived as follows. Defining $\boldsymbol{\Xi}(\mu) = K\hat{\mathbf{R}} + (\mu - N)\bar{\mathbf{R}}$, we have

$$\left|\bar{\boldsymbol{\Sigma}}_i(\alpha, \mu)\right| = \left|\mathbf{y}_i\mathbf{y}_i^H + \boldsymbol{\Xi}(\mu)\right| = |\boldsymbol{\Xi}(\mu)|\left(1 + \mathbf{y}_i^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{y}_i\right) \tag{20}$$

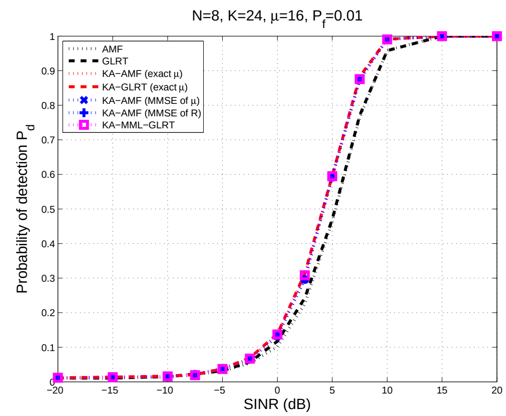

Fig. 3. Probability of detection versus SINR: sufficient training signals ($K = 24$) and reliable prior ($\mu = 16$). Two groups of performance curves can be observed. The conventional AMF and GLRT detectors are the inferior group, while the other group are the knowledge-aided detectors with slight advantage over the previous group in this case.

where $\mathbf{y}_0 = \mathbf{x}_0$ and $\mathbf{y}_1 = \mathbf{x}_0 - \alpha\mathbf{s}$. Then it is easy to show that

$$\hat{\alpha}_{\text{ML}} = \frac{\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{x}_0}{\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{s}}, \tag{21}$$

and the minimum cost function is

$$\min_{\alpha}\mathbf{y}_1^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{y}_1 = \mathbf{x}_0^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{x}_0 - \frac{\left|\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{x}_0\right|^2}{\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{s}}. \tag{22}$$

As a result, the test statistic reduces to

$$T = \frac{\max_{\mu>N}\eta(\mu)|\boldsymbol{\Xi}(\mu)|^{-L}\left(1 + \mathbf{x}_0^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{x}_0 - \frac{|\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{x}_0|^2}{\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{s}}\right)^{-L}}{\max_{\mu>N}\eta(\mu)|\boldsymbol{\Xi}(\mu)|^{-L}\left(1 + \mathbf{x}_0^H\boldsymbol{\Xi}^{-1}(\mu)\mathbf{x}_0\right)^{-L}}. \tag{23}$$

The final step is to find the MML estimates of μ under H_0 and H_1, which we resort to the one-dimensional Gauss-Newton algorithm. For implementation, we use *fminbnd* in MATLAB for the constrained maximization as μ is lower bounded by N [1].

Denoting $\hat{\mu}_{\text{ML},0}$ and $\hat{\mu}_{\text{ML},1}$ as the marginal likelihood estimates under H_0 and H_1, respectively, we have

$$\begin{aligned} T &= \frac{\eta(\hat{\mu}_1)|\boldsymbol{\Xi}(\hat{\mu}_1)|^{-L}\left(1 + \mathbf{x}_0^H\boldsymbol{\Xi}^{-1}(\hat{\mu}_1)\mathbf{x}_0 - \frac{|\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\hat{\mu}_1)\mathbf{x}_0|^2}{\mathbf{s}^H\boldsymbol{\Xi}^{-1}(\hat{\mu}_1)\mathbf{s}}\right)^{-L}}{\eta(\hat{\mu}_0)|\boldsymbol{\Xi}(\hat{\mu}_0)|^{-L}\left(1 + \mathbf{x}_0^H\boldsymbol{\Xi}^{-1}(\hat{\mu}_0)\mathbf{x}_0\right)^{-L}} \\ &\underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{KA-MML-GLRT}} \end{aligned} \tag{24}$$

where $\gamma_{\text{KA-MML-GLRT}}$ is a proper threshold to meet the given probability of false alarm.

[1]The upper bound specified in *fminbnd* is chosen to be an arbitrarily large integer. Particularly the upper bound of 500 is used in all our simulations in Section 4

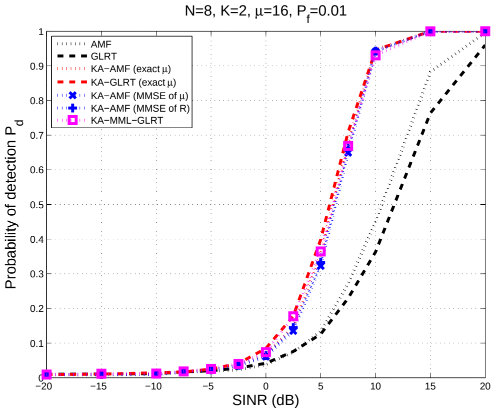

Fig. 4. Probability of detection versus SINR: limited training signals ($K = 2$) and reliable prior ($\mu = 16$).

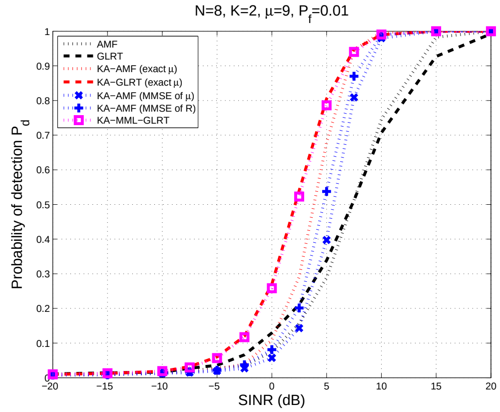

Fig. 5. Probability of detection versus SINR: limited training signals ($K = 2$) and less reliable prior ($\mu = 9$).

4. PERFORMANCE EVALUATION

In this section, simulation results are provided to demonstrate the efficiency of the proposed KA-MML-GLRT detector and also to compare it with other detectors in terms of probability of detection. The considered detectors include 1) the Kelly's GLRT [1]; 2) the conventional AMF [2]; 3) the KA-AMF with known μ [8]; 4) the KA-GLRT with known μ [8]; 5) the KA-AMF with the MMSE estimate of μ [15]; 6) the KA-AMF with the MMSE estimate of $\mathbf{R}$ [15]. To compute above KA-AMF detectors 5) and 6), we discretize the range of μ as $(9 : 1 : 100)$ with a stepsize of 1. In all simulation examples, we consider the case where $N = 8$ and the steering vector is given by $\mathbf{s} = [1, \cdots, 1]^T$. The *average* signal-to-interference-plus-noise ratio (SINR) is defined as

$$\text{SINR} = |\alpha|^2 \mathbf{s}^H \bar{\mathbf{R}}^{-1} \mathbf{s}, \tag{25}$$

where $\bar{\mathbf{R}}$ is the fixed prior covariance matrix generated by

$$[\bar{\mathbf{R}}]_{ij} = \rho^{|i-j|}, \tag{26}$$

where $\rho = 0.9$ is chosen. The simulated performance is obtained by using at least 10000 Monte Carlo trials for the probability of false alarm $P_f = 0.01$. For each Monte-Carlo trial, the covariance matrix $\mathbf{R}$ is generated from an inverse Wishart distribution as $\mathbf{R} \sim \mathcal{CW}^{-1}((\mu - N)\bar{\mathbf{R}}, \mu)$ of (3). Then, the disturbances $\mathbf{d}_k$, $k = 0, 1, \cdots, K$, are i.i.d. generated according to the generated covariance matrix $\mathbf{R}$ in each trial.

In the first example, we examine the detection performance with $K = 24$ and $\mu = 9$, a scenario of sufficient training signals $K \gg N$ and reliable prior covariance matrix $\mu > N$. As shown in Fig. 3, all considered knowledge-aided detectors give the same performance, which is slightly better than that of the the conventional AMF and GLRT detectors.

Fig. 4 shows the probability of detection versus the SINR when the training signals are limited, i.e., $K = 2$, and the prior $\bar{\mathbf{R}}$ is still relatively reliable, i.e., $\mu = 16$. In this case, the knowledge-aided detectors should put less weights on the sample covariance matrix from the training signals and more weights on the prior matrix $\bar{\mathbf{R}}$. As seen from Fig. 4, the KA-GLRT with the known μ (red dash lines) provides the benchmark performance over all detectors. The knowledge-aided detectors show very closed performance with the proposed KA-MML-GLRT detector giving slightly better performance.

Finally, we consider the most challenging scenario, where the training signals are limited $K = 2$ and, at the same time, the prior covariance matrix $\bar{\mathbf{R}}$ is less reliable, i.e., $\mu = 9$. Fig. 5 shows the evaluated probability of detection versus the SINR. As seen from Fig. 5, the proposed KA-MML-GLRT attains the detection performance close to the benchmark provided by the KA-GLRT with the known μ. Also, it is an evident performance improvement from the other two hyperparameter-free knowledge-aided detectors (i.e., the KA-AMF detectors with the MMSE estimate of μ and $\mathbf{R}$) to the proposed KA-MML-GLRT detector. Moreover, the proposed KA-MML-GLRT detector is also better than the KA-AMF with the known μ. The performance improvement of the proposed KA-MML-GLRT detector in this very limited training scenario may be due to the joint utilization of both test and training signals (effectively, $K = 3$ signals) for the estimation of the hyperparameter μ, whereas only $K = 2$ training signals are used for the MMSE estimates of μ or $\mathbf{R}$ in [15].

5. CONCLUSION

This paper considered the adaptive signal detection in a stochastic homogeneous model which treats the disturbance covariance matrix as a random matrix with mean given by a known prior covariance matrix. As the accuracy of the prior covariance matrix described by a hyperparamter, we developed a knowledge-aided hyperparameter-free GLRT detector by utilizing the MML estimation of the hyperparameter. Compared with the MMSE estimate of the hyperparameter, the MML estimation of the hyperparameter is free from a pre-discretization step and avoids the range specification. Numerical evaluation shows that the proposed KA-MML-GLRT provides further improvements over the KA-AMF detectors with the MMSE estimates, especially in the case of limited training signals and less reliable prior covariance matrix.

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