STOCHASTIC LOAD SCHEDULING FOR RISK-LIMITING ECONOMIC DISPATCH IN SMART MICROGRIDS

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ABSTRACT

In this work we present a novel scheme for load management in microgrids based on stochastic scheduling of loads under risk-limiting constraints. When trying to enforce adequate power supply in a microgrid, the volatility of renewable resources such as wind energy has to be considered. In the risk of inadequate power supply, loads have to be scheduled, which can be achieved by directly controlling individual loads or by setting pricing incentives to encourage beneficial behavior of the customers. A common drawback of conventional methods lies in the need of sophisticated control strategies and a significant amount of real-time signaling exchange between the microgrid and the central control unit. To address these issues, we propose a scheme that does not require a direct control of individual loads. Our method relies on sorting the appliances in the network into groups, and allowing these groups to schedule themselves stochastically according to broadcasted scheduling probabilities. In this paper, we propose an optimization problem to determine these group scheduling probabilities, as well as for choosing the best utilization of conventional generators, in a day-ahead planning scenario of an isolated microgrid. Using an outage-risk limiting constraint, we control the risk of inadequate power supply causing network outages. The proposed scheme can be easily implemented with unidirectional communication from a central control unit via simple broadcast messages.

***Index Terms*—** Microgrids, renewable energies, risk-limiting dispatch, economic dispatch, convex optimization

1. INTRODUCTION

The way in which electric power is generated and electrical grids are operated has experienced a significant paradigm shift in recent decades. Renewable energy resources have been integrated into the grid to increase its overall environmental compliance. Due to the volatile and mostly non-controllable nature of these resources, such as wind and solar energy, additional challenges for the reliability and economic operation of the power system arise. For example, additional uncertainties are caused by the need for intermediate storage, consumer participation in the market, and clustered operation in potentially standalone microgrids. Addressing these challenges with advanced technologies in network monitoring, communications and machine learning is an important subject of current research [1–4]. Smart microgrids are proposed as a promising way to increase the grid flexibility and reliability by decentralizing the energy generation. Meanwhile, smart microgrids bring about additional advantages, such as reducing carbon emissions and improving energy efficiency by incorporating renewable sources, utilizing waste heat of generators, and decreasing transmission distances. These microgrids may operate in standalone mode or connected to the main grid [5, 6]. The capability of standalone operation is essential for parts of the power grid. The reasons for this are manifold, such as reliability requirements or geographical circumstances on islands and in remote locations. A promising way to ensure the adequacy of generated power [1] is to schedule the service of loads, which are appliances that require electrical power, in the microgrid. This can be achieved by a hierarchical and centralized control structure or by enabling the consumer to participate in demand side management and setting incentives for beneficial behaviors [7, 8]. Finding an optimal plan of scheduling these loads is a topic of current interest in microgrid control research. However, the proposed approaches typically require either one of the following: A central controller has direct access to individual loads and can control them, or the number of customers that actively participate in the market and respond to incentives is sufficiently large [9–11].

Both requirements can cause significant communication- and computation overhead. Optimal power flow computation in a power grid that implements renewable energy sources and controllable loads leads to a nonlinear mixed-integer optimization problem. Heuristic solution approaches, based on load scheduling or incentive-based strategies, have been widely investigated, e.g. [7–10]. The common drawback of these methods lies in the need of sophisticated control strategies and a significant amount of real-time signaling exchange between the microgrid and the central control unit.

In this paper, we put forward a centralized strategy that does not require the central controller to have direct real-time access to the individual loads, and thus greatly decreases the amount of signaling required. Our proposed scheme relies on sorting the appliances in the network into groups, based on their average energy consumption. Individual devices are scheduled stochastically, using specific scheduling probabilities. These probabilities are computed by a central control unit for all groups of appliances and broadcasted over the network. The central controller only needs to know the number of appliances in each group, and their respective average energy consumption at certain times of the day. The scheduling probabilities for each appliance in the same group are the same. Distributing a list of scheduling probabilities for all groups in the network only requires unidirectional signaling. For example, broadcast signals from the central control unit could be used. In practical scenarios, the ability to receive such broadcast messages could be implemented easily, due to the increasing capabilities of smart meters and the connectedness of electric vehicle loading stations and industrial devices. An additional benefit of using this approach is that privacy concerns are respected, because appliances are not controlled individually. Our scheme aims to optimize the scheduling probabilities for multiple timeslots in a day-ahead scheduling scenario.

To the best of our knowledge, the system model and load scheduling method in this work has not been discussed in literature. Stochastic scheduling is mostly referred to in context of the stochastic nature of renewable energy sources [12–14], but not in the context of serving loads in a stochastic way.

The contributions of this paper are summarized in the following: A novel system model for the described scenario with volatile renewable energy sources and stochastic load scheduling is defined. A stochastic optimization problem for computing the optimal scheduling probabilities under risk-limiting constraints is formulated and solved using a proposed iterative optimization approach. The practicality of the approach is demonstrated in a microgrid setup.

The remainder of the paper is organized as follows: We provide the mathematical model of the considered microgrid scenario in Section 2. The corresponding stochastic optimization problem is formulated in Section 3, where a computationally efficient iterative optimization algorithm is proposed. The microgrid simulation to demonstrate the feasibility of the approach is discussed in Section 4. A summary of the results along with a discussion of further research prospects is provided in Section 5.

Notation: We use $\mathcal{N}(\mu, \sigma)$ to indicate a normal distribution with mean μ and standard deviation σ, and $\mathcal{B}(n,p)$ for a binominal distribution with n trials and success probability p. Furthermore, the normalized cumulative normal distribution function is indicated as $Q(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-y^2/2} \mathrm{d}y$.

2. SYSTEM MODEL

In our proposed microgrid system model several components, such as renewable energy sources and conventional generators, are implemented. We assume a standalone operation of the microgrid, where it is not possible to compensate inadequate power supply with resources from the main grid. The microgrid contains s ($s = 1, 2, \dots, S$) controllable generators with power output $P_s(\alpha_{s,t})$ at timeslot t ($t = 1, 2, \dots, T$). The magnitude of the output power is a linear function $P_s(\alpha_{s,t})$ of the control parameter $0 \leq \alpha_{s,t} \leq 1$, with $\alpha_{s,t} = 0$ and $\alpha_{s,t} = 1$ indicating minimal and maximum utilization, respectively. The total cost generated by utilizing generator s at level $\alpha_{s,t}$ is denoted by the known monotonically increasing function $\mathcal{C}_s(\alpha_{s,t})$. We define the sum of available power generated from wind turbines at timeslot t as Λ_t. Obviously this power source is non-controllable, and the forecast may be inaccurate. Since the probability density function of Λ_t can be modeled as a normal distribution [15, 16], we denote $\Lambda_t \sim \mathcal{N}(\lambda_t, \nu_t)$, where λ_t and ν_t represent the corresponding mean and standard deviation, respectively. A similar framework can be used for other renewable energy sources, such as solar energy. The portion of the generated load at timeslot t which cannot be controlled is denoted as M_t.

The controllable groups of loads are defined as follows: Let $\tilde{L}_{k,t}$ denote the average load created by an appliance in group k ($k = 1, 2, \dots, K$) at timeslot t. Group k contains N_k appliances in total. An appliance, in timeslot t, activates itself with a probability equal to the scheduling probability of its group k, which in the following is denoted as $p_{k,t}$. These scheduling probabilities $p_{k,t}$ are broadcasted to all appliances in all groups k. If each appliance is scheduled with a probability $p_{k,t}$, the probability density function of the total sum load $L_{k,t}$ created by this group follows a binominal distribution with $L_{k,t} \sim \mathcal{B}(N_k, p_{k,t})$. In the following, we assume that N_k, the number of appliances in group k, is sufficiently large to be approximated by the normal distribution $L_{k,t} \sim \mathcal{N}(\mu_{k,t}, \sigma_{k,t})$ with $\mu_{k,t} = \tilde{L}_{k,t} N_k p_{k,t}$ and $\sigma^2_{k,t} = \tilde{L}^2_{k,t} N^2_k p_{k,t}(1 - p_{k,t})$, using the de Moivre-Laplace Theorem [17]. For any $p_{k,t} < 1$, the service of a customer is delayed, which requires some form of financial compensation. This customer compensation cost is increasing with probability of not scheduling $(1 - p_{k,t})$, and is in the following denoted as $\mathcal{C}_k(p_{k,t})$. In the remainder of this section, we consider a particular fixed timeslot, and thus simply drop the time index t in the subscript.

In order to limit the risk of inadequate power supply, which may cause power outage events, a risk-limiting constraint has to be incorporated. This constraint is used to guarantee that the probability of the total generated power exceeding the total load is above a predefined threshold η, which therefore reflects the non-outage probability. We remark that the event that in a particular timeslot the total generated power exceeds the required power supply is a less critical problem than the opposite case, because microgrids with renewable energy sources typically have energy storage capabilities [9, 18]. Let C denote the sum of all deterministic powers and loads in the network, which is specified in the following section. In this case, the risk-limiting constraint takes the form of

$$\Pr\left(\sum_k L_k - \Lambda \leq C\right) \geq \eta \tag{1}$$

where η denotes the non-outage probability. Since L_k and Λ are independently normal distributed, we have $\sum_k L_k - \Lambda \sim \mathcal{N}\left(\sum_k \mu_k - \lambda, \sqrt{\sum_k \sigma_k^2 + \nu^2}\right)$. Therefore, the probability in (1) can be written as

$$Q\left(\frac{C - \sum_k \mu_k + \lambda}{\sqrt{\sum_k \sigma_k^2 + \nu^2}}\right) \geq \eta. \tag{2}$$

Using $\mu_k = \tilde{L}_k N_k p_k$ and $\sigma_k^2 = \tilde{L}_k^2 N_k^2 p_k(1-p_k)$, we obtain

$$Q^{-1}(\eta)\sqrt{\sum_k \tilde{L}_k^2 N_k^2 p_k(1-p_k)+\nu^2}+\sum_k \tilde{L}_k N_k p_k \leq C+\lambda. \tag{3}$$

3. SCHEDULING OPTIMIZATION

The optimization of the scheduling probabilities is carried out as an economic dispatch problem, which aims to minimize the total network utilization cost. This problem has been studied extensively in the context of microgrids, see e.g. [18–20]. We implement the constraint limiting the outage risk as shown in Eq. (3). The proposed outage risk limiting economic dispatch problem can be written as:

$$\begin{aligned}
\min_{p_{k,t},\alpha_{s,t}} \quad & \sum_t \left(\sum_k \mathcal{C}_k(p_{k,t}) + \sum_s \mathcal{C}_s(\alpha_{s,t})\right) && \text{(4a)}\\
\text{s.t.} \quad & Q^{-1}(\eta)\sqrt{\sum_k \tilde{L}_{k,t}^2 N_k^2 p_{k,t}(1-p_{k,t})+\nu_t^2} \\
& + \sum_k \tilde{L}_{k,t} N_k p_{k,t} + M_t + H_{t-1} \\
& \leq \sum_s P_s(\alpha_{s,t}) + \lambda_t \quad \forall t && \text{(4b)}\\
& |\alpha_{s,t}-\alpha_{s,t-1}| \leq r_s \Delta t \quad \forall s,t && \text{(4c)}\\
& 0 \leq \alpha_{s,t} \leq 1 \quad \forall s,t && \text{(4d)}\\
& 0 \leq p_{k,t} \leq 1 \quad \forall k,t && \text{(4e)}
\end{aligned}$$

In this problem, Eq. (4a) represents the total generated cost that has to be minimized. We assume convexity of the functions $\mathcal{C}_k(p_{k,t})$ and $\mathcal{C}_s(\alpha_{s,t})$. The outage-risk limiting constraint is formulated in Eq. (4b), which for a high non-outage probability threshold η ensures the adequacy of generated power, according to our derivations in Eqs. (1)-(3). The parameter $H_{t-1} = \sum_k \tilde{L}_{k,t-1} N_k(1-p_{k,t-1})$ in Eq. (4b) represents the sum demand of all controllable loads that have not been scheduled in the previous timeslot, and which are scheduled in the current timeslot. Ramping rates r_s for controllable generators are considered in Eq. (4c), where Δt is the duration of one timeslot. Eqs. (4d) and (4e) restrict the optimization parameters to their minimum and maximum bounds.

The risk-limiting constraint Eq. (4b) is a concave function of the optimization variables $p_{k,t}$, because the function $\phi(p_{k,t}) = p_{k,t}(1-p_{k,t})$ is concave and the square root function is concave and nondecreasing. Therefore, optimization problem (4) is nonconvex and cannot be solved easily. In the following, we introduce an iterative inner approximation technique based on the iterative Convex-Concave procedure [21–25] to approximate $p_{k,t}$ and $\alpha_{s,t}$ for each k and t over multiple iteration steps $i = 1,\ldots,I$, where I is the total number of iterations. Toward this aim, we define the function

$$f(p_{k,t}) = \sqrt{\sum_k \tilde{L}_{k,t}^2 N_k^2 p_{k,t}(1-p_{k,t})+\nu_t^2}. \tag{5}$$

The derivative of $f(p_{k,t})$ with respect to $p_{k,t}$ is given by

$$\begin{aligned}
g(p_{k,t}) &= \frac{\mathrm{d}\, f(p_{k,t})}{\mathrm{d}\, p_{k,t}} \\
&= \frac{\sum_k \tilde{L}_{k,t}^2 N_k^2\left(\frac{1}{2}-p_{k,t}\right)}{\sqrt{\sum_k \tilde{L}_{k,t}^2 N_k^2 p_{k,t}(1-p_{k,t})+\nu_t^2}}.
\end{aligned} \tag{6}$$

Using an initial point $\hat{p}_{k,t}^{(i-1)}$, we can compute the first-order Taylor approximation of $f(p_{k,t})$ around $\hat{p}_{k,t}^{(i-1)}$ as

$$\hat{f}\left(p_{k,t},\hat{p}_{k,t}^{(i-1)}\right) = f\left(\hat{p}_{k,t}^{(i-1)}\right) + g(\hat{p}_{k,t}^{(i-1)})\left(p_{k,t}-\hat{p}_{k,t}^{(i-1)}\right). \tag{7}$$

Using (7) in the outage-risk limiting constraint (4b), problem (4) can be reformulated as a convex optimization problem:

$$\begin{aligned}
\min_{p_{k,t},\alpha_{s,t}} \quad & \sum_t \left(\sum_k \mathcal{C}_k(p_{k,t}) + \sum_s \mathcal{C}_s(\alpha_{s,t})\right) && \text{(8a)}\\
\text{s.t.} \quad & Q^{-1}(\eta)\hat{f}\left(p_{k,t},\hat{p}_{k,t}^{(i-1)}\right) \\
& + \sum_k \tilde{L}_{k,t} N_k p_{k,t} + M_t + H_{t-1} \\
& \leq \sum_s P_s(\alpha_{s,t}) + \lambda_t \quad \forall t && \text{(8b)}\\
& |\alpha_{s,t}-\alpha_{s,t-1}| \leq r_s \Delta t \quad \forall s,t && \text{(8c)}\\
& 0 \leq \alpha_{s,t} \leq 1 \quad \forall s,t && \text{(8d)}\\
& 0 \leq p_{k,t} \leq 1 \quad \forall k,t && \text{(8e)}
\end{aligned}$$

If the constraint (8b) is satisfied, then (4b) is also satisfied, since $\hat{f}\left(p_{k,t},\hat{p}_{k,t}^{(i-1)}\right) \geq f(p_{k,t})$. The parameters $p_{k,t}$ and $\alpha_{s,t}$ are approximated over multiple iterations $i = 1,\ldots,I$ as follows: Firstly, a feasible point $\hat{p}_{k,t}^{(0)}$ is obtained. In references [23–26] iterative feasibility search procedures have been proposed based on the iterative inner approximation technique described above applied to the corresponding feasibility problems. This procedure can also be applied to find a feasible point $\hat{p}_{k,t}^{(0)}$ of our problem (if it exists). In the case that such a point $\hat{p}_{k,t}^{(0)}$ does not exist, a fundamental deficit of power supply is present in the network, which cannot be handled by the proposed approach. Secondly, problem (8) is solved to obtain $p_{k,t}^{(i)}$ and $\alpha_{s,t}^{(i)}$. This process is then repeated, using the optimal point $p_{k,t}$ of iteration i in the subsequent iteration $i+1$ as $\hat{p}_{k,t}^{(i)}$, until a convergence criterion is met. A popular convergence criterion relies on the vanishing update rule, i.e., the algorithm is stopped if $|p_{k,t} - p_{k,t}^{(i-1)}| \leq \epsilon \ \forall k,t$ and $|\alpha_{s,t} - \alpha_{s,t}^{(i-1)}| \leq \epsilon \ \forall s,t$ for some small threshold ϵ. The Convex-Concave procedure converges to a stationary point of the original problem (4) [21, 25]. With the described proce-

Table 1. Simulation Parameters

P^{MIN}	1000 kW
P^{MAX}	4000 kW
cost factors	$b = 40\ \$/(\mathrm{MWh})^2$ $c = 10\ \$/(\mathrm{MWh})$
ramping rate r	$0.3/\mathrm{h}$
Timeslot length Δt	$1\mathrm{h}$
Number of iterations I	10
Fixed load H	2500 kW
Wind power Λ	see Fig. 1
L_k	see Fig. 1
cost factors (in \$/kWh)	$d_1 = 0.2, e_1 = 0.04$ $d_2 = 0.2, e_2 = 0.03$ $d_3 = 0.2, e_3 = 0.02$

dure, it is possible to obtain the optimized scheduling probabilities $p_{k,t}$ and the generator utilization factors $\alpha_{s,t}$.

4. SIMULATION RESULTS

The parameters of a setup example for the microgrid simulation are displayed in Table 1. The considered example of a 12-hour cycle of varying group demands $L_{k,t}$ and mean wind power λ_t is illustrated in Fig. 1. We assume that the microgrid has one controllable generator, and its cost function $\mathcal{C}(\alpha) = bP(\alpha)^2(\Delta t)^2 + cP(\alpha)(\Delta t)$ is a quadratic function of the output power $P(\alpha) = P^{\mathrm{MIN}} + \alpha(P^{\mathrm{MAX}} - P^{\mathrm{MIN}})$ [27,28], where P^{MIN} and P^{MAX} denote the minimum and maximum output power of the generator, respectively. We model the sum power of wind turbines as described in Section 2 as a Gaussian process with a standard deviation ν_t, which is 5% of its mean λ_t [15, 16].

The cost function of scheduling probabilities is an increasing quadratic function $\mathcal{C}_k(p_{k,t}) = \Delta t \sum_t \left(L_{k,t} d_k (1 - p_{k,t})^2 + e_k(1 - p_{k,t})\right)$, where $(1 - p_{k,t})$ is the probability of not scheduling. This represents the cost of compensation to the customer, whose service is delayed for any $p_{k,t} < 1$. Since the cost function of controllable generators is also usually modeled as quadratic or piecewise linear, it makes sense from an economic standpoint to model the customer compensation accordingly. Three groups k of controllable loads, i.e. $k = 1, 2, 3$ are considered, that is commercial loads, plug-in electric vehicle loads and residential loads. The total generated load by these groups, as well as the wind power, are illustrated in Fig. 1. The timeframe under investigation is $t = 1, \ldots, 12$, representing the hours between 1 PM and 12 PM. The key performance criterion for the algorithm is how well it can manage the tradeoff between scheduling loads and regulating the power generator utilization, subject to risk-limiting constraints. Fig. 2 shows this tradeoff, using the iterative optimization method (8), for the timeframe under investigation. It can be observed that the scheduling of loads decreases whenever there is a risk of the power supply being

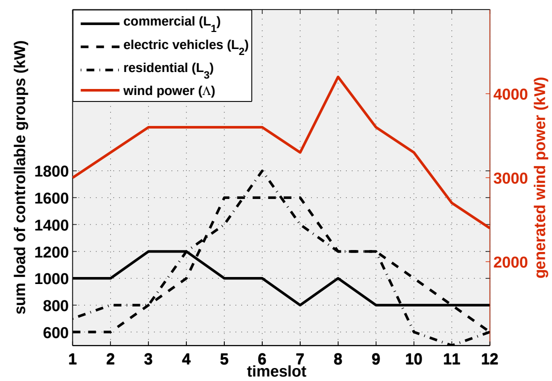

Fig. 1. Loads of controllable groups and wind power

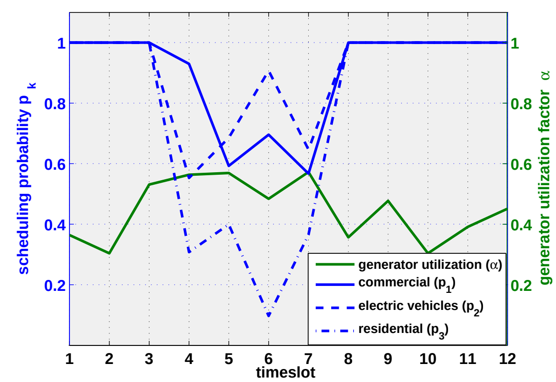

Fig. 2. Comparison of scheduling probabilities and generator utilization

inadequate. The algorithm selects, based on the defined cost functions, an optimized set of p_k and α for each timeslot. The practicality of this method for day-ahead planning is demonstrated by the scheduling probabilities “recovering” to a high level after the critical high-demand period between 4-7 PM.

5. CONCLUSION

An algorithm for stochastic load scheduling under outage-risk limiting constraints in smart microgrids has been developed. Load scheduling is performed by a central control unit which, transmits a broadcast message with scheduling probabilities to different groups of controllable applicances in the network. We formulated an optimization problem for scheduling probabilities and generator utilization based with the objective of economic dispatch. The volatile nature of renewable energy sources and stochastic loads was accounted for with an outage-risk limiting constraint. The feasibility of this method for day-ahead planning in a medium-large sized microgrid with renewable and conventional energy generators was demonstrated with a simulation.

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