SAR IMAGE TARGET RECOGNITION USING KERNEL SPARSE REPRESENTATION BASED ON RECONSTRUCTION COEFFICIENT ENERGY MAXIMIZATION RULE

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ABSTRACT

The typical classification rule for kernel sparse representation-based classifier (KSRC) is the reconstruction error minimization rule. Its computational complexity mainly depends on both the dimensionality of a subspace and the number of training samples. This paper presents an alternative classification rule, called reconstruction coefficient energy maximization, for KSRC and applies it to target recognition in synthetic aperture radar (SAR) images. The computational complexity of this rule is related to only the number of training samples, which is smaller than that of the reconstruction error minimization rule. Experimental results on the Moving and Stationary Target Acquisition and Recognition (MSTAR) public database indicate that KSRC is very promising in SAR image target recognition., and the reconstruction coefficient energy maximization rule outperforms the reconstruction error minimization rule in KSRC.

Index Terms— Sparse representation, kernel methods, reconstruction error minimization, synthetic aperture radar (SAR) image target recognition, Moving and Stationary Target Acquisition and Recognition

1. INTRODUCTION

Ssynthetic aperture radar (SAR) can generate all weather, 24-hour a day, high-resolution images with abundant information of amplitude, phase and polarization [1]. For military defense and civil applications, there are two interesting topics, or SAR image target recognition and SAR image classification. The goal of SAR image target recognition is to detect targets in SAR images each of which contain only a target [2, 3]. While SAR image classification is to separate different targets in an SAR image [4–6]. Here, we focus on the topic of target recognition in SAR images.

Many algorithms have been proposed for SAR image target recognition. These algorithms are divided into two groups, or template-based algorithms [3, 7] and feature-based classification algorithms [8–10]. Generally, feature-based classification algorithms can provide better generalization performance than template-based algorithms.

Recently, sparse representation-based classifier (SRC) and its kernel version (KSRC) were proposed in [11] and [12], respectively. Since SRC and its variants have good performance on high-dimensional face data, they have attracted substantial attention in many applications including target recognition in SAR images. In [13], a variant of SRC was proposed for automatic target classification in SAR images and obtained good performance. In [14], sparse representation is used to describe local features in sub-regions which are generated by using a spatial pyramid approach. To deal with multi-view automatic target recognition, a joint sparse representation method was proposed in [15]. It is known that pose estimation from SAR images itself is a very challenging problem. However, there is no need for any pose estimation when applying sparse representation to target recognition [13, 15], which is a quiet charming merit.

There are two important issues in SRC or KSRC. One issue is sparse representation methods, or how to sparsely represent a test sample using training samples. The other issue is about classification rules, or how to assign an estimated label to the test sample. At present, a lot of research focus on sparse representation methods [13, 15]. A reconstruction error minimization (REM) rule is adopted in SRC [11], which is the general rule in sparse representation methods including KSRC. We cannot ensure that the REM rule is the optimal one. In addition, the computational complexity of the REM rule is about $O(nd)$, where d is the dimensionality of a subspace and n is the number of training samples.

This paper proposes a reconstruction coefficient energy maximization (RCEM) rule for KSRC and applies it to target recognition in SAR images. The reconstruction coefficient energy maximization rule is inspired by the original idea behind SRC. Ideally, a valid test sample can be sufficiently represented using only the training samples of the same subject [11]. In other words, the reconstruction coefficients corresponding to the other classes are zero. Thus, we could directly use reconstruction coefficients to classify a test sample instead of using the reconstruction error minimization rule. Moreover, the reconstruction coefficient energy maximization rule has a computational complexity of $O(n)$ which is smaller

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than the REM rule.

The contribution here is to introduce KSRC to SAR image target recognition and propose an alternative classification rule for KSRC. The rest of this paper is arranged as follows. Section 2 introduces KSRC and presents the novel classification rule for KSRC. Section 3 shows experimental results and Section 4 concludes this paper.

2. RECONSTRUCTION COEFFICIENT ENERGY MAXIMIZATION FOR KSRC

KSRC is a nonlinear extension of SRC by introducing kernel tricks. In [12], KSRC shows its good performance on the application of face recognition. As far as we know, KSRC has been not applied to SAR image target recognition. The goal of this paper is to apply KSRC to SAR image target recognition. In the following, we first introduce KSRC and then present the alternative classification rule for KSRC.

2.1. Kernel sparse representation-based classifier

2.1.1. Kernel sparse representation

Consider a c-class classification task. Let the training set be $\{\mathbf{x}_i, y_i\}_{i=1}^n$, where n is the total number of training samples, $\mathbf{x}_i \in \mathcal{X} \subset \mathbb{R}^m$, $\mathcal{X}$ is an input space with the dimensionality of m, and $y_i \in \{1, 2, \cdots, c\}$. In KSRC, the data are mapped from the input space $\mathcal{X}$ into a high-dimensional (possibly infinite dimensional) kernel feature space $\mathcal{F}$ by using a nonlinear mapping function Φ.

In $\mathcal{F}$, we can linearly represent the image of a test sample in terms of the images of all training samples. Namely,

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i \Phi(\mathbf{x}_i) = \mathbf{\Phi}\boldsymbol{\alpha} \tag{1}$$

where $\Phi(\mathbf{x})$ is the image of $\mathbf{x}$ in $\mathcal{F}$, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_n]^T$ is the coefficient vector, α_i are the coefficients corresponding to the images $\Phi(\mathbf{x}_i)$ of training samples, and $\mathbf{\Phi}$ is the training sample matrix in $\mathcal{F}$.

It is necessary to reduce dimensionality of $\mathcal{F}$ since the space $\mathcal{F}$ has a very high or possibly infinite dimensionality. Let $\mathbf{P}$ be a transformation matrix. The both sides on (1) are multiplied by $\mathbf{P}$, and following equality results.

$$\mathbf{P}^T \Phi(\mathbf{x}) = \mathbf{P}^T \mathbf{\Phi}\boldsymbol{\alpha} \tag{2}$$

We introduce a pseudo-transformation matrix $\mathbf{B}$ and let $\mathbf{P} = \mathbf{\Phi}\mathbf{B}$. Thus, we have

$$\mathbf{B}^T \mathbf{k}(\cdot, \mathbf{x}) = \mathbf{B}^T \mathbf{K}\boldsymbol{\alpha} \tag{3}$$

where $\mathbf{k}(\cdot, \mathbf{x}) = [k(\mathbf{x}_1, \mathbf{x}), \cdots, k(\mathbf{x}_n, \mathbf{x})]^T = \mathbf{\Phi}^T \Phi(\mathbf{x})$, $\mathbf{K} = \mathbf{\Phi}^T \mathbf{\Phi} \in \mathbb{R}^{n \times n}$ is the kernel Gram matrix which is symmetric and positive semi-definite, and $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$.

Finally, kernel sparse representation can be cast into the following optimization problem:

$$\min_{\boldsymbol{\alpha}} \quad \|\boldsymbol{\alpha}\|_1 \tag{4}$$
$$\text{subject to} \quad \|\mathbf{B}^T \mathbf{k}(\cdot, \mathbf{x}) - \mathbf{B}^T \mathbf{K}\boldsymbol{\alpha}\|_2 \leq \varepsilon$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively denote the ℓ_1-norm and ℓ_2-norm, and ε is a small positive constant, say 10^{-3}.

2.1.2. Classification rule

The convex problem (4) can be efficiently solved [12,16]. The solution to (4) gives the reconstruction coefficient vector $\boldsymbol{\alpha}$. For a given test sample $\mathbf{x}$, we assign a label $\hat{y}$ to it by using the reconstruction error minimization rule. Namely,

$$\hat{y} = \arg \min_{i=1,\cdots,c} \quad r_i(\mathbf{x}) = \|\mathbf{B}^T \mathbf{k}(\cdot, \mathbf{x}) - \mathbf{B}^T \mathbf{K}\boldsymbol{\delta}_i\|_2 \tag{5}$$

where $r_i(\mathbf{x})$ is the reconstruction error generated from the i-th class, and

$$\boldsymbol{\delta}_i = [\delta_i(\alpha_1), \delta_i(\alpha_2), \cdots, \delta_i(\alpha_n)]^T \tag{6}$$

where the characteristic function δ_i can pick up the coefficients corresponding to the i-th class, or

$$\delta_i(\alpha_j) = \begin{cases} \alpha_j, & if \;\; y_j = i \\ 0, & otherwise \end{cases} \tag{7}$$

2.2. Reconstruction coefficient energy maximization

The reconstruction error minimization rule is a classical classification one in sparse representation methods. However, its optimality cannot be ensured. Moreover, it has a computational complexity of $O(nd)$ when ignoring the computation between $\mathbf{B} \in \mathbb{R}^{d \times n}$ and $\mathbf{K} \in \mathbb{R}^{n \times n}$, where d is the dimensionality of the subspace.

Here, we propose an alternative classification rule, or reconstruction coefficient energy maximization rule. This rule is inspired by the original idea behind SRC. Ideally, a valid test sample can be sufficiently represented using only the training samples of the same subject [11]. In other words, the reconstruction coefficients corresponding to the other classes are zero. Thus, we could directly use reconstruction coefficients to classify a test sample. The reconstruction coefficient energy maximization rule can assign a label $\hat{y}$ to a test sample $\mathbf{x}$ in terms of the following expression.

$$\hat{y} = \arg \max_{i=1,\cdots,c} \quad e_i(\mathbf{x}) = \|\boldsymbol{\delta}_i\|_2^2 \tag{8}$$

where $e_i(\mathbf{x})$ is the reconstruction coefficient energy of the i-th class on $\mathbf{x}$.

By (8), the reconstruction coefficient energy maximization rule is related to only reconstruction coefficients. Its computational complexity is about $O(n)$, which is independent of the dimensionality of the subspace. Obviously, the computational complexity of the reconstruction coefficient energy maximization rule is smaller than that of the reconstruction error minimization rule.

3. EXPERIMENTAL RESULTS

Usually, the Moving and Stationary Target Acquisition and Recognition (MSTAR) public database [17] is used to validate the performance of algorithms. Three targets considered are the T72 tank, the BTR70 personnel carrier and the BMP2 tank. This database contains the log-intensity images of them collected at 15-degree and 17-degree depression angles, respectively. We take the SAR images from the 17-degree set as the training set, and those from the 15-degree set as the testing set. The information about training and testing sets are shown in Table 1. SAR images in the MSTAR database are originally 128×128. We down-sample them by a factor of two in each dimension to 64×64. Therefore, the original features of each SAR image is obtained by stacking its columns and the number of feature is 4096.

We compare our scheme (KSRC with RCEM) with KSRC with REM and SRC. The quadratically constrained ℓ_1-minimization problem for both KSRC and SRC is solved by using ℓ_1-MAGIC software package [18]. In KSRC and SRC, let $\varepsilon = 0.001$. The RBF kernel $k(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\gamma\|\mathbf{x}_i - \mathbf{x}_j\|_2^2)$ is used in KSRC, where the parameter $\gamma > 0$ is the kernel parameter. For SRC, 4096 is still a large feature number, so we perform random projection to reduce dimensionality. For KSRC, the kernel Gram matrix depends on the number of training samples, here 1622. For fairness, random projection is also performed in KSRC.

All numerical experiments are performed on the personal computer with a 2.93GHz Inter(R) Core(T)2 Duo CPU and 2G bytes of memory. This computer runs on Windows XP, with MATLAB 7.01 and VC++ 6.0 compiler installed.

Table 1. Information on training and testing sets from the MSATR database

Target	Training (17-degree)	Testing (15-degree)
BMP2(sn−c9563)	233	194
BMP2(sn−c9566)	232	196
BMP2(sn−c21)	233	196
BTR70	233	196
T72(sn−132)	232	196
T72(sn−812)	231	195
T72(sn−S7)	228	191
SUM	1622	1364

3.1. Selection of kernel parameter

In this experiment, we consider the effect of the kernel parameter γ on KSRC with different classification rules. In [12], a median value of $1/(\|\mathbf{x}_i - \bar{\mathbf{x}}\|^2), i = 1, \cdots, n$ is adopted, where $\bar{\mathbf{x}}$ being the mean of all training samples. Let σ be this median value. We make a little change on this setting. Let $\gamma = p\sigma$ with p being a positive constant and selected from the set $\{2^{-3}, 2^{-2}, \cdots, 2^3\}$. Let $d = 100$ be the dimensionality of the subspace. We perform 10 runs because of the use of random projection and report the average results in Figure 1.

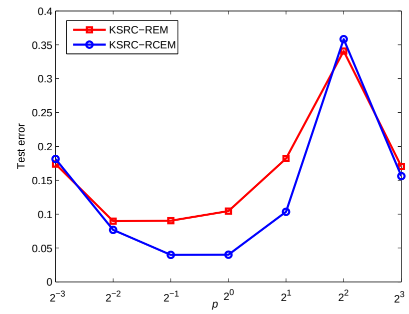


Fig. 1. Performance of KSRC with different classification rules under different kernel parameter views.

From Figure 1, we can see that KSRC with RCEM has much better performance than KSRC with REM when $2^{-2} \le p \le 2^1$. When $p = 2^{-2}$, KSRC with REM gets its best average performance, or a test error of 8.98%. For KSRC with RCEM, the best average performance (3.98%) is obtained at $p = 2^{-1}$. In the following experiments, the kernel parameter is set to be $2^{-2}\sigma$ and $2^{-1}\sigma$ for KSRC-REM and KSRC-RCEM, respectively.

3.2. Comparison of two classification rules

To compare KSRC-RCEM with KSRC-REM, we give a detail illustration. First, a test sample is selected from the target BMP2. Then, we use KSRC with two classification rules to classify it, respectively. Figure 2(a) shows the reconstruction coefficients obtained by KSRC, and Figure 2(b) gives selected training samples corresponding to the first seven large coefficients. The reconstruction errors and coefficient energy values of three targets for this test sample is shown in Table 2. The best performance is bolded in this table. Obviously, this test sample is misclassified by the REM rule. The RCEM rule can correctly recognize it.

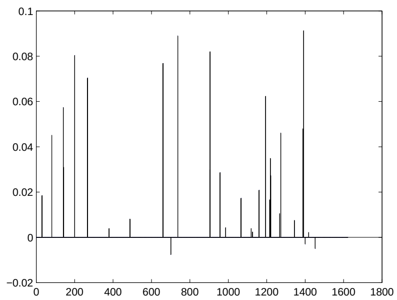
3.3. Effect of subspace dimensionality

To validate the effect of the subspace dimensionality d on the performance of algorithms, we vary d from 20 to 120 at intervals of 20. The other setting is the same as above. To eliminate the randomicity induced by random projection, we perform 10 trials for each d and report the average results in Figure 3. Since RCEM is a general rule, it could also be applied to SRC. Here, we have four compared methods, SRC with REM, SRC with RCEM, KSRC with REM, and KSRC with RCEM.

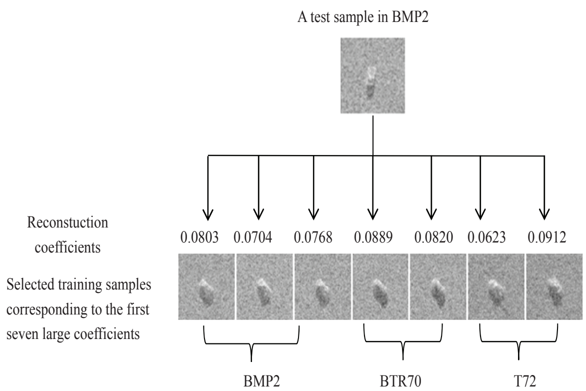
Observation on Figure 3 indicates that the four algorithms improve their performance when the subspace dimensionality

Table 2. Results of two classification rules on the test sample

	BMP2	BTR70	T72
Reconstruction error	0.6101	0.8070	**0.5837**
Reconstruction coefficient energy	**0.1549**	0.1248	0.1437



(a) Reconstruction coefficients



(b) Selected SAR images from three targets

Fig. 2. Kernel sparse representation based the test sample for BMP2. (a) Reconstruction coefficients, and (b) Selected SAR images from three targets.

is increasing. Moreover, we find that the novel rule is not efficient when applying to SRC. But, this rule does work well in KSRC. RCEM could get better performance than REM does when $d > 40$. In addition, KSRC always has better performance than SRC.

To see the CPU running time of four algorithms, Table 3 is given. The CPU running time means the running time for all test samples, including the time of both solving convex programming and classifying test samples. Comparison of KSRC and SRC shows that KSRC has faster running time when $d \geq 60$. On two classification rules, RCEM is much faster than REM in most cases, which supports the analysis of computational complexity.

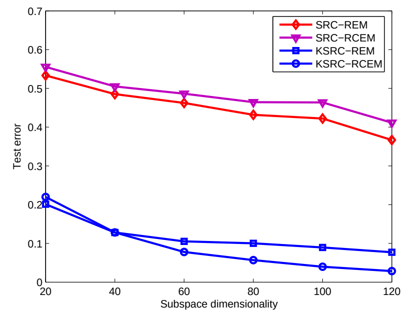


Fig. 3. Performance of four methods under different subspace dimensionality levels.

Table 3. CPU running time of four algorithms (sec.)

d	SRC-REM	SRC-RCEM	KSRC-REM	KSRC-RCEM
20	301.60	301.44	317.02	318.13
40	495.65	494.33	507.41	505.05
60	741.11	744.88	720.15	716.83
80	1037.46	1036.00	934.74	911.30
100	1386.36	1379.50	1105.40	1086.21
120	1648.69	1643.09	1217.57	1196.97

4. CONCLUSION

An alternative classification rule for KSRC is presented in this paper and applied to SAR image target recognition. The computational complexity of this rule is related to only the number of training samples, which is smaller than that of the reconstruction error minimization rule. Experimental results on the MSTAR database are carried out. In experiments, random projection is used to reduce the dimensionality of SAR images to a certain range. Under the our experimental conditions, the greater the dimensionality is, the lower the test error is. KSRC is much better than SRC from the dimensionality of 20 to 120. In addition, KSRC with RCEM outperforms KSRC with REM on the classification performance. In both SRC and KSRC, the RCEM rule has a faster running speed than the REM rule in most cases.

In the current work, the kernel Gram matrix obtained from training samples is directly taken as the dictionary of sparse representation. Dictionary learning is quiet popular in sparse representation. In the future, we plan to learn a compact dictionary from the training data and apply it to SAR image target recognition.

5. REFERENCES

[1] C. V. Jakowatz, D. E. Wahl, P. H. Eichel, D. C. Ghiglia, and P. A. Thompson, *Spotlight-mode Synthetic Aperture Radar: A Signal Processing Approach*, Kluwer Academic Publishers, 1996.

[2] B. Bhanu, D. E. Dudgeon, E. G. Zelnio, A. Rosenfeld, D. Casasent, and I.S. Reed, "Guest editorial introduction to the special issue on automatic target detection and recognition," *IEEE Transactions on Image Processing*, vol. 6, no. 1, pp. 1–6, 1997.

[3] G.J. Owirka, S.M. Verbout, and L.M. Novak, "Template-based sar atr performance using different image enhancement techniques," in *Proceedings of SPIE*, 1999, vol. 3721, pp. 302–319.

[4] L. Zhang, W. D. Zhou, T. T. Su, and L. C. Jiao, "Decision tree support vector machine," *International Journal on Artificial Intelligence Tools*, vol. 16, no. 1, pp. 1–16, 2007.

[5] V. V. Chamundeeswari, D. Singh, and K. Singh, "An analysis of texture measures in pca-based unsupervised classification of sar images," *IEEE GEOSCIENCE AND REMOTE SENSING LETTERS*, vol. 6, no. 2, pp. 214–218, 2009.

[6] J. Feng, L. C. Jiao, X. Zhang, , and D. Yang, "Bag-of-visual-words based on clonal selection algorithm for sar image classification," *IEEE GEOSCIENCE AND REMOTE SENSING LETTERS*, vol. 8, no. 4, pp. 691–695, 2011.

[7] R. Patnaik and D. Casasent, "SAR classification and confuser and clutter rejection tests on mstar ten-class data using MINACE filters," *Optical Pattern Recognition*, vol. 6574, pp. 657402.1ł657402.15, 2007.

[8] S. K. Rogers, J. M. Colombi, C. E. Martin, J. C. Gainey, K. H. Fielding, T. J. Burns, D. W. Ruck, M. Kabrisky, and M. Oxley, "Neural networks for automatic target recognition," *Neural Networks*, vol. 8, no. 7-8, pp. 1153–1184, 1995.

[9] Q. Zhao and J.C. Principe, "Support vector machines for sar automatic target recognition," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 37, no. 2, pp. 643–654, 2001.

[10] R. Huan and Y. Pan, "Decision fusion strategies for sar image target recognition," *IET Radar, Sonar and Navigation*, vol. 5, no. 7, pp. 747–755, 2011.

[11] J. Wright, A. Y. Yang, A. Ganesh, S. S. Sastry, and Y. Ma, "Robust face recognition via sparse representation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 31, no. 2, pp. 210–226, 2009.

[12] L. Zhang, W.-D. Zhou, P.-C. Chang, J. Liu, Z. Yan, T. Wang, and F.-Z. Li, "Kernel sparse representation-based classifier," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1684–1695, 2012.

[13] J. J. Thiagarajan, K. N. Ramamurthy, P. Knee, A. Spanias, and V. Berisha, "Sparse representations for automatic target classification in sar images," in *Proceedings of the 4th International Symposium on Communications, Control and Signal Processing*, Limassol, Cyprus, 3-5 March 2010.

[14] P. Knee, J. J. Thiagarajan, K. N. Ramamurthy, and A. Spanias, "Sar target classification using sparse representations and spatial pyramids," in *Proceedings of the 2011 IEEE Radar Conference*, 23-27 May 2011, pp. 294 – 298.

[15] H. Zhang, N. M. Nasrabadi, Y. Zhang, and T. S. Huang, "Multi-view automatic target recognition using joint sparse representation," *IEEE Transactions On Aerospace and Electronic Systems*, vol. 48, no. 3, pp. 2481–2497, 2012.

[16] L. Zhang, W.-D. Zhou, and F.-Z. Li, "Kernel sparse representation-based classifier ensemble for face recognition," *Multimedia Tools and Applications*, pp. 1–15, 2013.

[17] E. R. Keydel, "MSTAR extended operating conditions," in *Proceedings of SPIE*, 1996, vol. 2757, pp. 228–242.

[18] E. Candès and J. Romberg, "ℓ_1-magic: Recovery of sparse signals via convex programming," http://www.acm.caltech.edu/l1magic/, Oct. 2005.