DETECTION OF DROPS MEASURED BY THE TIME SHIFT TECHNIQUE FOR SPRAY CHARACTERIZATION

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ABSTRACT

Characterizing drops in a spray process is of high interest in many areas, such as car painting or spray drying. The Time Shift (TS) technique provides an efficient and accurate way to optically measure size and velocity of individual droplets in sprays. Its realization in practice is not wide spread, thus the necessary signal processing of the measured data has not yet been fully developed or optimized. However, the TS technique is the only technique deemed suitable for online spray monitoring. In this study, we derive a filtering concept by using only a single filter that can be used for detection of droplets measured by the TS technique. We show that our approach is optimal in terms of detection power. Additionally, we show that the average detection power does not exceed certain limits, close to the one of a conventional matched filter bank.

Index Terms— Time Shift Technique, Detection, Spray Characterization

1. INTRODUCTION

Spray characterization is important for a variety of applications, such as spray painting or spray drying, whereby the efficiency and quality of the spray process depends on atomization parameters like flow rate, injection pressure, airflow rate etc. that directly influence drop sizes and velocities. For instance, in coating processes, small droplets lead to overspray, whereas large drops lead to surface defects. Therefore, spray characterization methods are essential tools for quality assurance, development and optimization of these processes; a review of measurement methods and corresponding techniques for spray characterization is available in [1].

The time shift (TS) technique provides an efficient and accurate method to measure size and velocity of individual droplets in sprays in real time [2], [3], [4]. Up to now it is the only technique which is capable of measuring not only transparent but also non-transparent particles and droplets, which often occur in industrial applications. In addition, this technique does not require post-factory alignment and it can be operated in backscatter.

It was first introduced by Semidetnov in 1985 [5] and was further developed by Damaschke *et al.* in 2002 [6], [7]. The TS technique has also been called the pulse displacement technique and several variations have been discussed by Lin *et al.* [8]. The new developments and validation of this technique can be found in [9], [10] and [11]. This technique is based on the light scattering of drops passing a shaped laser beam. The scattered light is detected by at least two sensors located at different scattering angles. The time shift between two acquired signals and the characteristics of the measured signals directly depends on drop size and velocity.

Even though the TS principles go back to 1985, its realization in practice is not wide spread. As a consequence, the signal processing of the measured data has not yet been extensively investigated or developed. Dealing with high sampling rates due to the large number of drops per time, requires fast signal processing techniques. Additionally, a cheap sensor production that limits resources on the FPGA is necessary.

In this study, we introduce a mathematical framework to allow the detection of drops measured by the TS technique. Typically, a generalized matched filtering concept as in [12] can be applied to identify whether a drop is present in the measured signal or not. However, this concept requires a filter bank that is not feasible for the TS technique, since space and resources are strongly limited on the FPGA to allow a low-cost production. In this contribution, we replace the matched filter bank with a single filter that is applied for the detection of drops. The filter is designed in such a way that its detection power is maximized when a specific distribution of the expected drop characteristics is assumed.

The paper is structured as follows: in Section 2, we explain the measurement principle of the TS technique and the applied sensor setup. Section 3 describes the detection of drops measured by the TS technique. This includes a mathematical framework, the matched filter bank and its reduction to a single optimal filter. Section 4 presents the results by comparing the filter bank with the single optimal filter. The paper finishes with a conclusion and a short outlook in Section 5.

(a) (b)

Fig. 2: (a) Sensor setup, including two laser beams and four detectors. (b) Schematic sensor setup with laser distance b, beam width w and scattering angle Θ_s at a specific working distance WD.

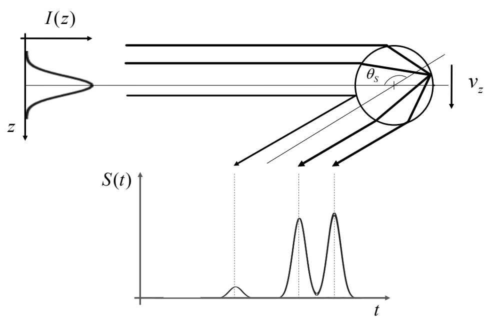


Fig. 1: Received signal $S(t)$ at scattering angle Θ_S caused by a transparent drop with velocity v_z in z direction, focused by a Gaussian laser beam with intensity $I(z)$.

2. MEASUREMENT PRINCIPLE

We will briefly summarize the measurement principle, while a detailed description of the TS technique can be found in Albrecht *et al.* [6]. To characterize sprays using the TS technique, a shaped (typically Gaussian) laser beam is focused at the measurement position in the spray. When a drop passes the shaped beam, it transforms the intensity of the laser in space into a time dependent signal on a detector. The light scattered from a single spherical particle can be interpreted as the superposition of all scattering orders present at the detector location. The intensity of the scattering orders are described by the Debye series [13] expansion of the Mie [14] scattering functions, or by using a geometric optics approach [15], [16] to compute the scattered field. When a particle passes through the focused light beam, the scattered light is detected by photodetectors focused onto the scattering center. Each photodetector provides a time signal known as a time-shift signal (Fig. 1). Depending on the scattering angle and relative refractive index, different scattering orders and their modes can appear at any one scattering angle [7], consequently, through placement of the detector, certain scattering orders can be selected. The signals from the detectors placed above or below the incident beam exhibit signals which are mirrored in time. The applied conventional sensor system uses four different sensors, displaced in space (Fig. 2a and Fig. 2b), resulting in four different time series each capturing the mentioned light intensities.

3. DETECTION OF DROPS

3.1. Framework

For simplicity, we assume our sensing system measures simultaneously four sampled time series, containing either the information of a present droplet or not. Let $\mathbf{r}^{(i)}$ be the sampled time series received from the i_{th} sensor, i.e. $i = 1, 2, 3, 4$ and $\mathbf{r} \in R^N$ with N denoting the number of samples. Furthermore, let $\mathbf{s} \in R^N$ be the ideal sampled signal, capturing the reflection and refraction of light when a droplet passes a focused (in our case Gaussian) laser beam. Exemplarily we focus on a spray process containing transparent droplets, e.g. water (Fig. 1). We know from light scattering physics that the ideal signal received at time t is given by

$$s(t) = \sum_{p=1}^{P} A_p \text{exp}\left(-(t-t_p)^2/\sigma_0^2\right), \tag{1}$$

where P, A_p, t_p and σ_0 are parameters, depending on the droplet characteristics, the sensor settings and the relative flight path between droplet and sensor [9], [2]. We define a parameter vector $\mathbf{\Theta} \in R^D$ containing all D unknown parameters that characterize the signal pattern caused by an individual droplet. Hence, we write $\mathbf{s}(\mathbf{\Theta})$, i.e. the ideal signal $\mathbf{s}$ is parameterized by an unknown parameter vector $\mathbf{\Theta}$. The following model is used to describe the received data $\mathbf{r}$, now for a fixed, individual sensor:

$$\mathbf{r} = k\mathbf{s}(\mathbf{\Theta}) + \mathbf{n}, \tag{2}$$

with noise vector $\mathbf{n} \in R^N$ and a constant $k > 0$ if a droplet is present or $k = 0$ if no drop occurs. In this work, the aim is to detect whether a droplet is present or not. Neglecting its complexity, the problem can be identified as a detection of a known signal with unknown parameters in noise. A common way to solve the described problem is to use hypothesis testing to decide whether $k > 0$ (hypothesis $\mathcal{H}_1$) or $k = 0$ (hypothesis $\mathcal{H}_0$).

3.2. Matched Filter

To begin with, we need to recap the matched filter concepts fitted to our scenario. Let us assume the two hypotheses

$$\mathcal{H}_1 : \mathbf{r} = \mathbf{s}(\mathbf{\Theta}) + \mathbf{n} \quad (3)$$
$$\mathcal{H}_0 : \mathbf{r} = \mathbf{n} \quad (4)$$

where $\mathbf{\Theta}$ is known and $\mathbf{n}$ is Gaussian noise $\mathbf{n} \sim \mathcal{N}(0, \sigma^2)$. For simplicity use $\mathbf{s}$ instead of $\mathbf{s}(\mathbf{\Theta})$. Having received a measured signal, a Neyman Person test [17] is applied to choose a particular hypothesis:

$$\frac{P(\mathbf{r}|\mathcal{H}_1)}{P(\mathbf{r}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda \quad (5)$$

where the left hand side is compared to some threshold λ that is found from constraints. Typically, Eq. (5) is written as

$$T(\mathbf{r}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda_0 \quad (6)$$

where $T(\mathbf{r}) = \mathbf{s}^{\mathbf{T}}\mathbf{r}$ is the test statistic that is compared to a threshold $\lambda_0 = \log(\lambda)$. However, the key point is that $\mathbf{\Theta}$ is generally unknown. A typical way out is the generalized matched filtering concept as in [12] that requires a matched filter bank (MFB) with the size related to the parameter space of $\mathbf{\Theta}$. Here, for each possible $\mathbf{\Theta}$ a filter is designed that matches the income. Being limited in resources on the FPGA for the detection part, a work-around is essential.

3.3. Single Optimal Filter

This section derives a work-around to avoid the large filter bank that is not realizable for the TS technique in practice. Suppose we do not receive $\mathbf{r}$ but $\mathbf{r}^*$:

$$\mathcal{H}_1 : \mathbf{r}^* = \mathbf{s}(\mathbf{\Theta}^*) + \mathbf{n} \quad (7)$$
$$\mathcal{H}_0 : \mathbf{r}^* = \mathbf{n} \quad (8)$$

where $\mathbf{\Theta}^*$ is a random variable, capturing the information of different drop characteristics. For simplicity we use $\mathbf{s}^*$ instead of $\mathbf{s}(\mathbf{\Theta}^*)$. Similar as in Section 3.2 the test statistic and its distributions are now given by

$$T^*(\mathbf{r}) = \mathbf{s}^T\mathbf{r}^* \quad (9)$$
$$T^*(\mathbf{r}|\mathcal{H}_1, \mathbf{s}^*) \sim \mathcal{N}(\mathbf{s}^T\mathbf{s}^*, \sigma^2\mathbf{s}^T\mathbf{s}) \quad (10)$$
$$T^*(\mathbf{r}|\mathcal{H}_0, \mathbf{s}^*) \sim \mathcal{N}(0, \sigma^2\mathbf{s}^T\mathbf{s}) \quad (11)$$

The probability of falsely detecting a drop α (false alarm rate) is then defined as

$$\alpha = P(T^* > \lambda_0|\mathcal{H}_0, \mathbf{s}^*) \quad (12)$$
$$= 1 - \Phi\left(\frac{\lambda_0}{\sigma\sqrt{(\mathbf{s}^T\mathbf{s})}}\right) \quad (13)$$

where Φ is the standard normal cumulative distribution function. Similarly, the probability of missing a droplet β (miss detection) is given by

$$\beta = P(T^* < \lambda_0|\mathcal{H}_1, \mathbf{s}^*) \quad (14)$$
$$= \Phi\left(\frac{\lambda_0 - \mathbf{s}^T\mathbf{s}^*}{\sigma\sqrt{(\mathbf{s}^T\mathbf{s})}}\right) \quad (15)$$

Since $\mathbf{s}$ and $\mathbf{s}^*$ are parameterized by $\mathbf{\Theta}$ and $\mathbf{\Theta}^*$, respectively, the detection power P_D is given by

$$P_D(\mathbf{\Theta}, \mathbf{\Theta}^*, \alpha) = 1 - \beta \quad (16)$$
$$= 1 - \Phi\left(\Phi^{-1}(1-\alpha) - \frac{\mathbf{s}^T\mathbf{s}^*}{\sigma\sqrt{(\mathbf{s}^T\mathbf{s})}}\right) \quad (17)$$

Note that P_D is identical to the one of a conventional matched filter when $\mathbf{s} = \mathbf{s}^*$, which is also known as a most powerful test [19]. However, since $\mathbf{s}$ is unknown in our scenario, $\mathbf{s} \neq \mathbf{s}^*$ and the detection power is decreased. To tackle the loss, we aim for an $\mathbf{s}$ that still guarantees detection power being close to the matched filter. In the described scenario, our goal is to find a $\mathbf{\Theta}$ to obtain an optimal P_D, independent of $\mathbf{\Theta}^*$. Treating $\mathbf{\Theta}^*$ as a random variable, we want to find a particular $\mathbf{\Theta}^*$ that maximizes the expected detection power of P_D:

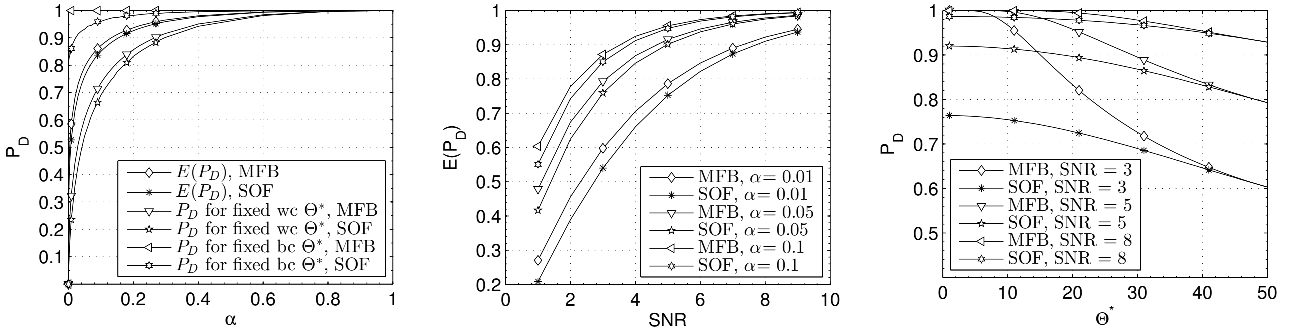
$$\hat{\mathbf{\Theta}}^* = \underset{\mathbf{\Theta}}{\operatorname{argmax}}\ E_{\mathbf{\Theta}^*}(P_D(\mathbf{\Theta}, \mathbf{\Theta}^*)) \quad (18)$$

where $E_{\mathbf{\Theta}^*}(\cdot)$ is the expected value with respect to the random variable $\mathbf{\Theta}^*$. We call the obtained filter from Eq. (18) the Single Optimal Filter (SOF).

4. RESULTS

The setup to analyze the performance of the SOF is the following. For simplicity, let $\mathbf{\Theta}$ contain only the velocity v of the droplet. The velocity v is related to Eq. (1) by $v = w\sigma_0/\sqrt{2}$, where $w = 5\mu m$. Note that similar to v, Θ will be measured in m/s. A reasonable assumption for the distribution of the drop velocity is a uniform distribution to not favor a particular drop velocity. Since a typical velocity range for flat fan nozzles at operation pressure does not exceed $50m/s$, we choose $\Theta \sim U(1, 50)$ that defines our measurement range. The data is sampled at $f_s = 40$Mhz and the SNR is defined by SNR $= A/\sigma$, where A is the amplitude of the signal.

To begin with, we show the performance of the SOF that we obtain by solving Eq. (18) numerically. Using Eq.



(a) (b) (c)

Fig. 3: (a) Average detection power ($E(P_D)$) for the matched filter bank (MFB) and the single optimal filter (SOF) over false alarm rates α, and worst case (wc) detection power (P_D) and best case (bc) detection power for MFB and SOF. (b) Detection power (P_D) over SNR for given false alarm rates α. (c) Detection power (P_D) over different drop velocities Θ^* at different SNRs.

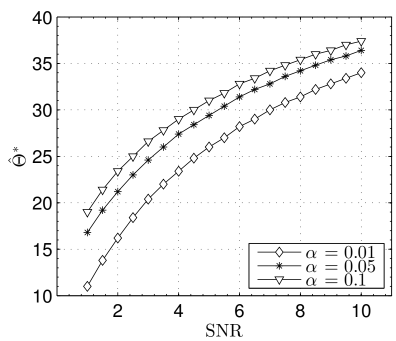


Fig. 4: Design of $\hat{\mathbf{\Theta}}^*$ depending on the SNR and the given false alarm rates α

(17), the Receiver Operating Characteristics (ROC) [18] are given in Fig. 3a. Here, we show the average detection power $E(P_D)$ obtained by averaging over the expected velocity range for both, the matched filter bank (MFB) and for the single optimal filter (SOF). Additionally, we show the worst case (wc) and the best case (bc) detection power, resulting from the hardest detectable drop type and the best detectable drop type, respectively. First, we observe that the detection power of the proposed filter (SOF) is close to a MFB that covers the complete measurement range from $1 - 50m/s$. This holds for both, the easiest detectable drop (bc) and the hardest detectable drop (wc). However, we save a lot of resources by using only a single filter instead of the complete filter bank. Fig. 3b depicts the detection power P_D for different SNR values over typical false alarm rates of $\alpha = 0.01, \alpha = 0.05$ and $\alpha = 0.1$. Again, we compare the MFB with the SOF. We observe that for an increasing SNR, the performance differences between SOF and MFB decrease. The next part of this contribution is essential for the design of the obtained SOF. Fig. 4 shows which $\hat{\Theta}^*$ to choose for a given SNR and the desired false alram rate. We observe that at higher SNR we have to choose a larger $\hat{\Theta}^*$ to design the SOF. Similar to the MFB, the SOF has different detection powers for different drop types $\mathbf{\Theta}$. Fig. 3c shows the detection power for different drop velocities in our measurement range at different SNR for a typical false alarm rate $\alpha = 0.01$. As expected, drops with lower velocity are easier to detect than faster drops. Being aware ot this fact, the following procedure is essential: Typically a distribution of all measured drop velocities is the complete outcome from spray measurement systems. Knowing the detectability of different drop characteristics, allows to adjust the overall measurement result. In particular the amount of a specific drop velocity needs to be weighted by its detectability.

5. CONCLUSION

We briefly described the concepts of the time shift technique to measure drops in a spray. We provided a mathematical framework that allows different concepts of detection. As a matched filter bank exceeds the resources and space on the FPGA, we reverted to a single filter. This filter is compared with the matched filter bank in terms of detection power. We showed that the single filter reaches a detection power that is close to the one of a complete filter bank. Future research involves the detection for a higher dimensional parameter vector $\mathbf{\Theta}$ and the extension to distributed detection [20]. Compressive sensing techniques may be considered to reduce the huge amount of data [21], [22].

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7. REFERENCES

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