INFRARED SMALL TARGET DETECTION WITH COMPRESSIVE MEASUREMENTS

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ABSTRACT

A novel scheme for infrared small target detection in compressive domain is presented. First, the original image is separated into two components, i.e., the target and the background. Next, we compress them individually. Finally, the compressed target image is utilized to construct the corresponding compressive detector to perform detection in compressive domain. Compared to conventional detection methods based on compressive sensing(CS), it not only can detect infrared small target images under diverse backgrounds in compressive domain without any reconstruction of original signal, but also possess faster detection speed and favorable detection performance, and require much less storage and computation. Experiment results indicate the scheme is effective.

Index Terms**—**compressive sensing, infrared small target detection, compressive measurements

1. INTRODUCTION

Infrared small target detection is an emerging topic in the field of defense. It plays a crucial role in infrared early warning and tracking systems. A variety of traditional detection methods have been proposed in past decades. With the emergence of compressive sensing (CS), target detection using CS catches intensive attention and a number of corresponding methods are presented as well. In existing methods using CS for target detection, such as compressive low rank and sparse decomposition method (CLSDM) [1], sparsity order detection (SOD) [2], detection is usually performed after reconstruction of the original signal. Actually, in certain cases, such as infrared small target detection, it is not necessary to recover the original signal before target detection [3-4]. However, for infrared small target images, the signal-to-noise ratio (SNR) of the compressive measurements is extremely low and hence, it is difficult to perform detection. Generally, Separation of the infrared small target from background image is considered as an effective approach to improve the SNR, such as the method proposed in [1]. It implements separation via recovery using low rank property of background and sparse property of target, respectively[5]. Motivated by [1-4] and inspired the idea of predistortion[6], a novel scheme for infrared small target detection using compressive measurements by applying compressive sensing to the background and target images separately is presented.

Different from conventional detection methods using CS, in the proposed one, target detection is performed in compressive domain without any reconstruction of signal and only the compressed target image is utilized for target detection. Hence, it has faster detection speed and requires much less storage and computation. The block diagram of this scheme is indicated in Fig. 1.

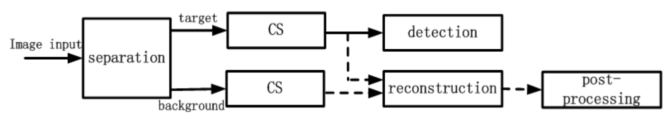

Fig. 1 Block diagram of the new scheme

2. INFRARED SMALL TARGET DETECTION WITH COMPRESSIVE MEASUREMENTS

An infrared image can be modeled as:

$$\mathbf{f} = \mathbf{f_b} + \mathbf{f_t} + \mathbf{n} \tag{1}$$

where $\mathbf{f} \in R^N$ is a vectorized form of the infrared image through row-tacking, $\mathbf{f_b}, \mathbf{f_t}, \mathbf{n} \in R^N$ are the background, target and noise vectors, respectively. Suppose that $\mathbf{\Phi} \in R^{M\times N}, M << N$ is a sensing matrix, the compressive measurements can be denoted as

$$\mathbf{y} = \mathbf{\Phi f} = \mathbf{\Phi}(\mathbf{f_b} + \mathbf{f_t} + \mathbf{n}) \tag{2}$$

The SNR (here, for target detection, the background can be viewed as noise) is defined as

$$SNR = \frac{\|\mathbf{\Phi f_t}\|_2^2}{\|\mathbf{\Phi}(\mathbf{f}_b + \mathbf{n})\|_2^2} = \frac{\|\mathbf{f_t}\|_2^2}{\|\mathbf{f}_b + \mathbf{n}\|_2^2}, \mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I}_M \tag{3}$$

Note that this SNR is very low since the target only occupies several image pixels, thus it is not suitable to perform target detection. To this end, Top-hat [7] is applied in our scheme to separate target from background image. As a result, the background component and the target component can be compressive sensed separately. The separated background and target vector can be written approximately as follows

$$\mathbf{f_t}' = \mathbf{f}_t + \mathbf{n}'$$
$$\mathbf{f_b}' = \mathbf{f_b} + \mathbf{n}'' \quad (4)$$

Where $\mathbf{f_t}' \in R^N$ consists of the target $\mathbf{f_t}$ and the noise $\mathbf{n}'$, $\mathbf{f_b}' \in R^N$ includes the background $\mathbf{f_b}$ and the noise $\mathbf{n}''$. Compressive sensing of these two signal on sensing matrix $\mathbf{\Phi_t} \in R^{M_t \times N}, \mathbf{\Phi_b} \in R^{M_b \times N} (M_t, M_b << N)$ are defined as

$$\mathbf{y_t}' = \mathbf{\Phi_t f_t}' = \mathbf{\Phi_t}(\mathbf{f_t} + \mathbf{n}')$$
$$\mathbf{y_b}' = \mathbf{\Phi_b f_b}' = \mathbf{\Phi_b}(\mathbf{f_b} + \mathbf{n}'') \quad (5)$$

Here, $\mathbf{y_t}'$ is selected to perform target detection since it has a much higher SNR than that of y. Assume that $\mathbf{n}' \sim N(0, \sigma^2 I_N)$ is the noise residual which has not been completely removed by the separation procedure, the detection problem is to distinguish the following two hypotheses

$$H_0 : \mathbf{y_t}' = \mathbf{\Phi_t n}'$$
$$H_1 : \mathbf{y_t}' = \mathbf{\Phi_t}(\mathbf{f}_t + \mathbf{n}') \quad (6)$$

where $\mathbf{\Phi_t}^{\mathbf{T}} \mathbf{\Phi_t} = \mathbf{I}_M$ if the sensing matrix is orthonormal.

The *False alarm rate* and *detection rate* can be defined respectively as follows

$$P_F = P_r(H_1 \text{chosen when } H_0 \text{ true})$$
$$P_D = P_r(H_1 \text{chosen when } H_1 \text{ true}) \quad (7)$$

In [3], the testing statistic is defined as

$$t = < \mathbf{y_t}', \mathbf{\Phi_t f_t} > \quad (8)$$

where $\mathbf{f_t}$ is assumed to be known. However, this does not always hold true in practice. Therefore, an alternative statistic t' is introduced instead as follows

$$t' = \|\mathbf{y_t}'\|_2^2 = < \mathbf{y_t}', \mathbf{y_t}' > = < \mathbf{y_t}', \mathbf{\Phi_t f_t} > + < \mathbf{y_t}', \mathbf{\Phi_t n}' > \quad (9)$$

It can be noted that [8]

$$t' \sim N(M\sigma^2, 2M\sigma^4) \quad \text{under } H_0$$
$$t' \sim N(M\sigma^2 + \|\mathbf{\Phi_t f_t}\|_2^2, 4\sigma^2 \|\mathbf{\Phi_t f_t}\|_2^2 + 2M\sigma^4) \quad \text{under } H_1 \quad (10)$$

and thus we have

$$P_F = P(t' > \gamma \mid H_0) = Q\left(\frac{\gamma - M\sigma^2}{\sqrt{2M}\sigma^2}\right)$$
$$P_D = P(t' > \gamma \mid H_1) = Q\left(\frac{\gamma - M\sigma^2 - \|\mathbf{\Phi_t f_t}\|_2^2}{\sqrt{4\sigma^2 \|\mathbf{\Phi_t f_t}\|_2^2 + 2M\sigma^4}}\right) \quad (11)$$

where γ can be determined for a prescribed false alarm rate $P_F = \alpha$ as follows

$$\gamma = \sqrt{2M}\sigma^2 Q^{-1}(\alpha) + M\sigma^2 \quad (12)$$

As a result, one gets

$$P_D(\alpha) = Q\left(\frac{Q^{-1}(\alpha)\sqrt{2M} - \frac{M}{N}\frac{\|\mathbf{\Phi_t f_t}\|_2^2}{\sigma^2}}{\sqrt{2M + 4\frac{M}{N}\frac{\|\mathbf{\Phi_t f_t}\|_2^2}{\sigma^2}}}\right), Q(z) = \int_z^\infty \exp(-\frac{\mu^2}{2}) du \quad (13)$$

Considering the RIP restricts on $\mathbf{\Phi_t}$, $P_D(\alpha)$ can be predicted as

$$P_D(\alpha) = Q\left(\frac{Q^{-1}(\alpha)\sqrt{2M} - \frac{M}{N}SNR}{\sqrt{2M + 4\frac{M}{N}SNR}}\right) \quad (14)$$

where $SNR = \frac{\|\mathbf{\Phi_t f_t}\|_2^2}{\sigma^2} = \frac{\mathbf{f_t}^T \mathbf{\Phi_t}^T \mathbf{\Phi_t f_t}}{\sigma^2} = \frac{\|\mathbf{f_t}\|_2^2}{\sigma^2}, \mathbf{\Phi}^T \mathbf{\Phi} = \mathbf{I}_M$,is much higher than that in Eq.(3). It can be seen that in the proposed scheme, the separation before sensing scheme has improved the SNR of compressive measurements for small target detection. Hence, it allows us to achieve a desirable detection probability without any reconstruction of original image at much faster speed in compressive domain.

3. EXPERIMENTAL WORK

First, we explore the detection performance of compressive detector t'. Assume that M/N=0.05, Fig.2 shows the operating characteristic curve (ROC), i.e., the relationship between P_F and P_D predicted by Eq.(14) at several SNR levels. It is noticeable that when SNR increases, the detection probability will increase rapidly. Note that separation for infrared small target image can generally improve SNR by more than 35 dB, therefore, it possesses robust performance.

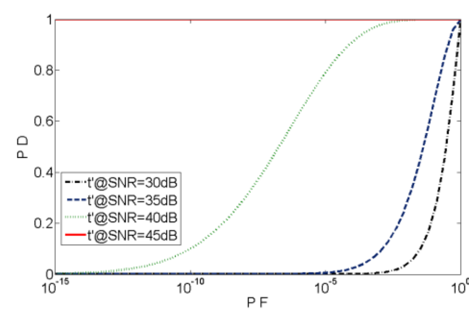

Fig. 2 Effect of SNR on P_D

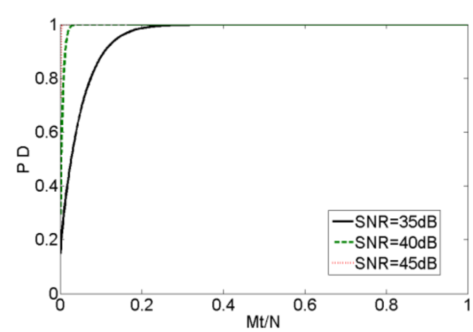

Fig. 3 Effect of M/N on P_D at several SNR levels

Then let $\alpha = 0.1$, Fig3 shows the effect of M / N on $P_D(\alpha)$ predicted by Eq.(14) at several different SNR levels. We can observe that the rate at which P_D approaches 1 is exponentially fast with a fraction of measurements at lower false alarm rate. Simultaneously, the number of measurements decreases as SNR becomes higher.

Furthermore, six hundred infrared small target images cropped from infrared devices are selected to demonstrate the detection performance of the presented scheme. In our experiments, we have $\alpha = 0.1, M / N = 0.05$ and a threshold is set on the foundation of Eq. (12) to identify the existing of target. The experimental results are shown in Fig4.We can observe that the proposed scheme has favorable performance and the detection probability is almost in accordance with the predicted one. Meanwhile, some typical infrared small target images which are capable of being detected by the proposed scheme are shown in Fig.5.

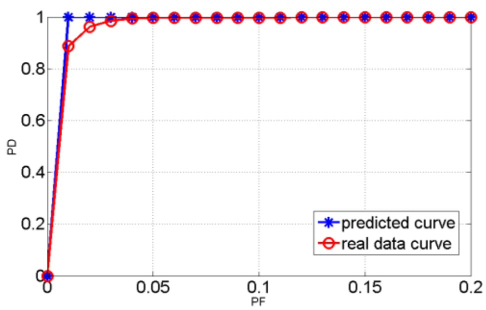

Fig. 4 Comparison of theoretical analysis and the real data experimental result

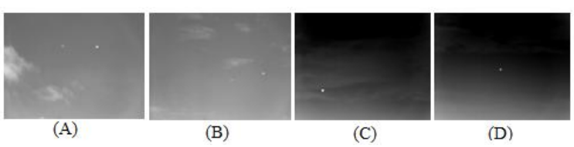

Fig. 5 Some typical infrared small target images under different backgrounds

Finally, we have the size of infrared image is $N = 256 \times 256$, $M / N = 0.2$, table1 demonstrates the execution time of several detection methods using CS. It is obvious that the execution time of our scheme is much less than the conventional methods since it gets rid of the curse of signal reconstruction, which contains some kinds of time-consuming iterative optimization algorithms.

Methods	SOD	CLSDM	Our scheme
A	7.271	9.458	0.061
B	0.765	9.099	0.060
C	0.865	8.329	0.059
D	0.205	8.323	0.062

Table1: Comparison of execution time(s) with different detection methods using CS

4. CONCLUSION

In this letter, a novel scheme for infrared small target detection in compressive domain is proposed. It is the first time to implement small target detection in compressive domain without any reconstruction of signal at much faster detection speed on the premise of ensuring high detection probability. The small target detection is easy to implement by setting a simple threshold to the norm of compressed target part. The data storage and execution time it required are much less than conventional methods. The proposed scheme can be used in infrared early warning and tracking systems.

5. ACKNOWLEDGEMENT

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