ESTIMATION EFFICIENCY, ACCURACY AND ROBUSTNESS IMPROVEMENT BY EXPLOITING THE GEOMETRY INFORMATION IN SAR-GMTI SYSTEM

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ABSTRACT

Suffering from ambiguous estimation or heavy computation complexity load, the radial velocity estimation of moving targets becomes the bottleneck of the synthetic aperture radar-ground moving target indication system. In order to improve the radial velocity estimation efficiency, we have proposed an efficient Radon transform (RT) estimation (ERTE) method by using the never exploited geometry information, which performs well in high SNR scenarios but bad for low SNR. Focusing on these, we propose the least square RT estimation (LSRTE) method to improve the estimation accuracy by utilizing the geometry information of multiple RT results. Given the geometry information determining measurement error, we modified the LSRTE into a weighted LSRTE (WLSRTE) method to improve the estimation robustness and accuracy. Experiments results validate the effectiveness of the proposed methods, and the proposed methods perform much more accurate and efficient than the conventional method.

Index Terms—SAR, GMTI, radial velocity estimation, Radon transform, geometry information

1. INTRODUCTION

Characteristic of all weather and all time working capability, radar has been widely used in civil and military applications [1-7]. Combined with the ground moving targets indication (GMTI), the synthetic aperture radar (SAR) based GMTI (SAR-GMTI) can achieve moving targets detection, motion parameters estimation and localization in the SAR image, supplying much more useful information [8-13]. The motion parameters, distinguishing the moving targets from clutter, should be estimated accurately and quickly, which even determines the capability level of the SAR-GMTI systems.

Playing an important role in moving targets localization, the radial velocity estimation can be mainly divided into phase based methods and amplitude based methods. By using the relationship between the radial velocity and the phase (or along track interferometry phase), the phase based method can estimate the radial velocity with the advantage of high accuracy. Every coin has two sides, and the phase based methods suffer from ambiguous estimation when the radial velocity is fast, because of the 2π wrapped phase. In order to solve the ambiguity, multiple phases corresponding multiple pulse repetition frequency, carrier frequencies, baselines or SAR velocities are used to estimate the radial velocity unambiguously [13, 14]. There is no free lunch, and the multiple relationships require complex system realization, resulting in low flexibility to the SAR-GMTI system. Independent from the ambiguity problem in principle, the amplitude method can estimate the radial velocity by utilizing the linear relationship between the moving targets trajectory and the radial velocity. Specialized in linear slope estimation, the Radon transform (RT) [15] is introduced to estimate the radial velocity by searching any possible value, which can be thought as the representative idea of the conventional Radon transform estimation (CRTE). Since the searching step size determines not only the computation complexity but also the estimation accuracy, the conventional amplitude based method should compromise between the estimation accuracy and the efficiency. In summary, the radial velocity suffers from ambiguous estimation (phase based method) or heavy realization / computation complexity load (the methods to solve the ambiguity), which even becomes the bottleneck of the SAR-GMTI system in application to achieve good performance.

In order to break this bottleneck, we have proposed some methods by using the never exploited geometry information [16], which performs well in high signal to noise ratio (SNR) scenarios but bad in low SNR. In this paper, we have modified our method to make it perform well even in low SNR. The least square (LS) RT estimation (LSRTE) method is presented firstly to improve the estimation accuracy in low SNR scenarios, and then the geometry information related weighted LS RT estimation (WLSRTE) method is proposed to improve the robustness to noise by analyzing the relationship between the geometry information and the measurement error.

2. SIGNAL MODEL

Based on the relationship between the range walk and the radial velocity, the radial velocity of the moving target can be estimated by [17]

$$v_r = c \cdot \tan \theta \cdot PRF/2f_s \tag{1}$$

where f_s and *PRF* denote the sampling frequency and the pulse repetition frequency, respectively, and θ is defined as the range walk angle of the moving target, and *c* is the speed of light. Thus, the radial velocity estimation turns into estimating range walk angle.

Accomplished in linear slope estimation, the RT is usually used to estimate the range walk angle and then the radial velocity. As aforementioned, the CRTE, as shown in Fig.1 (a), has to compromise between the accuracy and efficiency. In order to avoid these, we have proposed an efficient RT estimation (ERTE) method by utilizing the never exploited geometry information of the RT results [16]. The kernel can be summarized as: the parameters to be estimated are contained not only in the matched results but also in the mismatched results. The projection nature of the RT [15], as shown in Fig.1(b), is utilized to construct the geometry information model

$$L_{\theta} \left| \sin(\theta - \alpha) \right| = L_{1} \tag{2}$$

where α is the RT angle, L_{θ} and L_{1} denote the normalized length of the range walk trajectory (2-D) and of the RT result (1-D). The absolute operator can be removed as

$$L_{\theta}\sin(\theta - \alpha) = L_{1} \tag{3}$$

when $\theta > \alpha$. Since L_1 can be easily measured in the 1-D domain, twice RTs are used to cancel the unknown L_{θ} ,

$$L_{\theta} \sin(\theta - \alpha_1) / L_{\theta} \sin(\theta - \alpha_2) = L_1 / L_2$$
(4)

The measurement of L_1 can be mainly summarized as: measuring the maximum range whose normalized value is larger than the threshold of 0.5 in the RT domain [16], as shown in Fig.1(c). And then the range walk angle can be estimated via [16]

$$\tan\theta = (L_2 \sin\alpha_1 + L_1 \sin\alpha_2) / (L_2 \cos\alpha_1 + L_1 \cos\alpha_2) (5)$$

The radial velocity can be efficiently estimated by (5) and (1). From (5), we can see that the estimation accuracy is determined by the measured geometry information (L_1 and L_2). As is known, L_1 and L_2 can be measured accurately in high SNR but inaccurately in low SNR, however, the high SNR requirement cannot always be satisfied in actual SAR-GMTI system. Thus, much more practical methods should be proposed to meet the demands of estimation accuracy, efficiency and robustness.

3. THE LEAST SQUARE METHOD

In order to improve the estimation performance of the efficient RT in low SNR scenarios, we propose a lease square based RT estimation (LSRTE) method by using the multiple RT results in this section. Given the measurement error, we can modify the geometry information model as

$$L_{\theta}\sin(\theta - \alpha) + \varepsilon = L_1 \tag{6}$$

where ε denotes the measurement error of L_1 . To extend the once RT into multiple RTs, we can rewrite (6) as the following matrix form



Fig.1. Kernel of the LSRTE method. And then the geometry information mo

And then the geometry information model corresponding N times RT results, as shown in Fig.1 (a) (c) and (d), can be given by

$$\begin{pmatrix} \cos \alpha_{1} & -\sin \alpha_{1} \\ \cos \alpha_{2} & -\sin \alpha_{2} \\ \vdots & \vdots \\ \cos \alpha_{N} & -\sin \alpha_{N} \end{pmatrix} \begin{pmatrix} L_{\theta} \sin \theta \\ L_{\theta} \cos \theta \end{pmatrix} + \begin{pmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \vdots \\ \varepsilon_{N} \end{pmatrix} = \begin{pmatrix} L_{1} \\ L_{2} \\ \vdots \\ L_{N} \end{pmatrix}$$
(8)

where α_n , L_n and ε_n denote the *n* th $(n = 1, 2, \dots, N)$ RT angle, measured normalized length and measurement error, respectively. For convenience, (8) can be rewritten as

$$Ax + e = L \tag{9}$$

It is assumed that the measurement error of the normalized length obeys the Gaussian distribution. To minimize the residual sum of squares of the measurement error, we utilize the LS method to estimate the term containing the range walk angle of the moving target via

$$\begin{bmatrix} L_{\theta} \sin \theta & L_{\theta} \cos \theta \end{bmatrix}^{T} = \hat{x} = \begin{bmatrix} A^{T} A \end{bmatrix}^{-1} \cdot A^{T} L$$
(10)

The range walk angle can be estimated by the results of (10)

$$\tan \theta = L_{\theta} \sin \theta / L_{\theta} \cos \theta = \sin \theta / \cos \theta \qquad (11)$$

which can cancel the hardly measured normalized length of the moving target trajectory L_{θ} in the 2-D plane. And then the radial velocity can be calculated by (1) and (11).

Since the geometry information is sensitive to noise, a special processing (noise energy cancelling) has to be done to improve the SNR in the RT domain, which is proposed in [16]. The method can achieve SNR improvement by cancelling the noise energy in the RT domain. Before and after the noise energy cancelling process, the RT results are compared in Fig. 2 (b) and (c) or (f) and (g). Given the

geometry information sensitivity to noise, we combine the proposed LSRTE method with the noise energy cancelling process to achieve as high estimation accuracy as possible.

4. THE WEIGHTED LEAST SQUARE ESTIMATION

In the prior section, independent on the RT results, the measurement error is assumed to obey the Gaussian distribution, which simplifies the LSRTE to make it easy to understand. However, the assumption would be broken in low SNR scenarios, because the measurement error is related with the RT results, as shown in Fig.2. In this section, the relationship between the measurement error and the RT results will be analyzed, and a much more robust method is proposed to improve the estimation accuracy.



(d) RT result after noise energy cancelling (h) RT result after noise energy cancelling Fig. 2. Effect of the RT angles on the normalized length.

As shown in Fig.1 (a) and described in formula (2), the SNR in the RT domain increases along with the decreasing distance between the RT angle and the range walk angle, so different RT angles bring different SNR and different measurement error, which can be seen from the comparison of Fig. 2 (d) and (h). The relationship between measurement error and the normalized length can be described clearly as: the shorter the normalize length is, and the higher the SNR is, then the slighter the measurement error is; otherwise, much wider normalize length brings much worse estimation. Based on these analyses, we utilize the measurement error weighted by the normalized length to improve the estimation accuracy.

According to the theory of the weighted least square method, the larger the weight is, the more important the RT result deserves. So the weight can be selected as being inversely proportional with the SNR in the RT domain: the lower the SNR is, and the larger the measurement error is. And then the N times measurements can be defined as

$$Ax + Ke = L \tag{12}$$

where $K = 1/\eta_{RT}$ is the coefficient matrix determined by the reciprocal of the SNR (η_{RT}) in the RT domain.

In order to derive the value of *K*, we should analyze the SNR (η_{RT}) in the RT domain, which is caused by the noise energy cancelling and RT. In the range compression

domain, the power of the moving targets and noise can be set as p_s and p_n , respectively, and then the SNR η_{RC} in the range compression domain can be expressed via

$$\eta_{RC} = p_s / p_n \tag{13}$$

The power gain of the noise through the noise energy cancelling process is set as G, and then the power of the noise in the RT domain is $G \cdot p_n$. Given the law of energy conservation, we can derive the amplitude of the moving target as $p_s \cdot L_{\theta}/L_n^{\alpha}$, where L_n^{α} denote the theoretical normalized length of the *n* th RT. The SNR in the RT domain η_{RT} can be derived by

$$\eta_{RT} = \frac{p_s \cdot L_\theta}{G \cdot p_n \cdot L_n^\alpha} = \frac{L_\theta}{GL_n^\alpha} \frac{p_s}{p_n} = \frac{\eta_{RC} L_\theta}{G} \frac{1}{A_n x}$$
(14)

where $A_n = (\cos \alpha_n - \sin \alpha_n)$ represents the *n* th RT angle matrix. And then much more practical model of the *N* times RT results (12) can be rewritten as

$$Ax + \frac{G}{\eta_{RC}L_{\theta}} \left[diag(Ax) \right] e = L$$
 (15)

where $diag(\cdot)$ denotes the diagonalization. To estimate x, we intend to adopt the weighted LS method, however, Ax is unknown. Since the measured normalized length L can also express the model of the geometry information related measurement error, we use the following weight matrix

$$W = diag(L) \tag{16}$$

to estimate the range walk angle vector via

$$\begin{bmatrix} L_{\theta} \sin \theta & L_{\theta} \cos \theta \end{bmatrix}^{T} = \begin{bmatrix} A^{T} W^{-2} A \end{bmatrix}^{-1} \cdot A^{T} W^{-2} L \quad (17)$$

It should be note that the weight W is dependent on the parameter x, so the estimation by the WLSRTE would be different from that by the LSRTE. In this paper, we provide the experiments results to validate the estimation accuracy improvement, and the complicated theoretical proof would be discussed in the future research. Summarily, the geometry information model has been modified to adapt the real scenarios, and the WLSRTE method should improve the estimation accuracy than the LSRTE method in low SNR, because it utilizes the much more practical model.

5. EXPERIMENTS RESULTS AND ANALYSIS

In this section, experiments results are presented to demonstrate the effectiveness of the proposed methods. The range walk angle of the moving target is set as 1.3° , and the range of the RT angles can be set as $\left[-6^{\circ}, 6^{\circ}\right]$.

As we known, the estimation accuracy of the LSRTE and WLSRTE is related to the observing times (i.e. the RT times N), and that of the CRTE is determined by the RT angles searching step size (the ratio of 12° and N), so the estimation accuracy (relative error) versus the RT times is compared among the ERTE, LSRTE, WLSRTE and CRTE with the SNR of -10dB. The RT times are set in the range of $[2^1, 2^8]$, and the relative errors of the estimation methods are shown in Fig.3(a). Utilizing the geometry information of only two RT times, the ERTE method keeps the same estimation accuracy as the LSRTE with RT times of 2, which can be seen from Fig.3(a).



The high efficiency of the proposed methods can be demonstrated by the dotted lines in Fig.3(a): LSRTE and WLSRTE utilize 1/3 and 1/16 RT times of the CRTE to achieve the same estimation accuracy with the CRTE. It can be seen from Fig.3(a) that the proposed methods (LSRTE and WLSRTE) possess much higher accuracy than the ERTE method, and the estimation accuracy of the LSRTE and WLSRTE turns much higher along with the RT times increasing, which validates the estimation accuracy improvement of the proposed methods. As aforementioned, the estimation accuracy of the CRTE is determined by the RT angles searching step size (related with the RT times), which can also be demonstrated by Fig.3(a). It should be note that the estimation accuracy of the CRTE is higher than that of LSRTE, and this is because the searching step size (the ratio of 12° and RT times) turns small enough due to large RT times. Moreover, the proposed WLSRTE method utilizing much more practical measurement error model possesses much higher accuracy than the LSRTE method, which matches the theoretical expectancy mentioned above.

Now let's refocus on our original objective: to improve the estimation accuracy and the robustness in low SNR scenarios, and the relative error versus SNR is compared among the methods. The RT times is set as 64, and the comparison results are shown in Fig.3(b). It can be seen that the proposed LSRTE and WLSRTE method effectively improve the estimation accuracy than the ERTE method in low SNR scenarios, and the WLSRTE performs much more excellent because of its robustness capability. Moreover, the higher the SNR is, the slighter the measurement error affects, and then the higher estimation accuracy the proposed methods can achieve. Since the geometry information is sensitive to noise, the estimation accuracy of the LSRTE is lower than that of the CRTE in low SNR background. But the CRTE would not improve its accuracy along with the SNR, because its estimation accuracy is only determined by its angles searching step size when the SNR is high enough. Thus, the proposed methods possess much higher accuracy

and efficiency than the CRTE method not only in low SNR but also in high SNR.

The real data is also processed to verify the effectiveness of the proposed WLSRTE. As shown in Fig. 4(a), the radial velocity of the fast moving target in the clutter suppression result is estimated by WLSRTE. The moving target is transformed into the range compression and azimuth time domain (Fig. 4(b)), and the radial velocity is estimated by the WLSRTE as 9.49m/s. As shown in Fig. 4(c), range walk is well corrected by the estimated results.



6. CONCLUSIONS

Radial velocity estimation plays an important role in modern SAR-GMTI system. In order to improve the efficiency of the radial velocity estimation, we have proposed an efficient RT estimation method, which possesses acceptable accuracy in high SNR scenarios but performs worse in low SNR conditions. In this paper, our objective is to improve the estimation accuracy and robustness further to meet the requirement of application in low SNR, and two methods (LSRTE and WLSRTE) are proposed by exploiting much more geometry information. By utilizing much more practical measurement error model, the proposed WLSRTE method possesses much higher accuracy than the LSRTE and the conventional method. The experiment results are provided to verify the effectiveness of the proposed methods.

Instead of conventional searching process, the proposed methods can calculate rather than search for the parameter by utilizing the geometry information, which possess the advantage of high estimation accuracy and efficiency. Thus, we suggest that the geometry information should be promoted to improve the estimation performance in the signal processing field.

ACKNOWLEDGEMENTS

Qian Xuesan Laboratory of Space Technology, managed and operated by the China Academy of Space Technology, is a multi-program laboratory which is focusing on future space systems and applied fundamental research in multiple disciplines, including information, communication, radar, mechanics, energy transformation, materials, optics, and guantum technology. This work is supported by Qian Xuesan Laboratory of Space Technology and Natural Science Foundation of China (Grant No. 61401022).

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