GENERALIZED K-LEVEL CUTSET SAMPLING AND RECONSTRUCTION

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ABSTRACT

We propose a family of cutset sampling schemes and a generalized k-level image reconstruction approach formulated under a minimum mean squared error (MMSE) framework. The k-level reconstruction approach is a direct generalization of the recently proposed pattern-based approach, and can be applied to periodic samples either on a cutset or on a grid. Our experimental results indicate that the generalization of the k-level reconstruction approach results in only a small performance loss. For rectangular cutsets, we show that the proposed approach outperforms the cutset-MRF approach as well as two inpainting approaches. Moreover, we show that combining the cutset sampling with an additional point sample inside the periodic structure outperforms k-level reconstruction from cutset sampling and point sampling under comparable sampling densities.

***Index Terms*—** cutset, sampling, reconstruction

1. INTRODUCTION

The term *cutset* was defined in terms of a graphical model in the context of bilevel image compression [1, 2, 3, 4], and formed the basis for k-level image reconstruction [5], grayscale image reconstruction [3, 6, 7], and energy optimization in sensor networks [8, 9]. A cutset typically samples a two-dimensional field on rows and columns of a Cartesian grid (Figure 1 left panel). Cutset sampling results from physical constraints on data acquisition, where sensor deployment is restricted to narrow strips (e.g., city streets) or sensor movement is confined in pre-specified routes (e.g., taking water samples along the path of a ship).

Cutset sampling and reconstruction was first proposed for bilevel image reconstruction from a rectangular cutset based on a Markov random field (MRF) model; we refer to this as the *cutset-MRF* approach [1]. A hierarchical version of the cutset-MRF approach was proposed in [4]. The reconstruction of grayscale images from cutsets was considered in [6], which utilized a k-level reconstruction as an intermediate step, and in [7], which was based on an orthogonal gradient method. As an intermediate step for grayscale reconstruction, or as a reconstruction of image segments which are an intermediate step for semantic information extraction, k-level reconstruction is important on its own. A pattern-based k-level cutset reconstruction approach was proposed in [5], and was shown to outperform the cutset-MRF approach and several inpainting approaches, which are closely related because their goal is to fill in a missing region of an image. Most inpainting methods, such as [10, 11], focus on reconstructing missing regions of grayscale or color images. On the other hand, the k-level image reconstruction aims at reconstructing regional information or segments. The main challenge in cutset reconstruction is that a limited number of samples is available. In particular, the cutsets include one-pixel-wide lines of an image, as opposed to inpainting approaches, which typically aim at filling in gaps of otherwise dense image samples.

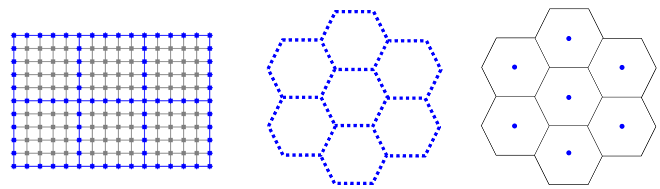


Fig. 1. Left: cutset (blue) on 2D Cartesian grid. Middle: hexagonal cutset. Right: hexagonal structure.

Previous work on cutset sampling assumes was restricted to rectangular cutsets. Little attention was given to more generalized cutsets. In this paper, we propose a family of generalized cutset sampling schemes and a generalized k-level image reconstruction approach that allow arbitrary periodic sampling regardless of the position and connectivity of samples. The main contributions include (i) a family of generalized cutset sampling schemes; (ii) a sampling technique that combines cutset sampling and point sampling, which provides additional information for reconstruction; and (iii) a MMSE approach for k-level cutset reconstruction that is suitable for arbitrary periodic sampling schemes.

The structure of this paper is as follows. Section 2 reviews related work. Section 3 presents generalized cutset sampling. Section 4 presents the MMSE framework and an algorithm for k-level image reconstruction. Section 5 demonstrates experimental results. Section 6 summarizes this paper.

2. RELATED WORK

Cutset sampling was first proposed in the cutset-MRF approach [1], where the samples were obtained densely on every N-th row and column of a 2D Cartesian grid. It was then extended to a hierarchical scheme that recursively subdivides the cutset into finer grids depending on image content [4]. In the cutset-MRF approach, a Markov random field (MRF) model is used to reconstruct bilevel images from rectangular cutsets [1]. The rectangular cutset is reconstructed block by block, where the block boundaries (cutset samples) are shared by adjacent blocks. Based on the properties of MRF, the optimal reconstruction of the interior of a cutset block can be obtained independently from each block.

A pattern-based approach to reconstruct k-level images from rectangular cutsets was proposed in [5]. It learns and utilizes the statistics from a dataset of k-level images, and has shown superior performance over the previous approaches. The algorithm consists of (i) the creation of a cutset pattern database; (ii) block-by-block reconstruction based on fuzzy retrieval of block boundary patterns; and (iii) a post-processing step that iteratively reduces local MRF energy given the initial reconstruction. A unique cutset pattern consists of a boundary pattern and a block interior pattern. The interior pattern is obtained as the average of a collection of patterns that have the same boundary. Thus, each unique boundary pattern is associated with one block interior pattern. The boundary pattern is represented as runs of consecutive pixels with value in the set $\{0, 1, \ldots, K-1\}$, and is further normalized to obtain a more compact representation. The specific properties of rectangular cutsets are utilized to obtain concise representations.

3. GENERALIZED CUTSET SAMPLING

Given an undirected connected graph $G = (V, E)$ comprising a set of vertices (nodes) V and a set of edges E, a cutset G_c is defined as a subgraph of G such that, when it is removed from G, it separates G into disconnected subgraphs. The cutset nodes (sampled nodes) enclose the unsampled nodes.

When embedded into a 2D Cartesian lattice, the rectangular cutset is formed by periodically sampling the rows and columns as shown in Figure 1, left panel. For a 2D Cartesian lattice, uniform sampling is the counterpart of rectangular cutset sampling. Their sampling densities are given by $d_u = \frac{1}{N_u^2}$ and $d_c = \frac{2N_c-1}{N_c^2}$, where N_u and N_c denote the uniform sampling step and the rectangular cutset sampling step, respectively. The sampling densities equalwhen $N_u = \sqrt{\frac{N_c^2}{2N_c-1}}$.

Hexagonal sampling is an alternative pixel tessellation of images that is more efficient than Cartesian sampling [12]. The natural cutsets one can define on a hexagonal lattice are shown in the middle panel of Figure 1. The uniform sampling counterpart on a hexagonal lattice is shown in the right panel of Figure 1. However, when the underlying dense image field is given on a Cartesian grid, obtaining the hexagonal uniform and cutset samples is not a trivial problem. Hexagonal uniform (sparse) sampling is often approximated by shifting the square pixels by a half pixel width [12]. Hexagonal cutset sampling can be approximated by rounding the locations of the samples to the nearest integer. Examples of actual uniform and cutset hexagonal sampling are shown in Figure 2. A generic cutset can be embedded with other topologies, such as triangles and randomly positioned lines and curves.

The main advantages of the cutset sampling are (i) the cutset nodes are connected in lines or curves, which reduces energy cost and transmission distance in cutset sensor networks [8, 9]; and (ii) the cutset nodes separate the cutset into disjoint subgraphs, each of which can be processed independently of the other subgraphs. The structure of the subgraphs is typically identical such that the overall topology of the cutset is periodic. Examples include triangular, rectangular, and hexagonal cutsets.

The main challenge of recovering unsampled nodes from the cutset is that the distance between non-neighboring nodes in cutsets could be relatively large as compared to conventional point sampling. To address this issue, we propose a mixture of cutset sampling and conventional sampling by taking an additional point sample at the center of each periodic structure, such that the cutset property of separation and enclosure is unchanged while the distance between the non-neighboring nodes is reduced by half.

4. GENERALIZED CUTSET RECONSTRUCTION

In periodic cutset sampling, the cutset can be subdivided into regions of interests (ROIs) with identical structure. Each ROI consists of the boundary and its interior. For example, the ROIs of rectangular cutsets sampled on Cartesian grid with sampling step N are $(N+1) \times (N+1)$ blocks, consist of $4N$ cutset nodes on the block boundary enclosing $(N-1) \times (N-1)$ unsampled nodes; the ROIs of hexagonal cutsets are hexagonal regions consisting of the hexagonal region boundary and its interior unsampled nodes. Let $\mathbf{B} \in [0, K)^m$ and $\mathbf{X} \in [0, K)^n$ denote the sample nodes and the region to be reconstructed (unsampled nodes) in the ROI, respectively, and $\mathbf{Y} \in [0, K)^{m+n} = \mathbf{X} \cup \mathbf{B}$ denotes the entire ROI, where K, m and n denote the number of levels, the number of sampled nodes and the number of samples to be reconstructed. Given a cutset ROI dataset $\mathbf{S}$, the distinct patterns in the ROIs are indexed as $\mathbf{Y}_i$, where $\mathbf{Y}_i = \mathbf{X}_i \cup \mathbf{B}_i$. Given the observations of $\mathbf{Y}_i \in \mathbf{S}_o$, a subset of $\mathbf{S}$, the objective is to find an estimator $\hat{\mathbf{X}}(\mathbf{B})$ such that the mean squared error of the estimate is minimized,

$$\text{argmin MSE}(\hat{\mathbf{X}}(\mathbf{B})) = \min E[(\hat{\mathbf{X}} - \mathbf{X})^2] \tag{1}$$

which yields the minimum mean squared error (MMSE) estimator

$$\hat{\mathbf{X}}(\mathbf{B}) = E[\mathbf{X}|\mathbf{B}] = \sum_{i \in \mathbf{S}} \mathbf{X}_i f(\mathbf{X}_i|\mathbf{B}_i) \approx \sum_{i \in \mathbf{S}_o} \mathbf{X}_i f(\mathbf{X}_i|\mathbf{B}_i) \tag{2}$$

where f denotes the frequency. The estimate of the hidden pattern $\hat{\mathbf{X}}$ given the cutset specification $\mathbf{B}$ reduces to the average of ROIs associated with unique cutset specifications based on the observations. When the observed subset is representative of the pattern population, the collected statistics can be used as the solution to the MMSE.

The pattern-based k-level cutset reconstruction algorithm proposed in [5] is a realization of the MMSE approach designed specifically for square cutsets sampled on 2D Cartesian lattices. Here, we propose a variant of the pattern-based algorithm that can reconstruct any periodically sampled k-level image. The proposed algorithm consists of three phases: (i) the construction of cutset ROI pattern database based on the observation of $\mathbf{Y}_i = \mathbf{X}_i \cup \mathbf{B}_i \in \mathbf{S}_o$; (ii) the fuzzy summarization of the cutset ROI patterns; and (iii) the retrieval of fuzzy patterns. The representation of the $(\mathbf{X}_i, \mathbf{B}_i)$ pair differs from that of [5] in that the conversion from the raw levels to the normalized levels does not depend on the position of the runs. Yet, it maps the distinct the raw levels to a consecutively numbered levels, starting from the first sample. The samples are ordered by their relative distances. Rotation is not required in consideration of non-symmetric cutset ROIs. The proposed reconstruction algorithm is capable of reconstructing any k-level regions given periodic samples from both cutset sampling and point sampling.

5. EXPERIMENTAL RESULTS

We illustrate our results on three datasets: (i) the k-level images obtained from human segmentations of natural images in BSDS500 [13]; (ii) the Brown bilevel image shape dataset (5578 images) [14]; and (iii) an in-house bilevel image dataset of 13 relatively complicated bilevel images. We used the training part of BSDS500 to construct the cutset ROI database. The test set of BSDS500 along with the other two datasets was used for the testing the k-level reconstruction algorithms.

Sampling: Figure 2 illustrates several typical sampling schemes, including point sampling, cutset sampling, and their combination. The sampling step N and radius R are chosen to yield comparable sampling densities and ROI sizes. Table 1 lists the parameters and the actual densities (discretized in the case of hexagonal sampling). In the rectangular cutset, the number of cutset samples is smaller than that of the nodes to be reconstructed when $N \geq 6$. We found that $N = 8$ is the most efficient for rectangular cutsets.

ROI: Examples of the fuzzy summarized patterns in the database are shown in Figure 3. Same ROIs are applied to cutset sampling and corresponding point sampling. The figure shows that point sampling is advantageous in keeping the coarse patterns, particularly when the number of samples are limited, while cutset sampling is better in capturing level partition details near the cutset samples. The mixture of cutset sampling and point sampling keeps both the structural information and the details, and reduces the ambiguity when multiple patterns are equally likely.

Table 1. Sampling parameter and density

N (2R)	random	uniform	cutset rect.	cutset rect. + pt	cutset hex.	cutset hex. + pt
8 (8)	.25	.25	.23	0.25	.27	.29
18 (16)	.11	.11	.11	.11	.11	.11

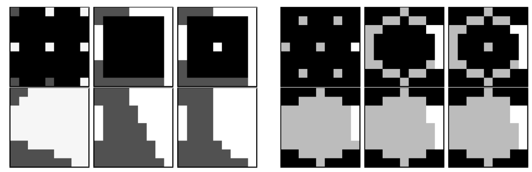
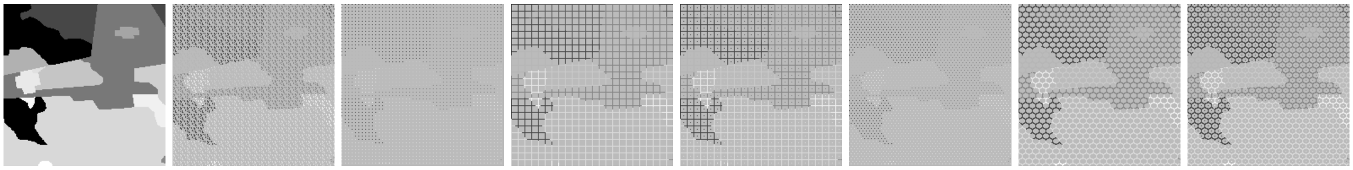


Fig. 3. Samples (top) and reconstructions (bottom). Black: unsampled nodes or non-ROI.

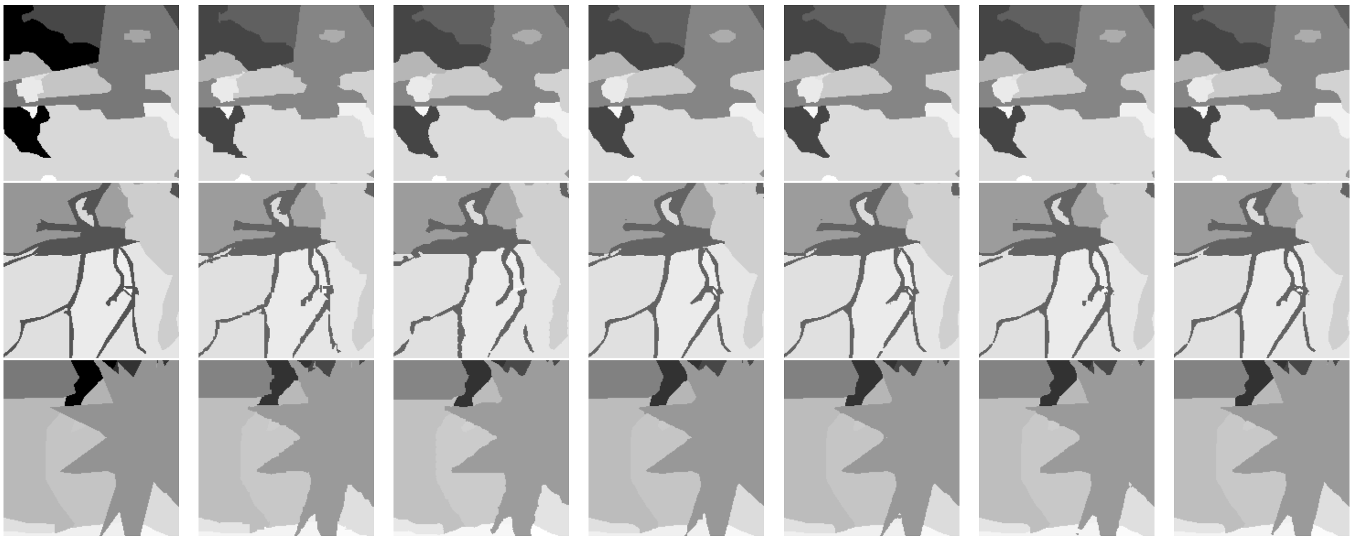
Visual Results: The reconstruction for several typical sampling schemes, including periodic random sampling, uniform sampling, rectangular cutset sampling (with additional center sample), and hexagonal sampling (with additional center sample), is illustrated in Figure 4. The proposed algorithm is general enough to handle other periodic sampling topologies as well. The corresponding sampling density and the average reconstruction error rate are given in Tables 1 and 2, with comparable sampling densities and ROIs. The results show that the structural information is preserved in the reconstruction of point sampling as well as cutset sampling under reasonable sampling steps. They also show that the reconstruction of cutset sampling is more piecewise smooth when compared to that of point sampling. For example, the sampling density of random point sampling, uniform sampling, and rectangular cutset sampling with an additional sample at the center are identical, while the rectangular cutset sampling has slightly lower sampling density. However, the cutset reconstructions have smoother segment contours.

Reconstruction Error: The average reconstruction error rates obtained in three datasets are given in Table 2. Note that the pixels in the k-level images are region labels rather than values, thus the reconstruction error for each pixel is 0 if it matches the ground truth and 1 otherwise. The comparison of the proposed reconstruction approach against the inpainting approaches, the cutset-MRF approach, and the pattern-based approach for square cutsets, are shown in Table 3. The comparison of the proposed reconstruction approach against the one in [5] under the same cutset specification shows that there is only little loss in terms of reconstruction error rate by generalizing the reconstruction algorithm. Yet, it still outperforms the cutset-MRF approach and two inpainting approaches. The generalized reconstruction approach enables reconstruction from general periodic cutset sampling as well as point sam-



original random uniform cutset rect. cutset rect. pt. hex. cutset hex. cutset hex. pt.

Fig. 2. Sampling. $N = 2R = 8$



groundtruth random uniform cutset rect. cutset rect. pt. cutset hex. cutset hex. pt.

Fig. 4. Reconstruction from different sampling schemes. $N = 2R = 8$

Table 2. Average reconstruction error rate with various sampling schemes. $N = 2R = 8$

Dataset	rand.	unif.	cutset rect.	cutset rect. pt	cutset hex.	cutset hex. pt
k-level [13]	.008	.007	.007	.006	.006	.005
bilevel [14]	.015	.013	.016	.013	.011	.009
our bilevel	.019	.016	.018	.014	.014	.011

Table 3. Comparison against other approaches in average reconstruction error rate of rectangular cutset $N = 8$

Dataset	[10]	[11]	[1]	[5]	proposed
k-level dataset [13]	.017	.025	-	.007	.007
bilevel dataset [14]	.032	.029	.018	.015	.016
our bilevel dataset	.038	.035	.027	.017	.018

pling, including both symmetric sampling topology, such as uniform sampling and hexagonal cutset sampling, and random sampling. The symmetric property is not utilized in the generalized k-level reconstruction approach for fair comparison among different sampling schemes. We believe that fully exploiting the specific pattern symmetries in each of the sampling schemes could further improve the result. Our results also shown that sampling an additional point at the center of the periodic cutset structure is efficient in reducing the reconstruction error, outperforming either point sampling or cutset sampling alone.

6. CONCLUSION

We proposed a family of generalized cutset sampling schemes and a MMSE framework for reconstructing k-level images. The k-level reconstruction approach can be applied to arbitrary periodic sampling schemes, including both cutset sampling and point sampling. We have shown that it outperforms the cutset-MRF approach and two inpainting approaches in k-level cutset reconstruction. The comparison of cutset sampling and point sampling demonstrates the strength of cutsets in capturing fine-grained structure. Moreover, sampling an additional point at the center of the periodic cutset samples improves the reconstruction.

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