

AN APPROXIMATE MESSAGE PASSING APPROACH FOR TENSOR-BASED SEISMIC DATA INTERPOLATION WITH RANDOMLY MISSING TRACES

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ABSTRACT

In this paper, we consider the reconstruction of a high-dimensional seismic volume with randomly missing traces. Seismic data in the frequency-space domain are represented via a high-order tensor. Applying the parallel matrix factorization model to the underlying seismic tensor, we propose an iterative approximate message passing (AMP) approach to seismic data interpolation based on loopy belief propagation. In particular, we extend the bilinear generalized AMP (BiG-AMP) approach to incorporate parallel low-rank matrix factorizations by using a “turbo” framework, enabling iterative message passing between the subgraphs of the all-mode unfoldings of the seismic tensor. The computational complexity of our algorithmic framework is low and scales linearly with the data size. Simulation results with synthetic seismic data suggest that the proposed algorithm yields better reconstruction performances relative to existing methods.

1. INTRODUCTION

Due to physical limitations and economic constraints, only a fraction of the prestack traces can be obtained in practical realities of acquiring seismic data. Reconstruction of the seismic data from incomplete spatial measurements is one of the key issues in seismic data acquisition. In frequency-space domain, high-dimensional seismic data can be described as a low-rank fifth-order tensor (including one frequency (or time) dimension and four spatial dimensions) in which a large amount of structure that can be exploited in order to recover the original, fully-sampled data effectively.

Given the high dimensionality and huge size of the seismic volume, recent research on seismic data acquisition has explored the use of dimensionality reduction (or rank-reduction) of tensor to recover the missing traces and increase the signal-to-noise-ratio of the seismic data effectively. For example, [1] considers using the high-order singular value

decomposition (HOSVD) based on the well-known Tucker model to implement rank reduction of the tensor. [2] proposes a tensor nuclear norm (TNN) algorithm, as a variant of the tensor singular value decomposition (tSVD), by relaxing the complexity measure term to a nuclear norm. [3] employs a parallel matrix factorization (PMF) model which then be formulated as a convex problem and solved using the alternating least squares method. [4] considers a convex optimization problem by penalizing the all-mode unfoldings of the tensor to have a low-nuclear norm, and applies the alternating direction method of multiplier (ADMM) method to solve the problem.

While a lot of approaches on tensor completion have been proposed and achieved considerably smaller reconstruction errors than before, ours is based on the approximate message passing (AMP) framework, an approximation of loopy belief propagation (BP) [5] that was recently developed to tackle generalized inference of linear inverse [6] and bilinear inverse (or matrix factorization) [7] problems. The AMP framework has been shown to perform rapid and highly accurate probabilistic inference with linear complexity scaling in the dimension of the data [6]. In this paper, we propose a novel approximate message passing (AMP)-based algorithm, called tensor completion AMP (TC-AMP), that leverages the recently proposed AMP-based “turbo” framework [8] to perform inference on a probabilistic model enforcing parallel matrix factorizations of the tensor. Specifically, we formulate our problem in a probabilistic way by simultaneously performing low-rank matrix factorizations to the all-mode unfoldings of the tensor. Then we partition the factor graph into several identical subgraphs and employ the bilinear generalized AMP (BiG-AMP) approach [7] to each subgraph, which enables soft posterior message to iteratively update between the modules. The key to our approach lies in combining the “turbo” framework of [8], where all the subgraphs corresponding to different low-rank matrix factorization modules are connected in the common hidden variable nodes and inference is performed on the augmented factor graph.

2. PROBLEM FORMULATION

The prestack seismic volume is generally represented by four spatial dimensions and frequency (or time), and has an ex-

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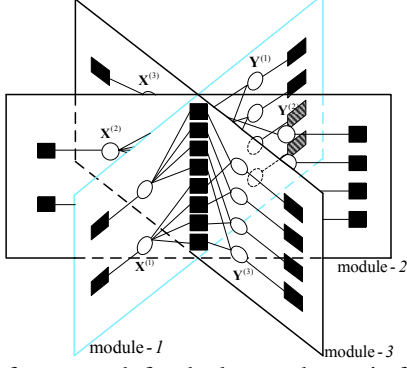


Fig. 1. The factor graph for the low-rank matrix factorization model with $N = 3, I_1 = I_2 = I_3 = 2$, and $R_1 = R_2 = R_3 = 1$.

tremely large data size and enormous redundancy. Therefore, we can safely identify the seismic data by a fifth-order (approximately) low-rank tensor $\mathcal{D} \in \mathbb{R}^{I_1 \times \dots \times I_N}$ with elements $\mathcal{D}_{i_1, \dots, i_n}$, where i_1, \dots, i_n are the indices for the frequency-space coordinates, $N = 5$ represents the order of the seismic tensor. In our problem, the goal is to recover \mathcal{D} from noisy and partial observations $\mathcal{B} = P_\Omega(\mathcal{D} + \mathcal{W})$, where \mathcal{W} is an additive white Gaussian noise (AWGN) tensor with unknown variance τ^w , Ω is the index set of observed entries, and P_Ω keeps the entries in Ω and zeros out others.

Similar to the PMF model [3] which simultaneously performs low-rank matrix factorizations to the all-mode unfoldings of tensor, we first denote the mode- n matricization (or unfolding) of \mathcal{D} , $\mathbf{D}^{(n)}$, as a matrix with the dimensions given by n reshaped along the rows, and the other dimensions reshaped into the columns of the matrix, i.e., $\mathbf{D}^{(n)} \in \mathbb{R}^{I_n \times \prod_{i \neq n} I_i}$. Then, we apply low-rank matrix factorization to each mode unfolding of \mathcal{D} by decomposing $\mathbf{D}^{(n)}$ as the product of two hidden variable matrices, i.e., $\mathbf{D}^{(n)} \triangleq \mathbf{X}^{(n)} \mathbf{Y}^{(n)}$, where $\mathbf{X}^{(n)} \in \mathbb{R}^{I_n \times R_n}$, $\mathbf{Y}^{(n)} \in \mathbb{R}^{R_n \times \prod_{i \neq n} I_i}$, $R_n \ll \min(I_n, \prod_{i \neq n} I_i)$ is the unknown rank of $\mathbf{D}^{(n)}$. Then, we have the following formulation:

$$\mathbf{B}^{(n)} = P_\Omega(\mathbf{D}^{(n)} + \mathbf{W}^{(n)}) = P_\Omega(\mathbf{X}^{(n)} \mathbf{Y}^{(n)} + \mathbf{W}^{(n)}), \forall n \quad (1)$$

where $\mathbf{B}^{(n)}$ and $\mathbf{W}^{(n)}$ are the mode- n unfolding of \mathcal{B} and \mathcal{W} , respectively.

For mode- n unfolding of \mathcal{D} , assuming independent entries for $\mathbf{X}^{(n)}$ and $\mathbf{Y}^{(n)}$, we obtain the separable probability density functions (PDFs) of $\mathbf{X}^{(n)}$ and $\mathbf{Y}^{(n)}$ as

$$p(\mathbf{X}^{(n)}) = \prod_{i_n, r_n} p(x_{i_n r_n}^{(n)}) = \prod_{i_n, r_n} \mathcal{N}(x_{i_n r_n}^{(n)}; \hat{x}_n, \nu^{x_n}), \quad (2)$$

$$p(\mathbf{Y}^{(n)}) = \prod_{r_n, t_n} p(y_{r_n t_n}^{(n)}) = \prod_{r_n, t_n} \mathcal{N}(y_{r_n t_n}^{(n)}; 0, 1), \quad (3)$$

where $\{x_{i_n r_n}^{(n)}\}_{i_n, r_n}$ are assumed to be i.i.d. Gaussian with unknown mean \hat{x}_n and variance ν^{x_n} in (2), $i_n \in \{1, 2, \dots, I_n\}$, $r_n \in \{1, 2, \dots, R_n\}$, and $t_n \in \{1, 2, \dots, \prod_{i \neq n} I_i\}$.

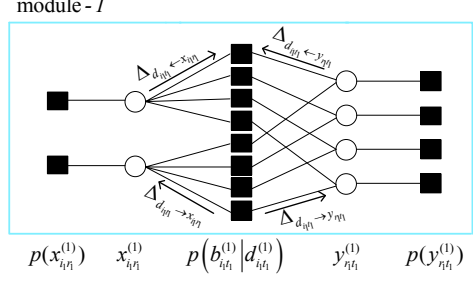


Fig. 2. The subgraph (painted blue in Fig. 1) for the low-rank matrix factorization of mode-1 unfolding of \mathcal{D} with $N = 3, I_1 = I_2 = I_3 = 2$, and $R_1 = R_2 = R_3 = 1$.

$\mathcal{N}(x; \hat{x}, \nu^x)$ denotes the PDF for a Gaussian random variable x with mean \hat{x} and variance ν^x . To avoid ambiguity and the unnecessary model parameters update, we assume that $\{y_{r_n t_n}^{(n)}\}_{r_n, t_n}$ follow i.i.d. Gaussian distribution with zero mean and unit variance in (3). Following [7], due to the assumption of possibly incomplete AWGN (PIAWGN) in (1), the likelihood function of $\mathbf{D}^{(n)}$ has the form

$$p(b_{i_n t_n}^{(n)} | d_{i_n t_n}^{(n)}) = \begin{cases} \mathcal{N}(b_{i_n t_n}^{(n)}; d_{i_n t_n}^{(n)}, \tau^w) & (i_n, t_n) \in \Omega, \\ \mathbb{1}_{b_{i_n t_n}^{(n)}} & (i_n, t_n) \notin \Omega, \end{cases} \quad (4)$$

where $\mathbb{1}_b$ denotes a point mass at $b = 0$.

3. THE TC-AMP ALGORITHM

We aim at estimating \mathcal{D} from noisy and partial observations \mathcal{B} of the low-rank matrix factorizations form $\mathbf{D}^{(n)} \triangleq \mathbf{X}^{(n)} \mathbf{Y}^{(n)}$, $\forall n$. As describe in Section 2, for a N -way tensor, we have

$$\mathcal{D} \triangleq \text{fold}_1(\mathbf{X}^{(1)} \mathbf{Y}^{(1)}) = \dots = \text{fold}_N(\mathbf{X}^{(N)} \mathbf{Y}^{(N)}), \quad (5)$$

where we define $\mathcal{D} = \text{fold}_n(\mathbf{D}^{(n)}) = \text{fold}_n(\mathbf{X}^{(n)} \mathbf{Y}^{(n)})$, $n \in \{1, \dots, N\}$. With the problem formulation in (1) and (5), our proposed method is to maximize the following posterior joint distribution

$$\begin{aligned} & p(\mathbf{X}^{(1)}, \mathbf{Y}^{(1)}, \dots, \mathbf{X}^{(N)}, \mathbf{Y}^{(N)} | \mathcal{B}) \\ &= \prod_n p(\mathbf{X}^{(n)}, \mathbf{Y}^{(n)} | \mathcal{B}) \\ &= \prod_n p(\mathcal{B} | \mathbf{X}^{(n)} \mathbf{Y}^{(n)}) p(\mathbf{X}^{(n)}, \mathbf{Y}^{(n)}) / p(\mathcal{B}) \\ &\propto \prod_n p(\mathcal{B} | \mathbf{X}^{(n)} \mathbf{Y}^{(n)}) p(\mathbf{X}^{(n)}) p(\mathbf{Y}^{(n)}), \end{aligned} \quad (6)$$

where \propto denotes equality up to a normalizing constant scale factor. Due to the exact inference of marginalizing (6) is typically intractable, we propose to solve an alternative problem that partitions the global factor graph (as shown in Fig. 1) into N identical bilinear subgraphs that mainly require local

information to complete their tasks. Specifically, each subgraph corresponds to a module that produces a posterior estimate of \mathcal{D} by considering maximizing its local posterior joint distribution, i.e., for each unfolding $\mathbf{D}^{(n)}$, $n \in \{1, \dots, N\}$, the posterior joint distribution of the local random variables can be factored as follows:

$$\begin{aligned} & p(\mathbf{X}^{(n)}, \mathbf{Y}^{(n)} | \mathcal{B}) \\ & \propto p(\mathcal{B} | \mathbf{X}^{(n)} \mathbf{Y}^{(n)}) p(\mathbf{X}^{(n)}) p(\mathbf{Y}^{(n)}) \\ & = \prod_{i_n, t_n} p(b_{i_n t_n}^{(n)} | d_{i_n t_n}^{(n)}) \times \prod_{i_n, r_n} p(x_{i_n r_n}^{(n)}) \times \prod_{r_n, t_n} p(y_{r_n t_n}^{(n)}), \end{aligned} \quad (7)$$

yielding the bilinear factor graph in Fig. 2, where circles denote random variables and squares denote posterior factors. Each factor node represents the conditional probability distribution between all variable nodes it connected [5]. In our case, N independent bilinear inverse problems are considered by performing the BiG-AMP approach [7] in parallel. Then, an efficient ‘‘turbo’’ framework [8] is used, i.e., after obtaining each subgraph’s posterior beliefs by performing the BiG-AMP approach, the prior beliefs of each subgraph are updated using the posterior beliefs passed from other subgraphs, then vice versa and repeat.

For each subgraph, given the bilinear construction of the factor graph in Fig. 2, we employ the BiG-AMP approach to the matrix completion problem and obtain the canonical sum-product BP iterative equations [5],

$$\Delta_{d_{i_n t_n} \leftarrow x_{i_n r_n}}^{(j)} = \frac{1}{C} p(x_{i_n r_n}) \prod_{b \neq t_n} \Delta_{d_{i_n b} \rightarrow x_{i_n r_n}}^{(j)}, \quad (8)$$

$$\Delta_{d_{i_n t_n} \rightarrow x_{i_n r_n}}^{(j+1)} = \frac{1}{C} \int_{\{x_{i_n c}\}_{c \neq r_n}, \{y_{r_n t_n}\}_{\forall r_n}} \left\{ \prod_{r_n} \Delta_{d_{i_n t_n} \leftarrow y_{r_n t_n}}^{(j)} \prod_{c \neq r_n} \Delta_{d_{i_n t_n} \leftarrow x_{i_n c}}^{(j)} p(b_{i_n t_n} | \sum_{r_n} x_{i_n r_n} y_{r_n t_n}) \right\}, \quad (9)$$

$$\Delta_{d_{i_n t_n} \leftarrow y_{r_n t_n}}^{(j)} = \frac{1}{C} p(y_{r_n t_n}) \prod_{h \neq i_n} \Delta_{d_{h t_n} \rightarrow y_{r_n t_n}}^{(j)}, \quad (10)$$

$$\Delta_{d_{i_n t_n} \rightarrow y_{r_n t_n}}^{(j+1)} = \frac{1}{C} \int_{\{x_{i_n r_n}\}_{\forall r_n}, \{y_{e t_n}\}_{e \neq r_n}} \left\{ \prod_{e \neq r_n} \Delta_{d_{i_n t_n} \leftarrow y_{e t_n}}^{(j)} \prod_{r_n} \Delta_{d_{i_n t_n} \leftarrow x_{i_n r_n}}^{(j)} p(b_{i_n t_n} | \sum_{r_n} x_{i_n r_n} y_{r_n t_n}) \right\}, \quad (11)$$

where constant C is an arbitrary normalization constant. $\Delta_{a \rightarrow b}$ denotes a message pass from some variable node a (factor node $p(a)$) to its adjacent factor node $p(b)$ (variable node b) in the factor graph. j is the local (intra-module) iteration index. For simplicity, here we have omitted the mark of mode- n , (n), in all the variables, such as $d_{i_n t_n}^{(n)}$, $x_{i_n r_n}^{(n)}$, and $y_{r_n t_n}^{(n)}$. Using Gaussian and Taylor-series approximations for (8)-(11), we then obtain three sets of approximately Gaussian

posterior messages, $\{\Delta_{d_{i_n t_n}}\}_{\forall i_n, t_n}$, $\{\Delta_{x_{i_n r_n}}\}_{\forall i_n, r_n}$, and $\{\Delta_{y_{r_n t_n}}\}_{\forall r_n, t_n}$ [7]. In particular,

$$\begin{aligned} \Delta_{d_{i_n t_n}} & \equiv p(b_{i_n t_n} | d_{i_n t_n}) \int_{\{x_{i_n r_n}, y_{r_n t_n}\}_{\forall r_n}} \prod_{r_n} \Delta_{d_{i_n t_n} \leftarrow y_{r_n t_n}} \Delta_{d_{i_n t_n} \leftarrow x_{i_n r_n}} \\ & \propto p(b_{i_n t_n} | d_{i_n t_n}) \mathcal{N}(d_{i_n t_n}; \hat{q}_{i_n t_n}, \nu_{i_n t_n}^q), \end{aligned} \quad (12)$$

where the parameters $\hat{q}_{i_n t_n}$ and $\nu_{i_n t_n}^q$ are obtained after the BiG-AMP iteration converges.

Then the posterior estimations of $\{d_{i_n t_n}\}_{\forall i_n, t_n}$ are facilitated by the following prior-dependent integrals

$$\hat{d}_{i_n t_n} = \int d_{i_n t_n} \Delta_{d_{i_n t_n}} d d_{i_n t_n}, \quad (13)$$

$$\nu_{i_n t_n}^d = \int d_{i_n t_n}^2 \Delta_{d_{i_n t_n}} d d_{i_n t_n} - \hat{d}_{i_n t_n}^2. \quad (14)$$

To enable effective implementation of ‘‘turbo’’ iteration, the posterior estimations of $\mathbf{D}^{(n)}$ obtained from the mode- n module is fed randomly to the other module as the a priori input, i.e., for a mode- n module, we use the updated prior distribution of $\mathbf{D}^{(n)}$, which is given by

$$p(d_{i_n t_n}^{(\sim n)}) = \mathcal{N}(d_{i_n t_n}^{(\sim n)}; \hat{d}_{i_n t_n}^{(\sim n)}, \nu_{i_n t_n}^{d(\sim n)}), \quad (15)$$

instead of the likelihood function of $\mathbf{D}^{(n)}$ in (4), where $\hat{d}_{i_n t_n}^{(\sim n)}$ and $\nu_{i_n t_n}^{d(\sim n)}$ are obtained from an arbitrarily module except the mode- n module.

Now we summarize the TC-AMP algorithm as follows: beginning at the initial iteration index, $i = 1$, the proposed algorithm first performs the BiG-AMP based on the priors and likelihood of (2)-(4) for N subgraphs in parallel. Then, for $i > 1$, the converged outgoing messages $\{\hat{d}_{i_n t_n}^{(n)}, \nu_{i_n t_n}^{d(n)}\}_{\forall i_n, t_n}$, $n \in \{1, \dots, N\}$ obtained from the mode- n module are treated as priori parameters for the other module and the updated prior distribution in (15) are used for all modules instead of the likelihood distribution in (4). This iteration continues until either a stopping condition or a maximum number of allowable iterations is reached. Finally, we obtain the final estimation of \mathcal{D} takes the form

$$\hat{\mathcal{D}} = \sum_n \alpha_n \text{fold}_n(\hat{\mathbf{D}}^{(n)}), \quad (16)$$

where the weight parameters $\alpha_1, \dots, \alpha_N$ are set as

$$\alpha_n = \frac{\|P_{\Omega}(\text{fold}_n(\hat{\mathbf{D}}^{(n)})) - \mathcal{B}\|_F^{-1}}{\sum_{k=1}^N \|P_{\Omega}(\text{fold}_k(\hat{\mathbf{D}}^{(k)})) - \mathcal{B}\|_F^{-1}}. \quad (17)$$

It is worth to note that our proposed algorithm described before relies on the specification of ranks $\{R_n\}_{\forall n}$ and model parameters $\{\tau^w, \{\hat{x}_n, \nu^{x_n}\}_{\forall n}\}$ which may not be known accurately in practical interpolation of seismic data. Here we adopt the ‘‘penalized log-likelihood maximization’’ rank selection strategy from [7] and the expectation-maximization (EM)-based method [6], [7], [9] to tune these parameters adaptively in each inter-module iteration. Since these two mechanisms can be performed directly in closed-form, and

thus do not significantly increase the complexity of the proposed algorithm.

In addition, the priors in (2),(3),(15) and likelihood function in (4) are all Gaussian, and hence, all expressions of the proposed algorithm have straightforward analytical expressions, and the computational complexity in each iteration is dominated by matrix multiplications [7], [9]. The complexity analyses in [7] indicate that, for a mode- n module, the BiG-AMP algorithm can be completed in $O(R_n \prod_i I_i)$ scalar multiplies per iteration. Thus, the complexity of each iteration of the proposed algorithm is bounded by $O(\sum_k R_k \prod_i I_i)$ scalar multiplies. Furthermore, in our simulation, we find that the proposed algorithm shows a good convergence in a small number of inter-module iterations (typically 3 to 5), implying that the total complexity is bounded by $O(\sum_k R_k \prod_i I_i)$ scalar multiplies.

4. PERFORMANCE ON SYNTHETIC SEISMIC DATA

In this section, we present synthetic seismic data results to compare the performance of the proposed TC-AMP algorithm with the TMac [3] and nuclear norm minimization (NNM) [4] algorithms.

The original seismic data \mathcal{D} is of size $512 \times 16 \times 16$, i.e., the spatial size of the data is 16×16 with 512 time samples per trace. We run different algorithms mentioned above to recover \mathcal{D} with 50%, 60%, ..., 90% traces missing. White Gaussian noise was added to achieve $\text{SNR} = 30\text{dB}$. The error in recovery is measured via the reconstruction quality

$Q = 10 \log_{10} (\|\mathcal{D}\|_F^2 / \|\hat{\mathcal{D}} - \mathcal{D}\|_F^2)$, where \mathcal{D} and $\hat{\mathcal{D}}$ represent the true noise-free complete data and reconstructed data, respectively. Fig. 3 shows the reconstruction quality averaged over 100 realizations, as a function of the percentage of missing traces. From Fig. 3 we see that when the percentage of missing traces is more than 70%, the proposed TC-AMP algorithm clearly provides the best recovery performance. In addition, the reconstruction qualities obtained by the proposed TC-AMP and the TMac algorithm perform much better than the NNM method.

A small subset of visual results of the recovered seismic data by various algorithms are presented in Fig. 4. As seen in Fig. 4, the NNM method shows the worst perceptual results than the TC-AMP and TMac algorithms. Obviously, our proposed algorithm preserves more fine details than the other competing methods.

5. CONCLUSION

We studied the problem of interpolating seismic data in the case of missing traces, assuming that the seismic tensor satisfies the PMF model. Then, we proposed an AMP-based algorithm, namely TC-AMP, to the tensor completion based on loopy BP. Finally, the numerical results with synthetic seismic data were presented to confirm the performance advantage of our proposed TC-AMP algorithm.

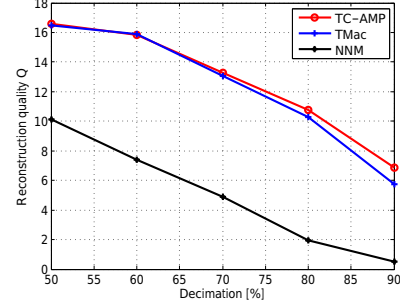


Fig. 3. Reconstruction quality Q versus percentage of missing traces for the TC-AMP, TMac, and NNM methods.

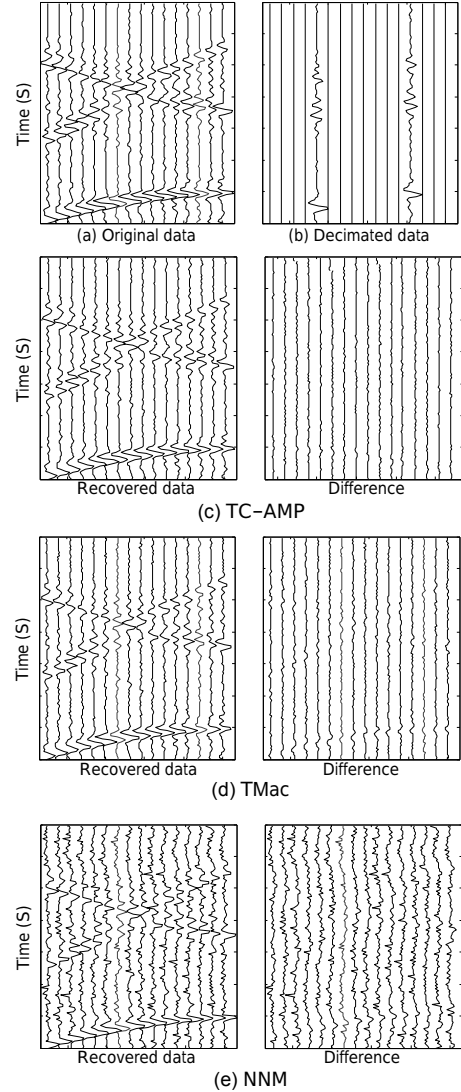


Fig. 4. A small subset of one slice of the data reconstruction results. $\text{SNR} = 20\text{dB}$. (a) The raw undecimated data, (b) 90% of the traces were removed at random. (c, d, and e) are the recovered results (left) and errors results (right) using the TC-AMP, TMac, and NNM, respectively.

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