HIGH DYNAMIC RANGE IMAGING VIA TRUNCATED NUCLEAR NORM MINIMIZATION OF LOW-RANK MATRIX

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ABSTRACT

We propose a ghost-free high dynamic range (HDR) image synthesis algorithm using a rank minimization framework. Based on the linear dependency among irradiance maps from low dynamic range (LDR) images, we formulate ghost-free HDR imaging as a low-rank matrix completion problem. The main contribution is to solve it efficiently via the augmented Lagrange multiplier (ALM) method, where the optimization variables are updated by closed-form solutions. Experiments on real image sets show that the proposed algorithm provides comparable or even better image qualities than state-of-the-art approaches, while demanding lower computational resources.

Index Terms— High dynamic range imaging, low-rank matrix completion, truncated nuclear norm minimization.

1. INTRODUCTION

The recent advancement of imaging technology has enabled a variety of devices to acquire high-resolution images. However, an ordinary digital camera still can only capture a limited dynamic range, which is significantly lower than that of a natural scene [1]. Therefore, a captured image typically contains under- or over-exposed regions due to a limited dynamic range. A lot of researches have been carried out to overcome this limitation and capture images of full dynamic ranges of the real scenes, which are called high dynamic range (HDR) images. One of the most popular HDR image acquisition approaches is to merge a set of conventional low dynamic range (LDR) images taken with different exposure times [2,3]. However, since the scene is often dynamic and has moving objects that introduce ghosting artifacts, a simple composition may fail to provide a high-quality HDR image. Therefore, ghost-free HDR imaging, which attempts to remove ghosting artifacts in the synthesized HDR image, has been an important research topic [4–10].

One approach to ghost-free HDR imaging is to register LDR images before HDR image synthesis, assuming the global camera motion. For example, Kang *et al*. [11] computed optical flow among the LDR images. To improve the accuracy of optical flow estimation, Zimmer *et al*. [12] employed a gradient domain correspondence estimation. However, optical flow-based correspondence estimation in [11, 12] may cause unreliable motion vectors due to information loss in poorly-exposed regions. Recently, Hu *et al*. [7] employed the PatchMatch algorithm due to its superior correspondence estimation performance. While this algorithm provides high-quality HDR images, the main disadvantage is its high computational complexity.

In addition to the global motion, moving objects cause ghosting artifacts as well; thus, attempts have also been made to alleviate the contributions of regions on moving objects detected by ghost region detection. For example, based on the assumption that the background irradiance with respect to exposure times are linear, Gallo *et al.* [4] used the deviation from linearity to measure ghost effects. Heo *et al.* [5] detected ghost regions employing the joint probability density between different exposure images and then further refined these regions using energy minimization. Recently, Lee *et al*. [9] employed a low-rank matrix completion framework, assuming a static background and sparsity of moving objects, but they used a less accurate approximation to the matrix rank.

In this work, we propose a ghost-free HDR image synthesis algorithm using a rank minimization framework. Specifically, assuming the linearity of irradiances among LDR images, we formulate background estimation as a rank minimization problem and then solve it efficiently using the augmented Lagrange multiplier (ALM) method, where the optimization variables are updated by closed-form solutions. Experimental results show that the proposed algorithm provides comparable or even higher image qualities than state-of-the-art approaches [5, 7, 9, 10], while providing substantial improvement in speed.

The rest of this paper is organized as follows. Section 2 briefly reviews related work. Section 3 describes the proposed HDR image synthesis algorithm. Section 4 provides experimental results. Finally, Section 5 concludes the paper.

2. RELATED WORK

Several ghost-free HDR imaging algorithms have been developed based on the assumption that pixels in the ghost regions show a variation over exposures and use the deviation from linearity to measure ghost effects [4–6]. Recently, to better exploit the linearity of irradiances, a rank minimization framework has been employed in ghost-free HDR imaging [8–10]. In [8], Oh *et al*. attempted to align input images and detect moving objects simultaneously using a rank minimization approach to compensate for both camera motion and object motion. Then, in [10], they extended their work employing the low-rank matrix completion [13]. Lee *et al.* [9] developed a low-rank matrix completion-based HDR imaging with multiple physical constraints on the properties of ghost regions. However, while recent researches on rank minimization-based ghost-free HDR imaging approaches provide high-quality results, such sophisticated algorithms require significant computational resources.

Relation to Prior Work. Motivated by the recent advancement on rank minimization framework [14–16], we focus on the application of the truncated nuclear norm minimization to HDR imaging, which is known to be a better approximation to the rank function.

The main difference between the proposed algorithm and a previous approach in [10] is that we develop a computationally more efficient algorithm by closed-form solutions in the optimization procedure, which was solved iteratively in [10].

3. PROPOSED ALGORITHM

3.1. Problem Formulation

We are given a set of images taken with different exposure times $\{\mathrm{vec}(Z_1), \mathrm{vec}(Z_2), \ldots, \mathrm{vec}(Z_n)\}$, where $\mathrm{vec}(Z_i) \in \mathbb{R}^{m\times 1}$ denotes a vector of pixel values, and m and n are the number of pixels in an image and the number of input images, respectively. Then, using the camera response function [2], we construct the observed irradiance matrix $\mathbf{D} = \{\mathrm{vec}(I_1), \mathrm{vec}(I_2), \ldots, \mathrm{vec}(I_n)\}$, where $\mathrm{vec}(I_i)$ is the irradiance vector for the ith image.

The scene irradiance can be decomposed into the underlying background and moving objects. Specifically, the irradiance matrix $\mathbf{D}$ can be represented as a sum of two matrices $\mathbf{X}$ and $\mathbf{E}$, which correspond to the underlying background scene and moving objects, respectively. Since matrix $\mathbf{X}$ represents a static scene, it has low rank, and $\mathbf{E}$ is a sparse matrix, *i.e.*, most elements in $\mathbf{E}$ are zero. In addition, we assume that only a limited number of observations can be made in HDR imaging in general due to under- and over-exposed regions in input images. However, note that we can also select preferable objects or regions that will appear in the synthesized HDR image by masking images as done in [10]. Then, irradiance estimation for ghost-free HDR imaging can be formulated as the following rank minimization problem

$$\begin{aligned} \underset{\mathbf{X},\mathbf{E}}{\text{minimize}} \quad & \mathrm{rank}(\mathbf{X}) + \lambda\|\mathbf{E}\|_0 \\ \text{subject to} \quad & \mathcal{P}_\Omega(\mathbf{X}+\mathbf{E}) = \mathcal{P}_\Omega(\mathbf{D}), \end{aligned} \tag{1}$$

where $\mathbf{X} \in \mathbb{R}^{m\times n}$, and $\mathcal{P}_\Omega$ denotes a sampling operator in the observed region Ω, *i.e.*,

$$[\mathcal{P}_\Omega(\mathbf{A})]_{ij} = \begin{cases} A_{ij}, & \text{if } (i,j) \in \Omega, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Because it is intractable in practice to solve the optimization problem in (1) directly, previous approaches use an approximate method via convex relaxation [13, 17]. For example, $\mathrm{rank}(\mathbf{A})$ is approximated by the nuclear norm $\|\mathbf{A}\|_* = \sum_{k=1}^{\min(m,n)} \sigma_k(\mathbf{A})$ [13] or the truncated nuclear norm [14] (also known as the partial sum of singular values [15]) $\|\mathbf{A}\|_r = \sum_{k=r+1}^{\min(m,n)} \sigma_k(\mathbf{A})$, where σ_k denotes the kth largest singular value of $\mathbf{A}$. Also, the ℓ_0-norm $\|\mathbf{E}\|_0$ is approximated by the ℓ_1-norm $\|\mathbf{E}\|_1$. In this work, we employ the truncated nuclear norm as an approximation for the rank function in (1), since it can exploit *a priori* target rank information about the problem in rank minimization. More specifically, given the target rank r, the optimization in (1) can be rewritten as

$$\begin{aligned} \underset{\mathbf{X},\mathbf{E}}{\text{minimize}} \quad & \|\mathbf{X}\|_r + \lambda\|\mathbf{E}\|_1 \\ \text{subject to} \quad & \mathcal{P}_\Omega(\mathbf{X}+\mathbf{E}) = \mathcal{P}_\Omega(\mathbf{D}), \end{aligned} \tag{3}$$

where λ controls the relative importance between the rank of $\mathbf{X}$ and the sparsity of $\mathbf{E}$. Based on the assumption that the underlying scene is static, we set the target rank to $r = 1$ in this work.

3.2. Optimization

We propose a computationally efficient algorithm to solve the optimization problem in (3) employing the ALM method [18, 19], which is known to be an efficient solver to the nuclear norm minimization due to its fast convergence and scalability. In [10], Oh *et al.* also solved the optimization in (3) using the ALM method. However, because of the sampling operator $\mathcal{P}_\Omega$, they could not update the optimization variable in a closed-form matter, thus solved it iteratively instead, which requires higher computational resources. The main novelty of the proposed algorithm over [10] is that we update the optimization variables by closed-form solutions, so that it is more efficient than the conventional algorithm [10]. Then, the main challenge is how to reformulate (3) to an ALM-oriented form, so that closed-form solutions can be obtained in the optimization procedure. To this end, we introduce slack variables as done in [16]. More specifically, we rewrite the optimization in (3) as

$$\begin{aligned} \underset{\mathbf{X},\mathbf{E},\mathbf{S}}{\text{minimize}} \quad & \|\mathbf{X}\|_r + \lambda\|\mathbf{E}\|_1 \\ \text{subject to} \quad & \mathbf{X}+\mathbf{E}+\mathbf{S} = \mathcal{P}_\Omega(\mathbf{D}), \\ & \|\mathcal{P}_\Omega(\mathbf{S})\|_F \le \delta, \end{aligned} \tag{4}$$

where $\mathbf{S}$ denotes a matrix of slack variables, and δ is the noise level.

The ALM method solves a series of unconstrained subproblems instead of the original constrained optimization problem. Here, we show how we adopt the ALM method to efficiently solve the optimization problem in (4). Specifically, for our problem in (4), we first define the augmented Lagrangian function $\mathcal{L}(\mathbf{X}, \mathbf{E}, \mathbf{S}, \mathbf{\Lambda}, \mu)$ as

$$\begin{aligned} \mathcal{L}(\mathbf{X}, \mathbf{E}, \mathbf{S}, \mathbf{\Lambda}, \mu) &= \|\mathbf{X}\|_r + \lambda\|\mathbf{E}\|_1 \\ &+ \langle \mathbf{\Lambda}, \mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X} - \mathbf{E} - \mathbf{S}\rangle + \frac{\mu}{2}\|\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X} - \mathbf{E} - \mathbf{S}\|_F^2, \end{aligned} \tag{5}$$

where $\mu > 0$ is a parameter to penalize the equality constraint, $\mathbf{\Lambda} \in \mathbb{R}^{m\times n}$ is the Lagrange multiplier matrix, and $\langle\cdot,\cdot\rangle$ denotes the matrix inner product. A solution to the original optimization problem in (4) can be obtained by minimizing its augmented Lagrangian $\mathcal{L}(\mathbf{X}, \mathbf{E}, \mathbf{S}, \mathbf{\Lambda}, \mu)$ for an estimate of $\mathbf{\Lambda}$ and a sufficiently large value of μ [20]. The ALM algorithm iteratively estimates both the optimal solution and the Lagrange multiplier until convergence. More specifically, we employ an alternating direction method [19], which separates an optimization over each variable and solves it successively. These sub-problems are described below.

Updating $\mathbf{X}$. In the first step, given estimates of $\mathbf{E}_k$, $\mathbf{S}_k$, and $\mathbf{\Lambda}_k$, we update $\mathbf{X}$ as

$$\begin{aligned} \mathbf{X}_{k+1} &= \arg\min_{\mathbf{X}} \mathcal{L}(\mathbf{X}, \mathbf{E}_k, \mathbf{S}_k, \mathbf{\Lambda}_k, \mu_k) \\ &= \arg\min_{\mathbf{X}} \|\mathbf{X}\|_r + \langle \mathbf{\Lambda}_k, \mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X} - \mathbf{E}_k - \mathbf{S}_k\rangle \\ &\quad + \frac{\mu_k}{2}\|\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X} - \mathbf{E}_k - \mathbf{S}_k\|_F^2 \\ &= \arg\min_{\mathbf{X}} \|\mathbf{X}\|_r + \frac{\mu_k}{2}\|\mathbf{X} - \mathcal{P}_\Omega(\mathbf{D}) + \mathbf{E}_k + \mathbf{S}_k - \mu_k^{-1}\mathbf{\Lambda}_k\|_F^2. \end{aligned} \tag{6}$$

We can obtain the closed-form solution to (6) by applying the partial singular value thresholding (PSVT) operator [15]. Specifically, let us consider the singular value decomposition (SVD) of a matrix $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, where $\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \ldots, \sigma_{\min(m,n)})$. Then, the PSVT operator is defined by $\mathbb{P}_{r,\tau}(\mathbf{A}) = \mathbf{U}(\mathbf{\Sigma}_1 + \mathcal{S}_\tau(\mathbf{\Sigma}_2))\mathbf{V}^T$, where $\mathbf{\Sigma}_1 = \mathrm{diag}(\sigma_1, \ldots, \sigma_r, 0, \ldots, 0)$ and $\mathbf{\Sigma}_2 = \mathrm{diag}(0, \ldots, 0, \sigma_{r+1}, \ldots, \sigma_{\min(m,n)})$, respectively, and $\mathcal{S}_\tau(\mathbf{A})$ denotes the element-wise soft-thresholding operator for $\tau > 0$, *i.e.*, $[\mathcal{S}_\tau(\mathbf{A})]_{ij} = \text{sgn}(A_{ij}) \cdot \max\{|A_{ij}| - \tau, 0\}$. The PSVT operator provides the closed-form solution to (6) [15], given by

$$\mathbf{X}_{k+1} = \mathbb{P}_{r,\mu_k^{-1}}\big(\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{E}_k - \mathbf{S}_k + \mu_k^{-1}\mathbf{\Lambda}_k\big). \tag{7}$$

Updating E. Next, we estimate $\mathbf{E}$, given $\mathbf{X}_{k+1}$, $\mathbf{S}_k$, and $\mathbf{\Lambda}_k$. Specifically, we solve the following optimization problem:

$$\begin{aligned}\mathbf{E}_{k+1} &= \arg\min_{\mathbf{E}} \mathcal{L}(\mathbf{X}_{k+1}, \mathbf{E}, \mathbf{S}_k, \mathbf{\Lambda}_k, \mu_k)\\ &= \arg\min_{\mathbf{E}} \lambda\|\mathbf{E}\|_1 + \langle \mathbf{\Lambda}_k, \mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{E} - \mathbf{S}_k\rangle\\ &\quad + \frac{\mu_k}{2}\|\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{E} - \mathbf{S}_k\|_F^2\\ &= \arg\min_{\mathbf{E}} \lambda\|\mathbf{E}\|_1\\ &\quad + \frac{\mu_k}{2}\|\mathbf{E} - \mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{S}_k + \mu_k^{-1}\mathbf{\Lambda}_k\|_F^2. \end{aligned} \tag{8}$$

The closed-form solution to (8) can be obtained by the soft-thretholding operator [21], given by

$$\mathbf{E}_{k+1} = \mathcal{S}_{\frac{\lambda}{\mu_k}}\big(\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{S}_k + \mu_k^{-1}\mathbf{\Lambda}_k\big). \tag{9}$$

Updating S. Also, we estimate $\mathbf{S}$ with fixed $\mathbf{X}_{k+1}$, $\mathbf{E}_{k+1}$, and $\mathbf{\Lambda}_k$, solving the following optimization problem.

$$\begin{aligned}\mathbf{S}_{k+1} &= \arg\min_{\|\mathcal{P}_\Omega(\mathbf{S})\|_F \le \delta} \mathcal{L}(\mathbf{X}_{k+1}, \mathbf{E}_{k+1}, \mathbf{S}, \mathbf{\Lambda}_k, \mu_k)\\ &= \arg\min_{\|\mathcal{P}_\Omega(\mathbf{S})\|_F \le \delta} \langle \mathbf{\Lambda}_k, \mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{E}_{k+1} - \mathbf{S}\rangle\\ &\quad + \frac{\mu_k}{2}\|\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{E}_{k+1} - \mathbf{S}\|_F^2\\ &= \arg\min_{\|\mathcal{P}_\Omega(\mathbf{S})\|_F \le \delta} \|\mathbf{S} - \mathcal{P}_\Omega(\mathbf{D}) + \mathbf{X}_{k+1} + \mathbf{E}_{k+1} - \mu_k^{-1}\mathbf{\Lambda}_k\|_F^2. \end{aligned} \tag{10}$$

In [16], we have shown that the optimization problem in (10) can be solved via a closed-form solution. Specifically, let $\mathbf{Y}_{k+1} = \mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{E}_{k+1} + \mu_k^{-1}\mathbf{\Lambda}_k$ for simpler notations. Then, the closed-form solution to (10) is given by

$$\mathbf{S}_{k+1} = \mathcal{P}_{\Omega^c}(\mathbf{Y}_{k+1}) + \min\left\{\frac{\delta}{\|\mathcal{P}_\Omega(\mathbf{Y}_{k+1})\|_F}, 1\right\}\mathcal{P}_\Omega(\mathbf{Y}_{k+1}). \tag{11}$$

Updating Λ. Finally, given $\mathbf{X}_{k+1}$, $\mathbf{E}_{k+1}$, and $\mathbf{S}_{k+1}$, the Lagrange multiplier $\mathbf{\Lambda}$ is updated as

$$\mathbf{\Lambda}_{k+1} = \mathbf{\Lambda}_k + \mu_k(\mathcal{P}_\Omega(\mathbf{D}) - \mathbf{X}_{k+1} - \mathbf{E}_{k+1} - \mathbf{S}_{k+1}). \tag{12}$$

In the optimization procedure, variables $\mathbf{X}$, $\mathbf{E}$, $\mathbf{S}$, and $\mathbf{\Lambda}$ are iteratively updated via (7), (9), (11), and (12), respectively, until convergence. Note that all those optimization variables are updated by closed-form solutions, so that the proposed algorithm is computationally more efficient than conventional rank minimization-based approaches [9, 10]

3.3. HDR Image Composition

We synthesize an HDR image by simply averaging the background irradiance maps in the low-rank matrix $\mathbf{X}$. Specifically, we compose an HDR image by

$$R_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}, \tag{13}$$

where R_i is the estimated radiance at pixel location i.

Table 1: The computation times of Heo *et al.*'s algorithm [5], Hu *et al.*'s algorithm, Lee *et al.*'s algorithm [9], Oh *et al.*'s algorithm [10], and the proposed algorithm for the "SculptureGarden" images.

	Heo *et al.*	Hu *et al.*	Lee *et al.*	Oh *et al.*	Proposed
Times (s)	390	392	91	149	52

4. EXPERIMENTAL RESULTS

We evaluate the performance of the proposed HDR image synthesis algorithm on two image data sets. The parameters δ and λ in (4) and (5) are set to 0 and $1/\sqrt{\max(m,n)}$, respectively. We define the observed region as a set of properly-exposed pixel locations, given by

$$\Omega = \{(i,j) | Z_{\text{th}} \le Z_{ij} \le 255 - Z_{\text{th}}\}, \tag{14}$$

where the threshold value is fixed to $Z_{\text{th}} = 2$ in this work. Note, however, that the observed region Ω can be chosen manually by a user as noted in [10]. We use the MATLAB function `tonemap` to display the results of the proposed algorithm, Hu *et al.*'s algorithm [7], Lee *et al.*'s algorithm [9], and Oh *et al.*'s algorithm [10], while Heo *et al.*'s algorithm [5] uses their own tone mapping technique. The results of the conventional algorithms are obtained by executing the codes provided by the respective authors.

Fig. 1 compares the synthesized results and their detailed parts on the "SculptureGarden" images. In Figs. 1(a)–(c), Heo *et al.*'s, Hu *et al.*'s, and Lee *et al.*'s algorithms provide HDR images with moving objects from one of the input images, whereas Oh *et al.*'s algorithm and the proposed algorithm yield the background scene in Figs. 1(d) and (e), respectively. This is because we define Ω as a set of properly-exposed pixels only, but note that we can also produce output HDR images that contain objects in an input image [10]. In Fig. 1(a), Heo *et al.*'s algorithm provides results with severe artifacts, *e.g.*, smeared textures on the ground and color distortions on the stairs, due to its incorrect ghost region detection. Hu *et al.*'s algorithm in Fig. 1(b) provides images without ghosting artifacts but yields blurring artifacts near boundaries of walking people, where correspondence matching fails due to poor exposure in the reference image. While Lee *et al.*'s algorithm in Fig. 1(c) preserves image details and removes ghosting artifacts effectively, it still provides color artifacts in highly saturated regions. Oh *et al.*'s algorithm and the proposed algorithm in Figs. 1(d) and (e), respectively, produces comparable results, but we see that the proposed algorithm removes ghosting artifacts more effectively.

Fig. 2 shows the synthesized HDR images and their magnified parts on the "Arch" images, obtained by each algorithm. Although all algorithms effectively remove ghosting artifacts, Hu *et al.*'s algorithm in Fig. 2(b) provides blurring artifacts in the ceiling because of failure of the correspondence estimation at poorly-exposed regions. We see that Heo *et al.*'s, Lee *et al.*'s, Oh *et al.*'s algorithms, and the proposed algorithm yield comparable results.

Table 1 compares the actual execution times for the "SculptureGarden" set with five images of resolution 1024×754. We use a PC with a 2.6 GHz CPU and 8 GB RAM. We see that the proposed algorithm is the most efficient in terms of execution time, especially compared with Oh *et al.*'s algorithm, which is also based on the low-rank matrix completion framework. This is because the proposed algorithm updates the optimization variables by closed-form solutions, whereas Oh *et al.*'s algorithm performs it iteratively. These results indicate that the proposed algorithm provides comparable or even better performance than conventional algorithms, while demanding lower computational resources.

(a) (b) (c) (d) (e)

Fig. 1: Synthesized results of the "SculptureGarden" image set by (a) Heo *et al*.'s algorithm [5], (b) Hu *et al*.'s algorithm [7], (c) Lee *et al.*'s algorithm [9], (d) Oh *et al*.'s algorithm [10], and (e) the proposed algorithm.

(a) (b) (c) (d) (e)

Fig. 2: Synthesized results of the "Arch" image set by (a) Heo *et al*.'s algorithm [5], (b) Hu *et al*.'s algorithm [7], (c) Lee *et al.*'s algorithm [9], (d) Oh *et al*.'s algorithm [10], and (e) the proposed algorithm.

5. CONCLUSIONS

We developed a ghost-free HDR image synthesis algorithm via truncated nuclear norm minimization for low-rank matrix completion in this work. Based on the assumption that the underlying background is static, we first represented the background and moving objects as a low-rank matrix and a sparse matrix, respectively. Then, we formulated the background estimation as the low-rank matrix completion problem and solved it efficiently using the ALM method. Experimental results demonstrated that the proposed algorithm provides image quality improvements, while requiring lower computational resources.

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