# Gammatone Filter Based on Stochastic Computation

Naoya Onizawa\*<sup>†</sup>, Shunsuke Koshita<sup>‡</sup>, Shuichi Sakamoto<sup>†</sup>, Masahide Abe<sup>‡</sup>, Masayuki Kawamata<sup>‡</sup>, and Takahiro Hanyu<sup>†</sup>

\* Frontier Research Institute for Interdisciplinary Sciences, Tohoku University, Sendai, Japan 980-8578

<sup>†</sup> Research Institute of Tohoku University, Sendai, Japan 980-8577

<sup>‡</sup> Department of Electronic Engineering, Graduate School of Engineering, Tohoku University, Sendai, Japan 980-8579

Email:nonizawa@m.tohoku.ac.jp

Abstract—This paper introduces a design of a gammatone filter based on stochastic computation for area-efficient hardware. The gammatone filter well expresses the performance of human auditory peripheral mechanism and has a potential of improving advanced speech communications systems, especially hearing assisting devices and noise robust speech recognition systems. Using stochastic computation, a power-and-area hungry multiplier used in a digital filter is replaced by a simple logic gate, leading to area-efficient hardware. However, a straightforward implementation of the stochastic gammatone filter suffers from significantly low accuracy in computation, which results in a low dynamic range (a ratio of the maximum to minimum magnitude) due to a small value of a filter gain. To improve the computational accuracy, gain-balancing techniques are presented that represent the original gain as the product of multiple larger gains introduced at the second-order sections. As a result, the proposed techniques maintain the original gain of the filter while improving the computational accuracy. The proposed stochastic gammatone filters are designed and evaluated using MATLAB that achieves a high dynamic range of 71.71 dB compared with a low dynamic range of 5.47 dB in the straightforward implementation.

*Keywords*-stochastic logic, gammatone filter, auditory filter, IIR filter, digital circuit implementation

#### I. INTRODUCTION

Brainware (brain-inspired) computing and LSI (BLSI) implementations have been recently studied that achieves a significant cognition capability compared to a traditional computation based approach [1]–[3]. For brainware auditory signal processing, a gammatone filter that has a similar response to the impulse responses of basilar membrane [4], [5] is a promising technique for advanced speech communications systems, such as cochlear implants [6]–[8] and noise robust speech recognitions [9].

However, the gammatone filter requires high computational power as the function of the gammatone filter is complex. Several VLSI implementations have been studied using analog [6], [7] or digital circuits. [8] In analog implementations, the complex function of the gammatone filter is efficiently designed, leading to low-power and low-area hardware, while they suffer from process variations, especially in advanced CMOS processes. In digital implementations, the gammatone filter is designed using a high-order infinite impulse response (IIR) filter. However, a large number of multipliers are required, causing large power dissipation and large area.

In this paper, we introduce a gammatone filter based on *stochastic computation*. Stochastic computation [10], [11] is a



Fig. 1. Frequency responses of five gammatone filters, where  $f_c$  are 0.5, 1, 2, 5, and 10 kHz.

purely-digital implementation technique that represents data as streams of random bits, while a power-and-area hungry multiplier used in a digital IIR filter is replaced by a simple logic gate, leading to area-efficient hardware. First, a stochastic gammatone filter is designed using a straightforward implementation technique and is then analyzed in terms of a dynamic range. Note that the dynamic range is used as a ratio of the maximum to minimum magnitude throughout the paper. Based on the analysis, the straightforward implementation suffers from significantly low computation accuracy due to a small value of a filter gain, which results in a low dynamic range. To increase the dynamic range, two gainbalancing techniques are proposed. The proposed gammatone filters are designed and evaluated using MATLAB, achieving a high dynamic range of 71.71 dB while the straightforward implementation has a very small dynamic range of 5.47 dB. To the best of our knowledge, this is the first hardware algorithm and architecture of a gammatone filter based on stochastic computation.

The rest of the paper is organized as follows. Section II reviews the gammatone filter and designs it using cascaded second-order sections. Section III describes the stochastic gammatone filter based on the gain-balancing techniques. Section IV evaluates the magnitude responses of the stochastic gammatone filter compared with a floating-point result. Section V concludes this paper.

#### II. GAMMATONE FILTER

# A. Transformation of gammatone filter

A gammatone filter is represented by an impulse response that is the product of a gamma distribution and a sinusoidal tone as follows:

$$g(t) = at^{n-1}e^{-2\pi b ERB(f_c)t}\cos(2\pi f_c t + \phi) \ (t > 0), \quad (1)$$



Fig. 2. Block diagram of four cascaded second-order sections for gammatone filters.

where a is a constant, n is the order of the filter, b is the bandwidth of the filter,  $f_c$  is the center frequency of the filter, and  $\phi$  is the steering phase. The equation can represent the human auditory filter when n is 4 and b is 1.019 times Equivalent Rectangular Bandwidth (ERB) [4]. The ERB can be approximated [5] as follows:

$$ERB(f_c) = 24.7(4.37f_c/1000 + 1).$$
<sup>(2)</sup>

In this paper, a is set to 1 and  $\phi$  is set to 0 as [7]. The frequency responses of the gammatone filters are shown in Fig. 1, where  $f_c$  are 0.5, 1, 2, 5, and 10 kHz.

The gammatone impulse response is converted to that in the frequency domain using the Laplace transform, which is then converted to a digital IIR filter using the bilinear transform with a  $f_c$  of 5 kHz and a sampling frequency  $f_s$  of 20 kHz used in this paper. The transfer function in digital domain, H(z), is described using an 8th-order digital IIR filter as follows:

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_8 z^{-8}}{1 + a_1 z^{-1} + \dots + a_8 z^{-8}},$$
(3)

where  $b_n \ (0 \le n \le 8)$  and  $a_m \ (1 \le m \le 8)$  are coefficients.

# B. Implementation using cascaded second-order sections

The 8th-order IIR filter for the gammatone response is factorized to form four second-order sections as follows:

$$H(z) = G \prod_{k=1}^{4} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}}, \qquad (4)$$

$$= G \prod_{k=1}^{4} H_k(z) = G \prod_{k=1}^{4} \frac{N_k(z)}{D_k(z)},$$
 (5)

where G is a gain, the transfer function of the feedforward block is defined as  $N_k(z)$  and that of the feedback block is defined as  $1/D_k(z)$ . The four second-order sections are described as shown in Fig. 2, where each section is designed using a second-order IIR filter.

Fig. 3 shows magnitudes of frequency responses in the four cascaded second-order IIR filter for a gammatone response, where the center frequency is  $0.5\pi$  rad. At each section, the maximum magnitude of the transfer function, max $|H_k(e^{j\omega})|$ , is larger than 1. In contrast, the maximum magnitude of the transfer function, max $|H(e^{j\omega})|$ , is 1 (0 dB) as G is  $6.795 \times 10^{-5}$ . In the next section, a gammatone filter based on stochastic computation is designed using the four cascaded IIR filter.



Fig. 3. Magnitudes of frequency responses in the four cascaded second-order IIR filter for a gammatone response, where the center frequency is  $0.5\pi$  rad.



Fig. 4. Stochastic computation blocks: (a) multiplier in unipolar coding, (b) multiplier in bipolar coding, and (c) scaled adder.

## III. STOCHASTIC IMPLEMENTATION OF GAMMATONE FILTER

## A. Stochastic computation

Stochastic computation has been recently exploited for several applications, such as low-density parity-code (LDPC) decoding [12], [13], image processing [14]–[16], and digital filters [17]–[19]. In stochastic computation, information is carried by the frequency of ones in a sequence in one of two formats: *unipolar* and *bipolar* coding. Note the probability of observing a '1' to be  $P_a = \Pr(a(t) = 1)$  for a sequence of bits a(t). A value a is  $a = P_a$ ,  $(0 \le a \le 1)$  in *unipolar* coding and is  $a = (2 \cdot P_a - 1), (-1 \le a \le 1)$  in *bipolar* coding.

A multiplier is simply designed using a simple logic gate [11], such as a 2-input AND gate in unipolar coding or a 2-input XNOR gate in bipolar coding shown in Fig. 4 (a) and (b). The output probability,  $P_c$ , is  $(P_a \cdot P_b)$  in unipolar coding. In the example shown in Fig. 4 (a), input values are represented using 10 bits and are multiplied with 10 clock cycles. Fig. 4 (c) shows a block diagram of a two-input scaled addition designed using a two-input multiplexor, unlike a binary full adder. The output probability,  $P_c$ , is  $P_s \cdot (P_a + P_b)$ , where  $P_s$  is a probability of a selector input to the multiplexor.

#### B. Circuit implementation

The gammatone filter is designed using the IIR filter with cascaded form consisting of four second-order sections shown in Fig. 2, where the magnitude responses are summarized in Fig. 3. The absolute values of several coefficients,  $b_{0k}$ ,  $b_{1k}$ ,  $b_{2k}$ , are larger than '1'. For example,  $b_{21}$  is -4.7077. These



Fig. 5. Second-order IIR filter based on stochastic computation in bipolar coding, where multiplications and additions are realized using stochastic logic.

coefficients needs to be normalized as stochastic computation can represent a value of -1 to 1. The normalizing factor,  $n_k$ , is defined as follows:

$$n_k = \frac{1}{m_k},\tag{6}$$

$$m_k \geq \max(|b_{0k}|, |b_{1k}|, |b_{2k}|),$$
 (7)

where  $m_k = \{2^l \mid l = 0, 1, ...\}$ . Using Eq. (6), the transfer function at each section is derived as follows:

$$H_k(z) = \frac{n_k(b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2})}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}} \cdot \frac{1}{n_k}.$$
 (8)

Fig. 5 shows a block diagram of a second-order IIR filter based on stochastic computation in bipolar coding. The stochastic IIR filter is designed based on [19]. In the stochastic IIR filter, multiplications are realized using XNOR gates and additions are realized using multiplexors. The input signal in binary format is converted to a stochastic bit stream using a binary-to-stochastic (B2S) converter, where the bit length is  $N_{sto}$  (the number of stochastic bits to represent a real value). Delay elements are realized using stochastic-to-binary (S2B) converters that are  $\lceil \log_2 N_{sto} \rceil$ -bit counter. The details of B2S and S2B are shown in [19]. As an output signal of the addition is scaled down, the output signal after S2B is scaled up in binary domain based on binary multiplication. For simple hardware, the binary multiplication is designed using a binary shifter.

A different stochastic implementation of a second-order IIR filter is described in Fig. 6. It is designed based on binary/stochastic hybrid logic, where additions are realized in binary domain instead of stochastic domain. In a specific application, the hybrid computation can improve computation accuracy with a small area overhead [20] compared with an implementation based on only stochastic computation. The hybrid IIR filter can be designed in bipolar coding as shown in Fig. 6 or unipolar coding that exploits a stochastic bit stream with a sign bit.



Fig. 6. Second-order IIR filter based on binary/stochastic hybrid computation, where only multiplications are realized using stochastic logic.



Fig. 7. Second-order IIR filter based on: (a) the globally gain balancing (GGB) technique and (b) the locally gain balancing (LGB) technique

#### C. Gain balancing

Using Eqs. (4), (8), a stochastic gammatone filter can be designed as a straightforward implementation. But, a small value of the gain, G, needs to be represented, causing low computation accuracy in stochastic computation. To avoid representing a small value, a globally gain balancing (GGB) technique is presented. In GGB, an input signal is scaled at each section as shown in Fig. 7 (a), while a multiplication with G as shown in Fig. 2 is removed. The scaling factor at each section,  $G_k$ , is determined based on the  $L_{\infty}$  norm of  $H_k(e^{j\omega})$  as follows:

$$G_k = \frac{1}{\max|H_k(e^{j\omega})|}.$$
(9)

Using Eq. (9), the transfer function of the gammatone filter based on GGB is derived as follows:

$$H(z) = \prod_{k=1}^{4} G_k H_k(z).$$
 (10)

In addition,  $G_k$ , is locally balanced to a feedback gain,  $g_{0k}$ , and a feedforward gain,  $g_{1k}$  at each section as shown in Fig. 7 (b). In the locally gain balancing (LGB) technique,  $g_{0k}$  is first determined based on the  $L_{\infty}$  norm of  $1/D_k(e^{j\omega})$  as follows:

$$g_{0k} = \frac{1}{\max|1/D_k(e^{j\omega})|} = \frac{G_k}{g_{1k}}.$$
 (11)

Then,  $g_{1k}$  is determined. Using Eq. (11), the transfer function



Fig. 8. Magnitude responses of gammatone filters based on bipolar coding, where Nsto is  $2^{16}$ . The proposed LGB technique increases the dynamic range to 42.77 dB compared with 5.47 dB of the straightforward implementation.



Fig. 9. Magnitude responses of gammatone filters based on the LGB technique, where Nsto is  $2^{16}$ . The proposed gammatone increases the dynamic range to 71.71 dB compared with 5.47 dB of the straightforward implementation.

of the gammatone filter based on LGB is derived as follows:

$$H(z) = \prod_{k=1}^{4} g_{0k} \frac{g_{1k}(b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2})}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}.$$
 (12)

## IV. EVALUATION

The gammatone filters based on stochastic computation are designed and evaluated using MATLAB, where  $f_c$  is 5 kHz and  $f_s$  is 20 kHz. Fig. 8 shows magnitude responses of gammatone filters in bipolar coding, where  $N_{sto}$  (the number of stochastic bits to represent a real value) is  $2^{16}$ . The magnitudes are averaged by 100-trial outputs obtained using a sinusoidal wave as an input signal, x. Based on the straightforward implementation, the dynamic rage is just 5.47 dB, showing a failure operation as gammatone filtering. It is because a small value of G reduces the computation accuracy. In contrast, using the GGB and the LGB techniques, the dynamic ranges are increased to 35.25 dB and 42.77 dB, respectively. However, the maximum magnitudes do not reach to 0 dB that a floating-point filter achieves.

Fig. 9 shows magnitude responses of gammatone filters based on the LGB technique, where  $N_{sto}$  is  $2^{16}$ . The filters are also designed based on binary/stochastic hybrid computation. Using the hybrid computation, the dynamic ranges are increased to 51.51 dB in bipolar coding and 71.71 dB in unipolar coding. In addition, the maximum magnitudes reach to almost 0 dB as well as the floating-point result. The



Fig. 10. Magnitude responses of the stochastic gammatone filter based on the hybrid computation in unipolar coding depending on  $N_{sto}$ .



Fig. 11. Ten transient responses of the stochastic gammatone filter based on the hybrid computation in unipolar coding, where Nsto is  $2^{16}$  and the input signal is a sinusoidal wave at 8 kHz.  $T (= 1/f_s)$  is the sample time of the gammatone filter.

magnitude responses are varied depending on  $N_{sto}$  as shown in Fig. 10.

Fig. 11 shows ten transient responses of the stochastic gammatone filter based on the hybrid computation in unipolar coding, where  $N_{sto}$  is  $2^{16}$  and the input signal is a sinusoidal wave at 8 kHz. Gammatone responses appear in the simulation, but, at each trial, they are slightly different because of stochastic (random) computation.

## V. CONCLUSION

In this paper, the gammatone filter based on stochastic computation has been presented for area-efficient hardware. The straightforward implementation of the stochastic gammatone filter designed using the cascaded second-order IIR filter is evaluated that causes the very low dynamic range due to the very small value of the filter gain. To increase the dynamic range, the gain-balancing techniques have been proposed that split the original small gain to multiple larger gains. Using the gain-balancing techniques, the computation accuracy at each IIR filter is improved, leading to a high dynamic range. As a result, the proposed stochastic gammatone filter achieves a high dynamic range of 71.71 dB compared with a low dynamic range of 5.47 dB in the straightforward implementation.

Future works include a reduction of the number of stochastic bits using different filter structures, such as a lattice structure and a state-space implementation, and performance evaluation in hardware.

#### ACKNOWLEDGMENT

This work was supported by Brainware LSI Project of MEXT and JSPS KAKENHI Grant Number 26700003.

#### REFERENCES

- G. Hinton, S. Osindero, and Y. Teh, "A fast learning algorithm for deep belief nets," *Science*, vol. 18, no. 7, pp. 1527–1554, July 1996.
- [2] P. A. Merolla, et al, "A million spiking-neuron integrated circuit with a scalable communication network and interface," *Science*, vol. 345, no. 6197, pp. 668–673, Aug 2014.
- [3] S. Park, K. Bong, D. Shin, J. Lee, S. Choi, and H.-J. Yoo, "1.93TOPS/W scalable deep learning/inference processor with tetra-parallel mimd architecture for big-data applications," in 2015 IEEE International Solid-State Circuits Conference (ISSCC), Feb 2015, pp. 1–3.
- [4] R. Patterson and J. Hoddsworth, "A functional model of neural activity patterns and auditory images," *Advance in Speech Hearing and Language Processing*, vol. 3, pp. 547–553, 1996.
- [5] B. Glasberg and B. Moore, "Derivation of auditory filter shapes from notched-noise data," *Hearing Research*, vol. 47, pp. 103–108, 1990.
- [6] A. Katsiamis, E. Drakakis1, and R. Lyon, "Practical gammatone-like filters for auditory processing," *EURASIP Journal on Audio, Speech,* and Music Processing, vol. 2007, pp. 1–15, 2007.
- [7] W. Ngamkham, C. Sawigun, S. Hiseni, and W. Serdijn, "Analog complex gammatone filter for cochlear implant channels," in 2010 IEEE International Symposium on Circuits and Systems (ISCAS), May 2010, pp. 969–972.
- [8] R. Karuppuswamy, K. Arumugam, and S. Priya, "Folded architecture for digital gammatone filter used in speech processor of cochlear implant," *ETRI Journal*, vol. 35, no. 4, pp. 697–705, Aug 2013.
- [9] R. Schlter, I. Bezrukov, H. Wagner, and H. Ney, "Gammatone features and feature combination for large vocabulary speech recognition," in *ICASSP*, 2007, pp. 649–652.
- [10] B. R. Gaines, "Stochastic computing systems," Adv. Inf. Syst. Sci. Plenum, vol. 2, no. 2, pp. 37–172, 1969.
- [11] B. Brown and H. Card, "Stochastic neural computation. I. computational elements," *IEEE Transactions on Computers*, vol. 50, no. 9, pp. 891– 905, Sep 2001.
- [12] V. Gaudet and A. Rapley, "Iterative decoding using stochastic computation," *Electronics Letters*, vol. 39, no. 3, pp. 299 – 301, Feb. 2003.
- [13] S. Sharifi Tehrani, S. Mannor, and W. Gross, "Fully parallel stochastic LDPC decoders," *IEEE Transactions on Signal Processing*, vol. 56, no. 11, pp. 5692 –5703, Nov. 2008.
- [14] P. Li and D. Lilja, "Using stochastic computing to implement digital image processing algorithms," in 29th IEEE International Conference on Computer Design (ICCD), Oct 2011, pp. 154–161.
- [15] P. Li, D. Lilja, W. Qian, K. Bazargan, and M. Riedel, "Computation on stochastic bit streams digital image processing case studies," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol. 22, no. 3, pp. 449–462, Mar. 2014.
- [16] N. Onizawa, D. Katagiri, K. Matsumiya, W. Gross, and T. Hanyu, "Gabor filter based on stochastic computation," *IEEE Signal Processing Letters*, vol. 22, no. 9, pp. 1224–1228, Sept 2015.
- [17] K. Parhi and Y. Liu, "Architectures for IIR digital filters using stochastic computing," in 2014 IEEE International Symposium on Circuits and Systems (ISCAS), June 2014, pp. 373–376.
- [18] N. Saraf, K. Bazargan, D. Lilja, and M. Riedel, "IIR filters using stochastic arithmetic," in 2014 Design, Automation and Test in Europe Conference and Exhibition (DATE), March 2014, pp. 1–6.
- [19] N. Onizawa, S. Koshita, and T. Hanyu, "Scaled IIR filter based on stochastic computation," in *IEEE 58th International Midwest Symposium* on Circuits and Systems (MWSCAS), Aug. 2015, pp. 297–300.
- [20] Y. Ji, F. Ran, C. Ma, and D. Lilja, "A hardware implementation of a radial basis function neural network using stochastic logic," in 2015 Design, Automation Test in Europe Conference Exhibition (DATE), March 2015, pp. 880–883.