# SUPER-RESOLUTION SPECTRAL ANALYSIS FOR ULTRASOUND SCATTER CHARACTERIZATION

*Konstantinos Diamantis*[1], *Maruf A. Dhali*[1], *Gavin Gibson*[2], *Yan Yan*[3], *James R. Hopgood*[3], *and Vassilis Sboros*[1]

[1]School of Engineering & Physical Sciences, Heriot-Watt University, Edinburgh, UK
[2]School of Mathematical & Computer Sciences, Heriot-Watt University, Edinburgh, UK
[3]School of Engineering, University of Edinburgh, Edinburgh, UK

**ABSTRACT**

Parametric Bayesian spectral estimation methods have been previously utilized to improve frequency resolution. Ultrasound signals have been tested in such methods resulting in higher precision frequency detection compared to common non-parametric spectral estimation methods based on the Fourier transform. Such a technique using a reversible jump Markov Chain Monte Carlo algorithm has been developed to fully characterize signals and in addition to frequency, to provide amplitude and noise estimation. The analysis of this method is demonstrated with a real copper sphere ultrasound scatter signal. Based on typical diagnostic ultrasound data between $1.2 - 4.5$ MHz the new spectral estimation achieves $110$ kHz minimum frequency resolution. This is at least twice the resolution of Fourier based methods, resulting in revealing new frequencies. The method may be used in the entire range of ultrasound imaging modalities and may help provide improved sensitivity, reproducibility and spatial resolution.

***Index Terms*—** spectral estimation, frequency resolution, Bayesian inference, ultrasound imaging, parametric model

## 1. INTRODUCTION

Medical ultrasound images are formed using signals that are band-width limited [1]. This is due to the current transducers that are utilized both for the transmission and the reception of the ultrasound pulses and have typically $100\%$ Full-Width-Half-Maximum (FWHM). Short pulse duration is utilized upon transmission in order to maximize spatial resolution of the image [2]. This combined with the usage of non-parametric, mainly Fourier-based, spectral analysis results in limited spectrum information [3].

*Yan et al.* have developed a novel parametric signal processing method for the measured ultrasound echo signal analysis within a Bayesian framework [4–7]. This is based on a reversible jump Markov Chain Monte Carlo (rjMCMC) algorithm [8], and provides significant resolution gains in terms of frequency detection [9–11]. This is accomplished by incorporating signal characteristics already known and by converting the problem of spectral estimation to the one of parameter estimation. The technique can be expanded to include amplitude (and hence phase) and noise estimation, which will enable thorough signal classification and open new opportunities in its usage in ultrasound signal processing. The current paper builds on the new spectral analysis tool [4–7] to achieve signal reconstruction through statistical post-processing of its output. A comparison with the classical Fourier Transform [12] and an initial performance assessment are also presented for a real ultrasound signal.

This work was supported by Heriot-Watt University.

## 2. BACKGROUND

Parametric spectral estimation relies on the selection of a particular signal model which is assumed to generate all data samples. The ultrasound pulses used in transmission consist of several cycles of sinusoids [6, 13], therefore the received responses can be represented as a sum of sines and cosines in white Gaussian noise. Bayesian inference [14,15] is then used to estimate the model parameters, which namely are the number of frequency components or model order ($k$), the frequencies ($\omega_k$), their amplitudes ($a_k$) and finally the noise variance ($\sigma_k^2$). The set of unknown parameters when the model order is also unknown is given by $\Psi = (k, \{\omega_k, a_k, \sigma_k^2\})$ where and as stated by Bayes theorem [16], the joint distribution of all $\Psi$ parameters, conditioned on the observed data sequence $y$, $p(\Psi \mid y)$ is defined as the joint posterior distribution, and can be calculated as:

$$p(\Psi \mid y) = \frac{p(\Psi)p(y \mid \Psi)}{p(y)} \propto p(\Psi)p(y \mid \Psi)\,, \qquad (1)$$

where $p(\Psi)$ denotes the joint prior distribution and $p(y \mid \Psi)$ the likelihood function. $p(y)$ is only a normalizing factor and can be omitted. The likelihood function is based on the current signal model where the noise is assumed to be independent and evenly distributed [7]. A single prior is selected for each unknown parameter [7, 17] and the joint prior distribution is just the product of all parameter priors since there is no dependence between them.

The parameter set $\Psi$ is then estimated by using MCMC algorithms, which draw samples from the estimated distribution and seek to find statistical averages using the law of large numbers [8, 16]. *Yan et al.* used a reversible jump Markov Chain Monte Carlo [8] algorithm to compensate for the number of frequency components (the model order) that is initially unknown [18, 19]. The reversible jump offers the ability to switch between different model orders from iteration to iteration [8, 11]. Ultimately, the most likely model order will be reached more frequently than the others based on an acceptance probability, indicating the most probable number of frequency components in this signal processing framework [5, 20]. Detailed information on the algorithm can be found in [7].

## 3. METHODS

### 3.1. Algorithm Development

For this frequency estimation technique where $k$ is also unknown, Bayesian analysis will lead to a highly multi-modal posterior distribution [21]. This will make more difficult the interpretation of the algorithm output and may even result in some meaningless parameter estimates. Imposing limitations such as $k \leq 20$ in [7] is a partial solution to this problem but higher performance could be achieved if additional statistical post-processing is applied to the rjMCMC results. Here, an attempt is presented to extract a reasonable summary of the posterior distribution through clustering, outlier rejection and signal comparison.

The algorithm is set to a large number of realizations ($N_{real}$) to ensure that there are sufficient data for analysis, since many estimates are ignored during a single-case study. A single realization also requires a high number of iterations ($N_{iter}$) until it converges to a specific model order. The output data from all realizations are considered for the current processing. They are separated based on the number of identified frequencies, so that the marginal posterior distributions of parameters of interest become unimodal. Previous allocation of estimated values, regardless of the model order, in histograms is no longer required to extract the final estimates [7]. In this work, realizations with the same number of estimated parameters are grouped and ordered and then it is straightforward to extract mean values from each frequency and associated amplitudes. Amplitude estimates refering to the same frequency (i.e. the first or the last) may contain values that differ greatly from realization to realization. For this reason they undergo further processing with all values significantly higher than two times the standard deviation, being removed. With all $\omega_k$ and $a_k$ known, the estimated signals for all the different $k$ can be reconstructed. The parameter set for which initial and reconstructed signal present the highest correlation coefficient is considered to be the best approximation. From the reconstructed signal's power and with the estimate of the noise variance, the Signal-to-Noise-Ratio (SNR) can also be calculated by:

$$SNR_{(dB)} = 10 \log_{10} \frac{P_s}{P_n} , \qquad (2)$$

where $P_s$ is the reconstructed signal power and $P_n$ is the noise power (variance). The complete parametric spectral estimation method as described in Section 2 including the supplementary features of the current Section results in signal reconstruction that quantifies the method's performance and enables comparisons with classical non-parametric methods.

The rjMCMC method requires a rough initial spectral estimate on which the first iteration of each run will be based. The multitaper method [3] that relies on the averaging of modified periodograms obtained by the same data to produce a spectrum, has been used in the past [4]. Although it is generally accepted that it is superior than the single periodogram or the Fast Fourier Transform (FFT) due to averaging, it is still subject to the major limitations of all non-parametric methods [3]. Here, the Multiple Signal Classification (MUSIC) algorithm [22] has also been used to provide a starting point for the algorithm. MUSIC belongs to the sub-space methods, also known as high-resolution methods and is particularly appropriate for signals consisting of sinusoids in noise. The description matches with the nature of the under study ultrasound signals and the signal model selected for the current spectral estimation method. There is no specific advantage noticed in favor of any of the two methods in terms of reconstructed and initial signal resemblance. Yet for sinusoidal signals with closely spaced frequency components, MUSIC parameters can be set accordingly to produce a spectrum that may help reducing the number of initial samples that are discarded as inaccurate (burn-in period). This way estimates convergence can be reached faster, reducing the computational burden that is one of the major disadvantages of this technique.

### 3.2. Ultrasound Data Acquisition

A modified ultrasound transducer (Sonos5500 Philips Medical Systems, Andover, MA, USA) was used to acquire echo signals from solid copper spheres (SCSs) as described previously [23]. Briefly the scanner has a transmit and receive bandwidth between 1.2 MHz and 4.5 MHz. Raw echo signals from the scatterers as shown in Fig. 1, are preamplified and stored in a computer using 20 MHz sampling rate in A/D convertion after receive. The experimental setup consists of a water tank and tubing that allows the drop of SCSs by gravity. Their path coincides to the centre of the ultrasound beam, which is calibrated using a membrane hydrophone. The echo acquisition was made at a distance between $7-8$ cm from the face of the transducer. The method is described in detail in [23].

## 4. RESULTS AND DISCUSSION

The received response of a 6-cycle SCS signal, where the transmit frequency is 1.62 MHz, is given as input to the rjMCMC sampler. The sinusoidal sphere signal is short with a length that does not surpass 90 samples making the conven-

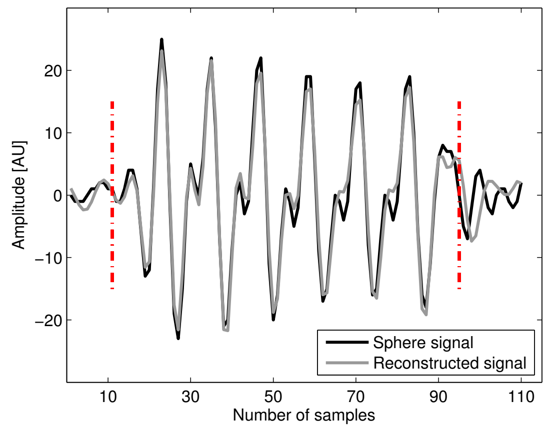


**Fig. 1**. Comparison of the sphere signal with the reconstructed one as obtained after the post-processing of the algorithm's output. The resemblance between the two is quantified by the correlation coefficient that is measured to 0.987 if only the signal is considered (inside red dash/dotted line).

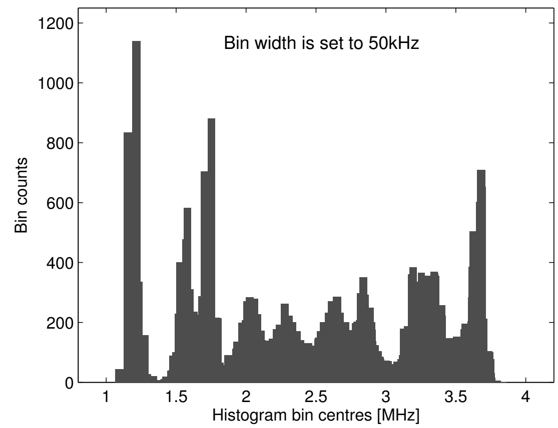


**Fig. 3**. Histogram displaying the cumulative distribution of the estimated frequency components from $5000$ realizations. A 50 kHz bin width was used. Nine peaks can be clearly distinguished while four of them may correspond to two peaks merged to one.

tional spectral analysis more difficult. The algorithm is set to $10000$ iterations to ensure that convergence is achieved and to $5000$ realizations. Each realization results in different number of frequency components varying from $12$ to $17$ as shown in Fig. 2 with numbers $13$ and $14$ being the most frequent ones. Output frequencies from all realizations are put into the aggregate histogram of Fig. 3 with a bin size of 50 kHz that also defines the theoretical frequency separation limit ($\Delta$f). This is the minimum difference between neighboring frequencies that can both be identified from the frequency spectrum. The corresponding $\Delta$f of the FFT which is the standard method to extract the spectrum of a signal, is defined as the ratio of the sampling frequency divided by the number of signals samples [12] that in this case it would be 20 MHz$/90$ samples $=$ $222$ kHz. However, it must be noted here that the ability to differentiate two very closely spaced components also depends strongly on relative amplitudes, which, in practice, significantly reduces the above spectral resolution.

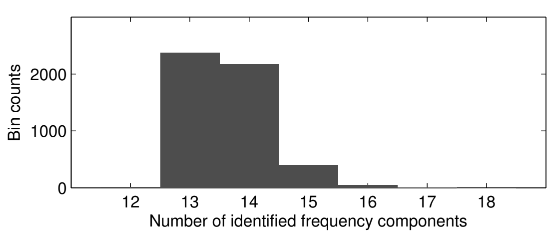


**Fig. 2**. Histogram showing number of identified frequency components for the $5000$ realizations of the rjMCMC method.

By following the steps as outlined in Section 3.1 for distinct $k$'s, all $\omega_k, a_k$ and $\sigma_k^2$ are extracted. The advantage of the rjMCMC over non-parametric spectral estimation is that it returns individual frequency and amplitude values indicating where the energy of a signal may be concentrated. Fig. 4 shows the FFT spectrum together with the results of the proposed method. The method's output presents good resemblance with the FFT where the spectrum peaks match with the ones achieved by the parametric method. The $111$ kHz distance between the $4^{th}$ and the $5^{th}$ components is the smallest one that is noticed and is therefore the actual minimum frequency separation achieved. This demonstrates that there is a minimum of 2-fold spectral resolution improvement compared to the FT. An investigation on spectral resolution performance of the method is beyond the scope of this paper but needs to be addressed in the context of all the parameters that affect it (eg. amplitude).

Further, there are $6$ more instances where the distance between $\omega_k$ and $\omega_{k+1}$ does not exceed $190$ kHz that couldn't have been identified by the FFT. These distances are all highlighted in Fig. 4 (oval shapes) where at the same time the FFT presents one peak instead of two or two instead of three, etc. There is also a more clear view compared to Fig. 3 regarding the frequency locations and number. With the reconstructed signal, the estimation accuracy can be checked by comparing it with the sphere signal, providing the confidence that the final estimates are correct. Fig. 1 shows the reconstructed signal plotted over the initial one. The correlation coefficient is $0.987$ for the signal part between the red lines. The visual inspection shows the close similarity of the signals. The noise variance is also estimated to $24.2$ dB. Importantly, the reconstructed signal in Fig. 1 is noise-free, and as a consequence a correlation coefficient of $1$ between the $2$ signals is unrealistic.

The above observations require synthetic signal confir-

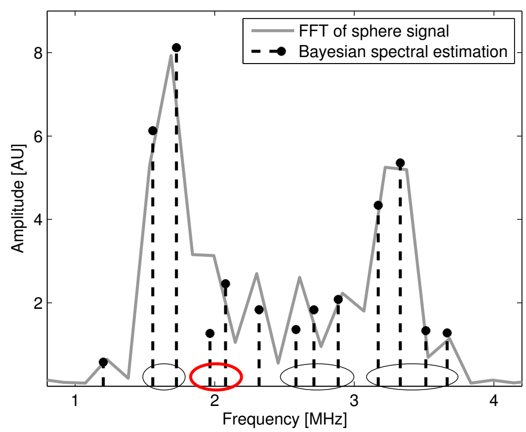


**Fig. 4**. Comparison of the FFT of the initial sphere signal with the output of the parametric spectral estimation. The Bayesian method results in individual amplitude and frequency values instead of a spectrum.

mation, and therefore a simplified typical ultrasound signal is also studied. The signal includes only 3 frequency components and assigned values are $\omega_k = \{1.2, 1.42, 1.97\}$ MHz, $a_{k,s} = \{3.7, 1.1, 1.7\}$ for the sine amplitudes and $a_{k,c} = \{2.5, 0.8, 1.2\}$ for the cosine amplitudes. To replicate the conditions of the real signal acquisition, white Gaussian noise with SNR= 25 dB was added to the signals and sampling frequency and number of samples were kept the same. The difference between $\omega_2$ and $\omega_1$ is almost the same as the FT resolution limit (222 kHz). The corresponding algorithm frequency estimates are $\omega_k^{'} = \{1.2, 1.42, 1.97\}$ MHz with only several kHz deviation from the true values whereas estimated amplitudes are $a_{k,s}^{'} = \{3.6, 1.1, 1.7\}$ and $a_{k,c}^{'} = \{2.6, 0.8, 1.1\}$ . The SNR is calculated to 24.4 dB and the correlation coefficient between synthetic and reconstructed signals is 0.995. In Fig. 5 the FFT of the synthetic signal compared to the results of the proposed method are shown. The FFT includes only two peaks while all 3 frequency components have been identified by the parametric estimation. The displayed amplitudes are extracted just by converting the estimates from the trigonometric form to amplitude ($a_k$) and phase. The use of the synthetic signal here confirms the improvement provided by the new method. However, this is not conclusive and a future extensive parametric study will help extract the performance and limitations of this technique.

Considering that the FT is widely used in the post-processing in nearly all ultrasound imaging modes, including Doppler velocity measurements, elastography, contrast enhanced ultrasound (CEUS) and others, it is intriguing to investigate the benefits of the new method in all these areas. An important consideration is the high computational burden associated with this technique. Even supposing that the MUSIC will provide a computational advantage over the multitaper technique (a 1000 iteration burn-in period compared to 3000 for 10000 iterations in total in this work), the algorithm remains far from real time application, which is often required in ultrasound imaging. Although more work may achieve improved computational cost, it is also worth investigating whether a large improvement in spectral clarity helps provide such improvement that the offline option should be taken into consideration. Here, linear scatter signals are used and similarly nonlinear contrast microbubble signals can be used. Current CEUS pulse sequences provides excellent linear tissue signal cancellation, but does very little in enhancing microbubble signals. A study of microbubble signature using a high resolution spectral analysis may aid in the development of new coded sequences that help generate higher resolution and SNR CEUS images. Such an advance may be acceptable as an offline post-processing mode if it really helps reveal new diagnostic information or significantly improves the sensitivity and specificity of existing diagnostic examinations.

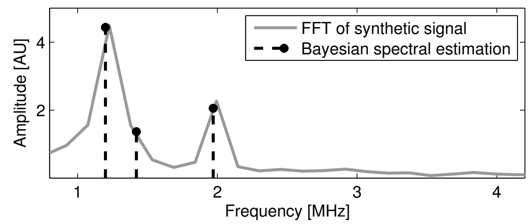


**Fig. 5**. Similar to Fig. 4, the FFT of the synthetic signal is compared with the output of the parametric spectral estimation.

## 5. CONCLUSION

A parametric Bayesian spectral estimation technique that outperforms conventional methods for ultrasound signal analysis has been presented. This is achieved through an MCMC algorithm where a reversible jump approach is employed since the number of singal components is unknown. The algorithm is based on prior knowledge of signals that are being analyzed, and statistical post-processing of its output. The reconstructed signals are represented as a sum of sinusoids and initial results on synthetic or real scatter signal show significantly improved spectral resolution compared to the FT. This method can be tested using linear and nonlinear (microbubbles) ultrasound scatter signals in order to inform future signal processing methods for ultrasound imaging.

## 6. REFERENCES


[1] P. Hoskins, K. Martin, and A. Thrush, "Diagnostic ultrasound: physics and equipment," (2nd. ed.) Cambridge University Press, 2010.

[2] M. L. Oelze, and W. D. O'Brien, Jr., "Defining optimal axial and lateral resolution for estimating scatterer properties from volumes using ultrasound backscatter," *J. Acoust. Soc. Am.*, vol. 115, no. 6, pp. 3226-3234, 2004.

[3] D. Manolakis, V. Ingle, and S. Kogon, "Statistical and adaptive signal processing: spectral Estimation, signal modelling, adaptive filtering and array processing," McGraw-Hill, New York, USA, 2003.

[4] Y. Yan, J. R. Hopgood, and V. Sboros, "Analysis of echo signal from single ultrasound contrast microbubble using a reversible jump MCMC algorithm," *Proc. IEEE Eng. in Med. and Biol. Soc.*, pp. 1273-1276, 2007.

[5] Y. Yan, J. R. Hopgood, and V. Sboros, "A novel estimation system for multiple pulse echo signals from ultrasound contrast microbubbles," *Proc. IEEE Int. Conf. on Acoustics, Speech and Sig. Proc. (ICASSP)*, pp. 601-604, 2008.

[6] Y. Yan, J. R. Hopgood, R. Steel, and V. Sboros, "Statistical spectral analysis for echo signals from microbubbles and solid spheres," *Proc. IEEE Ultrason. Symp.*, pp. 1382-1385, 2008.

[7] Y. Yan, J. R. Hopgood, and V. Sboros, "Bayesian spectral estimation applied to echo signals from nonlinear ultrasound scatterers," *EURASIP Journal on Advances in Sig. Proc.*, vol 2011, no. 146175, Available from: 10.1155/2011/146175, 2010.

[8] P. J. Green, "Reversible jump MCMC computation and Bayesian model determination," *Biometrika*, vol. 82, no. 4, pp. 711-732, 1995.

[9] C. Andrieu and A. Doucet, "Joint model selection and estimation of noisy sinusoids via reversible jump MCMC," *IEEE Trans. on Signal Processing.*, vol. 47, no. 10, pp. 2667-2676, October 1999.

[10] A. Roodaki, J. Bect, and G. Fleury, "Comments on joint Bayesian model selection and estimation of noisy sinusoids via reversible jump MCMC," *IEEE Trans. on Signal Processing.*, vol. 61, no. 14, pp. 3653-3655, 2013.

[11] D. Ustundag, "Recovering sinusoids from data using bayesian inference with RJMCMC," *7th Int. Conf. Nat. Comp. (ICNC)*, vol.4, pp.1850-1854, July 2011.

[12] M. S. Lawrence, "Digital spectral analysis," Englewood Cliffs, NJ: Prentice-Hall, 1987.

[13] T. Misaridis and J. A. Jensen, "Use of modulated excitation signals in medical ultrasound. Part I: basic concepts and expected benefits" *IEEE Trans. Ultrason., Ferroelec., Freq. Contr.*, vol. 52, no. 2, pp. 177-191, February 2005.

[14] P. C., Gregory, "Bayesian logical data analysis for the physical sciences," Cambridge University Press, New York, USA, 2005.

[15] A. Gelman, J. B. Carlin, H. S.Stern, and D. B.Rubin, "Bayesian data analysis (Texts in Statistical science)," Chapman & Hall/CRC, 2004.

[16] G. Bretthorst, "Bayesian spectrum analysis and parameters estimation," Springer, New York, USA, 1988.

[17] M. Hong, M. F. Bugallo, and P. M. Djuric, "Joint model selection and parameter estimation by population Monte Carlo simulation," *IEEE J. Sel. Topics Sig. Proc.*, vol. 4, no. 3, pp. 526-539, 2010.

[18] P. M. Djuric, "A model selection rule for sinusoids in white Gaussian noise," *IEEE Trans. on Signal Processing.*, vol. 44, no.7, pp. 1744-1751, 1996.

[19] D. V. Rubtsov and J. L. Griffin, "Time-domain Bayesian detection and estimation of noisy damped sinusoidal signals applied to NMR spectroscopy," *J. Magn. Reson.*, vol. 188, no. 2, pp. 367-379, 2007.

[20] D. Gamerman, "Markov chain monte carlo: stochastic simulation for Bayesian inference," Chapman & Hall/CRC, Boca Raton, Florida, USA, 1997.

[21] M. Stephens, "Dealing with label switching in mixture models," *J. R. Statist. Soc.*, vol. 62, part 4, pp. 795-809, 2000.

[22] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagation*, vol. AP-34, pp.276-280, 1986.

[23] V. Sboros, S. D. Pye, C. A. MacDonald, J. Gomatam, C. M. Moran, and W. N. McDicken, "Absolute measurement of ultrasonic backscatter from single microbubbles," *Ultr. Med. Biol.*, vol. 31, no. 8, pp. 1063-1072, 2005.