

TRANSIENT MODEL OF EEG USING GINI INDEX-BASED MATCHING PURSUIT

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ABSTRACT

We introduce a novel, transient model for the electroencephalogram (EEG) as the noisy addition of linear filters responding to trains of delta functions. We set the synthesis part as a parameter-tuning problem and obtain synthetic EEG-like data that visually resembles brain activity in the time and frequency domains. For the analysis counterpart, we use sparse approximation to decompose the signal in relevant events via Matching Pursuit. We improve this algorithm by incorporating the Gini Index as a stopping criteria; in this way, we promote sparse sources while, at the same time, eliminating one of the free parameters of Matching Pursuit. Results are presented using synthetic EEG and BCI competition data. Statistics of the model parameters are more informative and possess finer temporal resolution than classical methods such as Power Spectral Density (PSD) estimation.

Index Terms— EEG, Gini Index, Matching Pursuit, Sparsity, Transient

1. INTRODUCTION

EEG is a noninvasive technique used to record brain activity at the scalp level. In terms of digital signal properties, it presents appropriate temporal resolution due to its relatively high sampling frequency. Usually, it is utilized as a tool to quantify sleep stages [1,2,3], states of consciousness [4,5], physiological and pathological conditions [6,7,8] and even as a rehabilitation technique through Brain-Machine Interfaces [9,10]. Some of these applications, however, operate under the strong assumptions of stationarity and linearity. Furthermore, it is well known that the structure and nature of this type of brain signals rely on nonlinearities and transient events coming from the basic signal processing performed in the brain via spikes or action potentials. Therefore, the utilized statistical approaches are incompatible with the actual nature of EEG. In fact, electroencephalographers and neurophysiologists learn to extract clinical information from the EEG by eye balling in time the phasic events and rhythms of the signal [1].

In this work we introduce a novel, transient model for EEG signals where the relevant events of the recording are posed as marked point processes activating temporal filters. Specifically, the filters are grouped according to the well-known, functional EEG rhythms. The transient nature of the marked point processes allows us to translate this problem to a more convenient, mathematical framework where sparsity must be encouraged. For the analysis part of the model, we pose it as a sparse approximation problem where Matching Pursuit is used as a greedy, fast approximation of the optimal solution. The final results not only encourage sparsity by introducing a Gini Index-based Matching Pursuit decomposition, but they also highlight the increase in the number, and therefore, richness of features available in comparison to classical PSD analysis.

The rest of the paper is organized as follows: Section 2 introduces the novel, transient model; section 3 provides the necessary tools for the analysis part of the system, including Matching Pursuit and its improvement by using the Gini Index. Section 4 presents results on synthetic data and BCI Competition recordings available online, and, finally, section 5 provides conclusions and further work.

2. TRANSIENT MODEL FOR EEG

Following the clinical interpretation, we will model the EEG as the result of transient events over time that encode information concerning a particular physiological or pathological state, immersed in a noisy background. When comparing to other methods [11], this assumption is closer to reality because it produces a non-stationary time series. Additionally, it is well known that the human brain is exquisitely regulated to provide a constant level of neural excitatory activity, which means a constant change of state because of the interaction with a non-stationary world; a good example is the compromise between selective attention and habituation that takes place in the brain in response to stimuli [12].

Fig. 1 illustrates the block diagram of the proposed transient model, described by, the following equations, where $x(t)$ is the resulting EEG-like signal.

$$x(t) = n(t) + \hat{x}(t) = n(t) + \sum_{i=1}^L y_i(t) \quad (1)$$

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$$y_i(t) = \sum_{j=1}^{n_i} \int_{-\infty}^{\infty} \alpha_{i,j} \delta(t - \tau_{i,j}) h_{i,\omega_j}(u) du \quad (2)$$

where L is the number of filter banks, α is the amplitude, $n(t)$ is the noise and $h_{i,\omega}(t)$ is also known as an atom in a dictionary $H_i = \{h_{i,\omega}\}$.

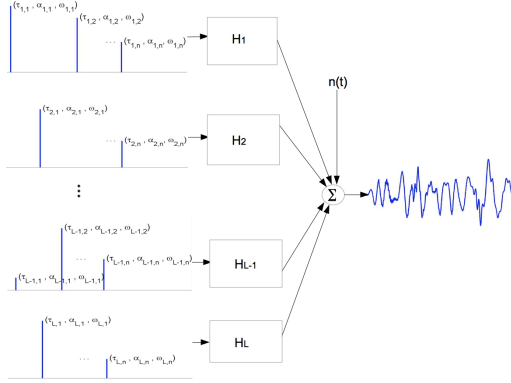


Fig. 1. Transient model for EEG. Each pulse has an associated feature vector with its timing, amplitude, duration, Q factor and index with respect to its corresponding filter bank. A sequence of pulses is known as a source. $H_i(t)$ denotes the filter bank for source i . $n(t)$ represents a single realization of a Gaussian, pink noise stochastic process.

In this way, each pulse activates a particular filter from its corresponding filter bank by indexing an element via ω_j . Also, $y_i(t)$ constitutes the contribution of all the convolutions of the pulses for filter bank i . Additionally, we will refer to the sequences of pulses as sources of the system, and to the filter's impulse responses, as events.

We must emphasize the fact that the proposed system introduces sparse sources via the logical connection of each pulse as a sample of a marked point process [13] where the features constitute the amplitude and index with respect to the filter bank. In addition, the filter bank can have properties on its own that can provide richer and more specific features to the model; for instance, each filter may possess a particular Q factor, central frequency and number of taps in the case of FIR band pass temporal filters.

As will be mentioned in Section 4, the importance of this synthetic model is not only the fact that the output signal visually resembles EEG in the time and frequency domains but that it can also be used as ground truth when performing studies that deal with analysis or spectral estimation for the analysis part of the problem.

3. TRANSIENT ANALYSIS OF EEG

3.1. Sparse Approximation

Our main goal is the analysis or decomposition of brain electrical activity recordings, i.e. estimate all the unknown parameters from a single channel EEG recording. This can be identified as an undercomplete blind source separation problem where tools such as ICA [14,15] could be applied

exploiting the nonstationary nature of the EEG [16]. However, the scope of this paper deals with a restricted version of source separation known as sparse approximation [17] where we have access to an overcomplete dictionary of atoms, or events, that will decompose the signal using a particular criterion.

In general for vectors in a finite-dimensional Hilbert space, C^A , the sparse approximation problem can be posed as follows:

$$\min_{A \subset \Omega} \min_{b \in C^A} \left\| x - \sum_{\lambda \in A} b_\lambda \varphi_\lambda \right\|_2 \quad \text{subject to } |A| \leq m \quad (3)$$

Where x is the input vector, $D = \{\varphi_\lambda\}$ is the dictionary matrix which is indexed by Ω . Finally, m is a fixed, positive integer that will determine the number of atoms extracted from the dictionary. The inner minimization is a least square problem and a classic example is the Fourier decomposition of vectors where the dictionary atoms are complex sinusoids in a discrete frequency space for the case of sampled signals. The outer minimization, however, is combinatorial, also known as NP-hard. This type of situations is well known to have intractable solutions. Moreover for the time series case, it is necessary to take into account all the possible shifts of every single atom in the dictionary, which makes the problem even more complex. Possible solutions include L1 relaxations such as Basis Pursuit [18]; yet, we use here a greedy method known as Matching Pursuit (MP).

3.2. Matching Pursuit

MP was proposed by Mallat and Zhang [19] as a greedy alternative to solve (3). We modified the initial cost function in order to accommodate the time structure of the signal by performing cross-correlation instead of inner product. In this way, we are able to efficiently compute the decomposition amplitudes and timings via FFT implementations of correlation. For the continuous time case, the algorithm is detailed next:

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 $r(t) \leftarrow x(t)$ 
for  $i = 1, \dots, m$  do
   $b_q(t) = \text{xcorr}(\varphi_q(t), r(t)) \quad q = 1, \dots, P$ 
   $p_i \leftarrow \text{argmax}_q \max_t |b_q(t)|$ 
   $\tau_i \leftarrow \text{argmax}_t |b_{p_i}(t)|$ 
   $\alpha_i \leftarrow b_{p_i}(\tau_i)$ 
   $r(t) \leftarrow r(t) - \int_{-\infty}^{\infty} \alpha_i \delta(t - \tau_i - u) \varphi_{p_i}(u) du$ 
end for

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Fig. 2. Matching Pursuit. Time Series Implementation. P atoms in $D = \{\varphi_\lambda\}$

Although fast and efficient, there is one major conceptual problem with this implementation: the number of events must be known beforehand. This is a strong assumption, especially when dealing with transient events. For instance, if the free parameter m is chosen too low, the decomposition events will most likely correspond to the low

frequencies (because of the inverse relationship between EEG power and frequency) and leave some relevant events out of the decomposition at high frequency. On the other hand, if m is chosen too high, the sources will start to overpopulate and the sparsity assumption becomes meaningless. This last case is the one most utilized in the literature [20,21], where other criteria such as incoherence of the residue with the dictionary or threshold on the reconstructed signal power are mere proxies of the parameter m . It is also worth noting that, for our case, the free parameters escalate because we are running Matching Pursuit for each available frequency band.

For our case, we cannot set a particular value of the parameter m without explicitly affecting the sparsity of the sources and the reconstructed signal. For this reason, we decided to use a different measure of sparsity as stopping criteria for Matching Pursuit: the Gini Index.

3.3. Gini Index-based Matching Pursuit

The Gini Index, also known as Gini Coefficient, is vastly used to measure inequality in wealth distribution. For vectors with positive support and elements sorted in an ascending fashion, it is calculated according to (4):

$$G(\vec{c}) = 1 - 2 \sum_{k=1}^N \frac{c(k)}{\|\vec{c}\|_1} \left(\frac{N - k + 0.5}{N} \right) \quad (4)$$

Furthermore, Hurtley and Rickard [22] performed an exhaustive study of measures of sparsity under 6 basic principles and concluded that the only one that satisfies all of them is the Gini Index. They also made a compelling argument regarding the analogies between distribution of wealth and distribution of power in signal processing, and proposed that sparsity is equivalent to inequity of wealth distribution, e.g. an equitable distribution of wealth is a low sparsity case close to zero; and, an inequitable distribution of wealth is analogue to an increment in sparsity, with the limit being equal to one when only one coefficient carries all the power of a signal.

Our proposed modified version of Matching Pursuit is detailed next.

```

 $r(t) \leftarrow x(t)$ 
 $G(0) = 0$ 
 $i \leftarrow 1$ 
 $G(i) = 0$ 
while  $G(i-1) \leq G(i)$  do
   $b_q(t) = \text{xcorr}(\varphi_q(t), r(t)) \quad q = 1, \dots, P$ 
   $p_i \leftarrow \arg\max_q \max_t |b_q(t)|$ 
   $\tau_i \leftarrow \arg\max_t |b_{p_i}(t)|$ 
   $\alpha_i \leftarrow b_{p_i}(\tau_i)$ 
   $r(t) \leftarrow r(t) - \int_{-\infty}^{\infty} \alpha_i \delta(t - \tau_i - u) \varphi_{p_i}(u) du$ 
   $G(i) = \text{Gini}(\|r(t)\| / \|x(t)\|)$ 
   $i \leftarrow i + 1$ 
end while

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Fig. 3. Gini Index-based Matching Pursuit Algorithm

Basically, we keep track of the normalized reconstructed signal power and stop the decomposition when the maximum in the Gini Index is achieved; this implies that the next element after the maximum will not contribute relatively much to the decomposition; hence, it is not necessary to be taken into account.

Even though a simpler method can be implemented where a threshold is set on the normalized reconstructed signal power, it would still need a parameter and it would not explicitly promote sparsity. With this new implementation we have a parameter-free Matching Pursuit stopping criteria that promotes sparse sources.

4. RESULTS

The first results were obtained using the proposed model to effectively generate time series that visually resemble EEG in the time and frequency domain. Specifically, the parameters of the sources were modeled as samples from exponentials and uniform distributions for the amplitudes and timings, respectively. Additionally according to previous studies [20,23], it has been shown that sinusoid segments, spikes, and Gabor filters provide a suitable sparse representation of EEG signals. The sparsity is mainly accomplished by the induced overcompleteness of the dictionary. Hence, a particular EEG structure can be represented efficiently with fewer components. We decided to use temporal Gabor filters dictionaries with a 0.5 Hz-interval discrete frequency grid. As an additional restriction, we set the maximum duration of the filters to 1 second resulting in a total of 834 atoms. The final step was selecting the number of components for each band; for this synthetic type of data, we allocated 3, 4, 4 and 5 atoms for the theta, alpha, beta and gamma bands, respectively.

Fig. 4 depicts the average PSD estimator of these signals for several SNR values under the same parameter conditions. It is evident that, for low SNR values, the signal behaves similar to the power distribution of pink noise. However, as the SNR increases, more peaks at discrete frequencies start to appear. This is a consequence of the discrete frequency grid of the Gabor filters in the dictionaries of each band.

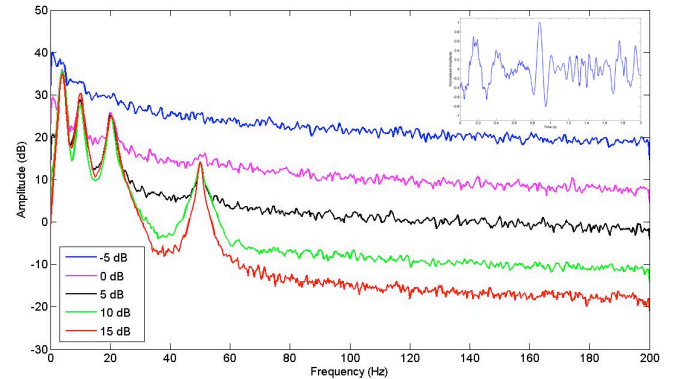


Fig. 4. Average PSD estimator of synthetic EEG-like data generated by the proposed model. 100 different trials were generated per SNR value. The right corner shows a sample time series for SNR = 15 dB.

We ran the novel Gini Index-based Matching Pursuit implementation on the theta, alpha, beta and gamma bands separately with each corresponding synthesis dictionary. The number of events was recorded as the number of iterations it took to arrive to the first maximum of the Gini Index sequence. A total of 100 2-second long trials were simulated with different SNR levels. Fig. 5 shows the average number of events found for each band and it confirms that our method strives to recognize the same number of original events that synthesized the signal. Specifically, our method stabilizes after a particular SNR value and remains detecting the accurate number of events present in each frequency band. It is also evident that gamma atoms are harder to find when the SNR level is low due to the inverse relationship between amplitude and frequency in the synthetic signal.

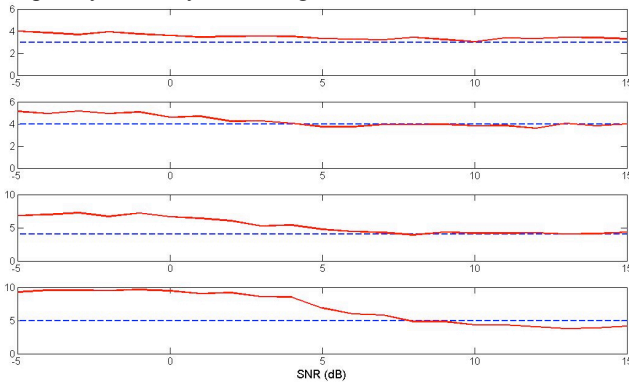


Fig. 5. Number of events detected by Gini-Index Matching Pursuit algorithm. From top to bottom: Theta band, Alpha band, Beta band, Gamma band. Blue corresponds to the ground truth and red indicates the average estimated number of atoms for different levels of SNR

The next results use BCI competition data. Specifically, dataset IIIb was used where the subject is asked to imagine a motor task in order to move a cursor left or right in a screen in front of him. Additionally, two bipolar electrodes were provided from the somatosensory region of the scalp corresponding to C3 and C4 according to the 10-20 EEG system; details about the data and a similar experiment can be found at [24,25].

In order to analyze different data modes, we segmented it into a rest stage from 0 to 2 seconds and an action stage from 2 to 4 seconds. The action stage is further subdivided according to the left and right classes. After performing PSD analysis, there is a clear difference between left and right classes when comparing modulated power in the alpha band for both electrodes (image not shown). However, by windowing the data, all the information contained in the time domain is lost and one of the main properties and advantages of EEG recordings is wasted: high temporal resolution. On the other hand, by performing a sparse decomposition analysis, we can obtain information regarding the particular number of relevant events for each task alongside all their features. Fig. 6 illustrates the number of events and the potential of this new method; whereas PSD analysis does not have the notion of relevant events,

our algorithm finds a sparse representation that provides even more information than classical analysis. Again, an appropriate Gabor filter-based dictionary was utilized.

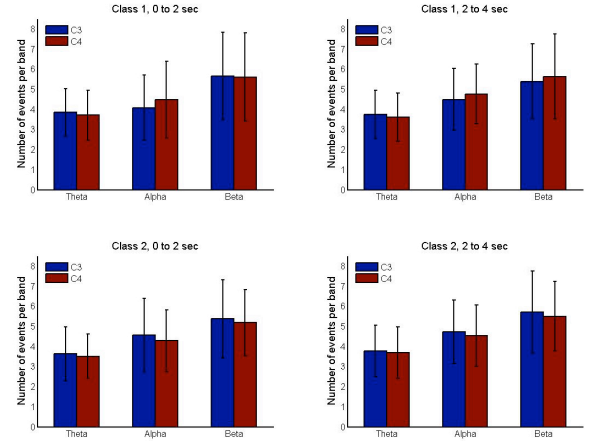


Fig. 6. Average number of events per band and electrode detected by Gini-Index Matching Pursuit algorithm. Class 1: right task. Class 2: left task.

Lastly, Table I shows the average values of the amplitude and inter-event times of alpha-events for both tasks. It is remarkable that power discrimination is still preserved even with a reduced number of atoms; also, inter-event timings might not show discriminability between tasks, however, they do differ to the rest stage inter-event timing. This is reasonable because each condition is an active state, so we do not expect necessarily a difference between classes because of intercortical connectivity, but both differ from rest. It is also possible to perform a similar analysis for all the additional features obtained after decomposition.

TABLE I. Average values of amplitude (adimensional) and inter-event times (seconds) for both classes and electrodes. Alpha band. Average inter-event timing for rest stage = 0.3090 s.

	Class 1		Class 2	
	Amplitude	Inter-event	Amplitude	Inter-event
C3	10.69	0.3120	8.45	0.3200
C4	8.37	0.3200	10.80	0.3040

5. CONCLUSIONS AND FURTHER WORK

We introduced a novel, transient model for EEG signals where the synthesis part provided time series visually similar in time and power distribution to real brain activity. The analysis counterpart was handled as a sparse approximation problem where Matching Pursuit was regulated using the Gini Index in order to promote sparsity and eliminate the free parameter of this greedy method. The final result is a set of sparse sources with fine time resolution and richer information regarding frequency, amplitude and shape of the relevant events of brain activity.

Further work includes statistical inference, discrimination and relationship between the different features of the sparse sources across time, location (such as electrode locations) and frequency bands.

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