FURTHER RESULTS ON MAINLOBE ORIENTATION REVERSAL OF THE FIRST-ORDER STEERABLE DIFFERENTIAL ARRAY DUE TO MICROPHONE PHASE ERRORS

Xiaoguang Wu^{*†} and Huawei Chen^{*}

* College of Electronic and Information Engineering,
 Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
 [†] College of Computer Science and Technology,
 Nanjing Tech University, Nanjing 211816, China

ABSTRACT

In our previous work, we have revealed the phenomenon of mainlobe orientation reversal (MOR) suffered by the firstorder steerable differential array (FOSDA) due to microphone phase errors. However, only a deterministic analysis is conducted in that work. To provide useful guidance for practical FOSDA design, this paper further studies the problem of MOR in the presence of uncertain microphone phase errors from two perspectives, i.e., interval analysis and statistical analysis. In particular, two criteria are derived to facilitate the FOSDA design in order to avoid undesirable MOR.

Index Terms— Differential microphone array, superdirective beamforming, mainlobe orientation reversal.

1. INTRODUCTION

Differential microphone arrays (DMAs) have many attractive virtues such as small size, high directivity, and frequency-invariant beampattern [1, 2]. Therefore, continuous efforts have been devoted to study and design DMAs in the past two decades [3–10]. Typical applications of DMAs include speech enhancement [11, 12], hearing aids [13], assistive listening headsets [14], hands-free communication [15], and automatic speech recognition [16].

The traditional DMAs are usually non-steerable, i.e., their mainlobe orientation may not be easily steered to an arbitrary desired direction. For example, the mainlobe orientation of a first-order DMA is always fixed along the array axis [4]. To combat this problem, several steerable DMAs have been proposed [4,17,18]. Among them, the first-order steerable differential array (FOSDA) whose spacial response is constructed via the linear combination of a monopole and two orthogonal dipoles using a four-element square array, is particularly interesting [17], since it has frequency-invariant equi-shaped beampatterns for the orthogonal dipoles for all frequencies.



Fig. 1. The configuration of the FOSDA.

It has been noted that although DMAs have some attractive virtues mentioned above, they are very sensitive to microphone mismatches [19]. Recently, we have found the phenomenon of mainlobe orientation reversal (MOR) suffered by the FOSDA in the presence of microphone phase errors [20], which implies the failure of FOSDA design. In contrast, the FOSDA with microphone gain errors is immune to the MOR. In [20], only a preliminary deterministic analysis, i.e., for deterministic microphone phase errors, has been presented. In practice, however, microphone phase errors are usually uncertain and may change over time [21]. Therefore, further work is indispensable to provide a useful guidance to guarantee the FOSDA being steered around the desired directions in the presence of uncertain microphone phase errors. In this paper, we further studies the MOR problem from two perspectives, i.e., interval and statistical analysis. In particular, two criteria have been derived to guide practical FOSDA design to avoid the MOR. Moreover, as a byproduct, this paper also presents a proof for the statement in [20] that the FOSDA is more susceptible to suffering from MOR at low frequencies.

2. MATHEMATICAL MODEL

As shown in Fig. 1, the FOSDA consists of four microphones in a square, where the distance between two nondiagonal microphones is d. For a unit-amplitude incident plane wave with frequency f and incident angle (θ, ϕ) ($\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$ denote the elevation and azimuth angles, respectively), the *i*th microphone signal is given by

$$E_{i} = \exp\left[j\omega t + j\omega\sin\theta\left(p_{x_{i}}\cos\phi + p_{y_{i}}\sin\phi\right)/c\right] \quad (1)$$

This work was supported by the National Natural Science Foundation of China under Grant No. 61471190.

where $\omega = 2\pi f$, t denotes the time, p_{x_i} and p_{y_i} refer to the x- and y- coordinates of the *i*th microphone, respectively, c is the speed of sound, and $j = \sqrt{-1}$.

The *i*th microphone signal with microphone phase errors can be expressed as $E_i^{(p)} = E_i e^{j\psi_i}$, where ψ_i denote microphone phase errors. Then the normalized array response of the FOSDA with its mainlobe oriented toward $\phi = \varphi_s$ is

$$\overline{E}_{s(\alpha)}^{(p),\varphi_s}(\theta,\phi) = \alpha \overline{E}_m^{(p)}(\theta,\phi) + (1-\alpha)\overline{E}_d^{(p),\varphi_s}(\theta,\phi)$$
(2)

where $\alpha \in [0,1]$ is the directivity controlling parameter, $\overline{E}_m^{(p)}(\theta,\phi)$ and $\overline{E}_d^{(p),\varphi_s}(\theta,\phi)$ represent the normalized responses of monopole and steered dipole, respectively. In (2), the high-pass frequency response and $\pi/2$ phase shift have been compensated out of the dipole responses.

The normalized response of the monopole is

$$\overline{E}_{m}^{(p)}(\theta,\phi) = \frac{1}{4} \sum_{i=1}^{4} E_{i}^{(p)}$$
$$= \frac{1}{2} \left\{ e^{j\upsilon_{24}} \cos\left(\Theta_{24} + \tau_{24}\right) + e^{j\upsilon_{31}} \cos\left(\Theta_{31} + \tau_{31}\right) \right\}$$

where $v_{24} = (\psi_2 + \psi_4)/2$, $v_{31} = (\psi_3 + \psi_1)/2$, $\tau_{24} = (\psi_2 - \psi_4)/2$, $\tau_{31} = (\psi_3 - \psi_1)/2$, $\Theta_{24} = \sqrt{2}\Omega \sin\theta \cos(\phi - \pi/4)$, $\Theta_{31} = \sqrt{2}\Omega \sin\theta \cos(\phi + \pi/4)$, $\Omega = \omega d/(2c)$.

The steered dipole is constructed by two orthogonal dipoles oriented toward $\pm \pi/4$, i.e., $E_d^{(p),-\pi/4} = E_3^{(p)} - E_1^{(p)}$, and $E_d^{(p),\pi/4} = E_2^{(p)} - E_4^{(p)}$. The normalized response of the steered dipole can be expressed as

$$\begin{split} \overline{E}_{d}^{(p),\varphi_{s}}\left(\theta,\phi\right) \\ &= \frac{c}{j\sqrt{2}\omega d} \left[\cos\left(\varphi_{s} + \frac{\pi}{4}\right) E_{d}^{(p),-\pi/4}(\theta,\phi) \right] \\ &+ \sin\left(\varphi_{s} + \frac{\pi}{4}\right) E_{d}^{(p),\pi/4}(\theta,\phi) \right] \\ &\approx e^{j\upsilon_{31}} \cos\left(\varphi_{s} + \frac{\pi}{4}\right) \left[\sin\theta\cos\left(\phi + \frac{\pi}{4}\right) + \frac{\tau_{31}}{\sqrt{2}\Omega} \right] \\ &+ e^{j\upsilon_{24}} \sin\left(\varphi_{s} + \frac{\pi}{4}\right) \left[\sin\theta\sin\left(\phi + \frac{\pi}{4}\right) + \frac{\tau_{24}}{\sqrt{2}\Omega} \right] \end{split}$$

3. MAIN RESULTS

In this section, we first revisit the MOR problem of the FOSDA with deterministic microphone phase errors. Then, we further studies the MOR with uncertain microphone phase errors from interval and statistical analysis perspectives, respectively.

3.1. The MOR of the FOSDA Revisited

Recall from [20] that the mainlobe orientation of the FOSDA is at $\tilde{\varphi}_s \approx \varphi_s$ if the array directivity controlling parameter α

satisfies

$$\alpha \ge \frac{|\kappa|^2 - |\gamma|^2}{||\kappa|^2 - |\gamma|^2 + 2\Re\{\gamma - \kappa\}|} \triangleq \alpha_T \tag{3}$$

where $\gamma = \overline{E}_{d}^{(p),\varphi_{s}}(\theta,\varphi_{s})$, and $\kappa = \overline{E}_{d}^{(p),\varphi_{s}}(\theta,\varphi_{s}+\pi)$. Otherwise, we have $\widetilde{\varphi}_{s} \approx \varphi_{s} + \pi$, i.e., the MOR occurs.

To proceed, we need to revisit the above finding. Notice that, for small microphone phase errors, it follows that $|\kappa| \approx \Re\{\overline{E}_d^{(p),\varphi_s}(\theta,\varphi_s+\pi)\} \triangleq \kappa_r$, and $|\gamma| \approx \Re\{\overline{E}_d^{(p),\varphi_s}(\theta,\varphi_s)\} \triangleq \gamma_r$. Consequently, α_T in (3) can be reformulated as

$$\alpha_T = \frac{(\kappa_r - \gamma_r) (\kappa_r + \gamma_r)}{|\kappa_r - \gamma_r| |\kappa_r + \gamma_r - 2|}.$$
(4)

Further note that

$$\dot{n}_r - \gamma_r \approx -2\sin\theta \tag{5}$$

$$\kappa_r + \gamma_r \approx \frac{\sqrt{2}}{\Omega} \left[\cos\left(\varphi_s + \frac{\pi}{4}\right) \tau_{31} + \sin\left(\varphi_s + \frac{\pi}{4}\right) \tau_{24} \right].$$
 (6)

By (5) and (6), Eq. (4) can be reduced to

$$\alpha_T = -\frac{\kappa_r + \gamma_r}{|\kappa_r + \gamma_r - 2|} = \begin{cases} -\frac{1}{|1 - \frac{2}{\kappa_r + \gamma_r}|}, & \kappa_r + \gamma_r > 0; \\ 0, & \kappa_r + \gamma_r = 0; \\ \frac{1}{1 + \frac{2}{|\kappa_r + \gamma_r|}}, & \kappa_r + \gamma_r < 0. \end{cases}$$
(7)

Since $\alpha \in [0, 1]$, by (7) we deduce that the MOR occurs if and only if $\kappa_r + \gamma_r < 0$. Moreover, by (7) we have the following relationships:

$$f \downarrow \Longrightarrow \Omega \downarrow \Longrightarrow |\kappa_r + \gamma_r| \downarrow \Longrightarrow \alpha_T \uparrow \tag{8}$$

which implies that the FOSDA is more susceptible to suffering from MOR at low frequencies.

3.2. Interval Analysis

Suppose that microphone phase errors are unknown-butbounded, i.e., $\psi_i \in [-\Delta_{\psi}, \Delta_{\psi}]$ with $\Delta_{\psi} > 0$, and that the frequency range of interest $f \in [f_l, f_h]$, i.e., $\Omega \in [\frac{\pi d}{c}f_l, \frac{\pi d}{c}f_h] \triangleq [\underline{\Omega}, \overline{\Omega}]$. From the analysis above, α_T will take values over some interval $[\underline{\alpha}_T, \overline{\alpha}_T]$. To guarantee no MOR occurs, the array directivity controlling parameter α should satisfy $\alpha > \overline{\alpha}_T$. Therefore, we are particularly interested in finding the lowest bound of α , i.e. $\overline{\alpha}_T$.

To this end, first we can rewrite (6) as

$$\kappa_r + \gamma_r \approx \frac{\sqrt{2}}{\Omega} \left[\cos\left(\varphi_s + \frac{\pi}{4}\right) \tau_{31} + \sin\left(\varphi_s + \frac{\pi}{4}\right) \tau_{24} \right]$$
$$= \frac{1}{\Omega} \sqrt{2\left(\tau_{31}^2 + \tau_{24}^2\right)} \sin\left(\beta + \varphi_s + \frac{\pi}{4}\right) \tag{9}$$

where β satisfies $\tan \beta = (\tau_{31}/\tau_{24})$. Recall that the mainlobe orientation of the FOSDA may be reversed if and only if $\kappa_r + \gamma_r < 0$. Thus, we have

$$\overline{\alpha}_T = \frac{1}{1 + \frac{2}{\max\{|\kappa_r + \gamma_r|\}}} \tag{10}$$

By (9), we deduce that $\kappa_r + \gamma_r$ attains its maximum value when either one of the following conditions holds: 1) $\tau_{31} = \Delta_{\psi}, \tau_{24} = \Delta_{\psi}, \beta = \pi/4, \varphi_s = \pi; 2) \tau_{31} = \Delta_{\psi}, \tau_{24} = -\Delta_{\psi}, \beta = 3\pi/4, \varphi_s = \pi/2; 3) \tau_{31} = -\Delta_{\psi}, \tau_{24} = -\Delta_{\psi}, \beta = 5\pi/4, \varphi_s = 0; \text{ and } 4) \tau_{31} = -\Delta_{\psi}, \tau_{24} = \Delta_{\psi}, \beta = 7\pi/4, \varphi_s = 3\pi/2$. Accordingly, we have

$$\overline{\alpha}_T = \frac{1}{1 + \frac{\Omega}{\Delta_{\psi}}} = \frac{1}{1 + \frac{\pi f_I d}{c \Delta_{\psi}}}.$$
(11)

Recall that, to avoid the MOR, it should satisfy $\alpha > \overline{\alpha}_T$. Therefore, reconsidering (11), we have

$$\alpha \pi f_l d + (\alpha - 1)c\Delta_{\psi} > 0. \tag{12}$$

3.3. Statistical Analysis

Suppose that the microphone phase errors ψ_i are independent and each follows a Gaussian distribution with zero mean and variance σ^2 , i.e., $\psi_i \sim \mathcal{N}(0, \sigma^2)$. In this circumstance, α_T will be a random variable, and we are interested in determining the probability of absence of MOR for a given directivity controlling parameter α .

Since $\psi_i \sim \mathcal{N}(0, \sigma^2)$, it follows that $\tau_{31}, \tau_{24} \sim \mathcal{N}(0, 2\sigma^2)$. To simplify notation, we denote $\chi \triangleq \kappa_r + \gamma_r$. Then by (6), we have $\chi \sim \mathcal{N}(0, \sigma^2/\Omega^2)$. Recall that the mainlobe orientation may be reversed if and only if $\chi < 0$. Therefore, the probability of no MOR for a given $\alpha \in [0, 1]$ can be derived as follows

$$P(\alpha > \alpha_T) = P(\alpha_T < 0) + P(0 \le \alpha_T < \alpha)$$

$$= \frac{1}{2} + P\left(\frac{\chi}{\chi - 2} < \alpha\right)$$

$$= \frac{1}{2} + P\left(\chi > \frac{2\alpha}{\alpha - 1}\right)$$

$$= \frac{1}{2} + \int_{\frac{2\alpha}{\alpha - 1}}^{0} \frac{\Omega}{\sqrt{2\pi\sigma}} \exp\left(-\frac{\chi^2 \Omega^2}{2\sigma^2}\right) d\chi$$

$$= \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[\frac{\sqrt{2\Omega\alpha}}{\sigma(\alpha - 1)}\right]$$
(13)

where $\operatorname{erf}[\cdot]$ denotes the Gaussian error function. Note that in the derivation above, we have only considered a given frequency. For a given frequency range of interest $f \in [f_l, f_h]$, by using the relation (8), we can deduce that the probability of no MOR for a given $\alpha \in [0, 1]$ can be expressed as

$$P(\alpha > \alpha_T) = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[\frac{\sqrt{2\pi} df_l \alpha}{c\sigma(\alpha - 1)}\right].$$
 (14)

By (14), to guarantee no MOR with probability one, it should satisfy

$$\operatorname{erf}\left[\frac{\sqrt{2\pi}df_{l}\alpha}{c\sigma\left(\alpha-1\right)}\right] + 1 = 0.$$
(15)

Table 1. Design Specifications of the FOSDA	
lowest frequency	200 Hz
highest frequency	3700 Hz
maximum signal attenuation	-3 dB
maximum noise sensitivity	20 dB



Fig. 2. Simulation results of α_T with random microphone phase errors in [-0.05, 0.05] rad.

4. NUMERICAL EVALUATION

In this section, we apply the theoretical results presented above to the FOSDA design for speech communication applications. The design specifications are shown in Table I, same as considered in [17]. Note that the array size should satisfy $2.03 \le d \le 2.28$ cm under the considered specifications [17].

Firstly, we perform the interval analysis on the FOSDA. Assume the size of the FOSDA d = 2.28 cm and microphone phase errors $\psi_i \in [-0.05, 0.05]$ radian. By (11), it follows that the theoretical lowest bound of α is $\overline{\alpha}_T = 0.5427$, which implies that the array directivity controlling parameter α should be chosen greater than 0.5427 to guarantee that no MOR occurs. To justify this theoretical finding, several simulations have been conducted. Fig. 2 shows the simulation results of α_T for 1000 random trials with different microphone phase errors, and Figs. 3(a) and 3(b) show the normalized array responses of the FOSDA with $0 < \alpha < 0.5427$ and $0.5427 < \alpha < 0.7$, respectively. Both Fig. 3(a) and Fig. 3(b) are the results of 50 Monte Carlo simulations with random array directivity controlling parameter α and microphone phase errors ψ_i . As we can see from Fig. 2, the simulation results of α_T are well bounded below the theoretical bound $\overline{\alpha}_T = 0.5427$. Moreover, from Fig. 3 we can see that it may suffer from MOR when $\alpha < \overline{\alpha}_T$, while it does not when $\alpha > \overline{\alpha}_T$. For clarity, the cases with MOR are highlighted with red dotted lines in Fig. 3(a).

Having demonstrated the validity of our theoretical analysis, now we provide a useful guidance for practical FOSDA design to avoid the MOR from a interval-analysis perspective. In Fig. 4, the lowest bounds of α to guarantee absence



Fig. 3. Normalized array responses of the FOSDA with random microphone phase errors in [-0.05, 0.05] rad., where $\overline{\alpha}_T = 0.5427$. (a) $0 < \alpha < \overline{\alpha}_T$. (b) $\overline{\alpha}_T < \alpha < 0.7$.



Fig. 4. Lowest bounds of α to guarantee no MOR versus array size and microphone phase error.

of MOR of the FOSDA with varying array size and microphone phase errors are presented. Alternatively, given a specific array directivity controlling parameter α , we can also determine the tolerance for microphone phase errors. In particular, for the two well-known FOSDAs with cardioid and hypercardioid responses, it shows that the microphone phase errors should be less than 0.042 rad. or 2.41° for $\alpha = 0.5$ (cardioid-response FOSDA), and less than 0.014 rad. or 0.8° for $\alpha = 0.25$ (hypercardioid-response FOSDA).

Next, we conduct the statistical analysis on the FOSDA. Assume the size of the FOSDA d = 2.28 cm and all microphone phase errors follow a Gaussian distribution with zero mean. Fig. 5 shows the probability of no MOR, i.e., $P(\alpha > \alpha_T)$, as a function of α with $\sigma = 0.001, 0.01$, and 0.1 rad., respectively, where 10000 Monte Carlo simulation trials have been carried out, and the results based on which are denoted as "simulation results". To verify our theoretical statistical analysis, the theoretical results obtained by (14) are also presented in Fig. 5. As we can see from the figure, the theoretical results are well consistent with the simulation results. Moreover, as we can expect, the probability of no MOR tends to increase with α increasing or σ decreasing.



Fig. 5. The probability of no MOR versus α .



Fig. 6. Lowest bounds of α to guarantee no MOR with probability one versus array size and standard deviation of microphone phase error.

To provide a guidance for avoiding the MOR from statistical-analysis perspective, Fig. 6 shows the lowest bounds of α to guarantee no MOR with probability one under various array sizes and standard deviations of microphone phase error. On the other hand, given specific array directivity controlling parameter α , we can also determine the tolerance for microphone phase errors to guarantee no MOR with probability one. Specifically, it follows that the standard deviation of microphone phase errors should satisfy $\sigma < 0.014$ rad. or $\sigma < 0.8^{\circ}$ for the cardioid-response FOSDA, and $\sigma < 0.0047$ rad. or $\sigma < 0.27^{\circ}$ for the hypercardioid-response FOSDA.

5. CONCLUSIONS

In this paper, we have further studied the MOR problem suffered by the FOSDA in the presence of uncertain microphone phase errors from two perspectives, i.e., interval and statistical analysis. Two design criteria have been derived to avoid the MOR with uncertain microphone phase errors, which are helpful to practical FOSDA design. Several design examples have demonstrated the effectiveness of the theoretical results.

6. REFERENCES

- [1] J. Benesty and J. Chen, *Study and Design of Differential Microphone Arrays*, Springer, 2013.
- G. W. Elko, "Superdirectional microphone arrays," in Acoustic Signal Processing for Telecommunication, S. L. Gay and J. Benesty, Eds. Boston, MA: Kluwer, 2000, ch. 10, pp. 181–237.
- [3] G. W. Elko and A. T. N. Pong, "A simple first-order differential microphone," in *Proc. IEEE WASPAA*, pp. 169– 172, Oct. 1995.
- [4] G. W. Elko and A. T. N. Pong, "A steerable and variable first-order differential microphone array," in *Proc. ICASSP*, pp. 223–226, 1997.
- [5] H. Teutsch and G. W. Elko, "First- and secondorder adaptive differential microphone arrays," in *Proc. IWAENC*, pp. 35–38, 2001.
- [6] T. D. Abhayapala and A. Gupta, "Higher order differential-integral microphone arrays," *Journal of the Acoustical Society of America*, vol. 127, pp. EL227– EL233, Apr. 2010.
- [7] M. Kolundzija, C. Faller, and M. Vetterli, "Spatiotemporal gradient analysis of differential microphone arrays, *J. Audio Eng. Soc.*, vol. 59, pp. 20–28, Feb. 2011.
- [8] E. De Sena, H. Hacihabiboğlu, and Z. Cvetković, "On the Design and Implementation of Higher Order Differential Microphones," *IEEE Trans. on Audio, Speech, and Language Processing*, vol. 20, pp. 162–174, Jan. 2012.
- [9] L. Zhao, J. Benesty, and J. Chen, "Design of robust differential microphone arrays," *IEEE/ACM Trans. on Audio, Speech, and Language Processing*, vol. 22, pp. 1455–1465, Oct. 2014.
- [10] C. Pan, J. Chen, and J. Benesty, "Theoretical analysis of differential microphone array beamforming and an improved solution," *IEEE/ACM Trans. on Audio, Speech, and Language Processing*, vol. 23, pp. 2093–2105, Nov. 2015.
- [11] M. Ihle, "Differential microphone arrays for spectral subtraction," in *Proc. IWAENC*, pp. 259–262, 2003.

- [12] J. Benesty, M. Souden, and Y. Huang, "A perspective on differential microphone arrays in the context of noise reduction," *IEEE Trans. on Audio, Speech, and Language Processing*, vol. 20, pp. 699–704, Feb. 2012.
- [13] H. Puder, E. Fischer, and J. Hain, "Optimized directional processing in hearing aids with integrated spatial noise reduction," in *Proc. IWAENC*, pp. 1–4, Sep. 2012.
- [14] S. Nordholm, A. Davis, P. C. Yong, and H. H. Dam, "Assistive listening headsets for high noise environments: Protection and communication," *Proc. ICASSP*, pp. 5753–5757, Apr. 2015.
- [15] B. Geiser, H. Krüger, P. Vary, and D. Wiese, "A differential microphone array with input level alignment, directional equalization and fast notch adaptation for handsfree communication," in *Proc. 11. ITG Symposium on Speech Communication*, pp. 1–4, Sep. 2014.
- [16] E. Messner, H. Pessentheiner, J. A. Morales-Cordovilla, and M. Hagmuller, "Adaptive differential microphone arrays used as a front-end for an automatic speech recognition system," *Proc. ICASSP*, pp. 2689–2693, Apr. 2015.
- [17] R. M. M. Derkx and K. Janse, "Theoretical analysis of a first-order azimuth-steerable superdirective microphone array," *IEEE Trans. on Audio, Speech, and Language Processing*, vol. 17, pp. 150–162, Jan. 2009.
- [18] B. De Schuymer and H. Brouckxon, "Steerable microphone array system with a first order directional pattern," European Patent Application, Application No.: 10151106.1, Aug. 24, 2011.
- [19] M. Buck, "Aspects of first-order differential microphone arrays in the presence of sensor imperfections," *European Transactions on Telecommunications*, vol. 13, pp. 115–122, Mar.-Apr. 2002.
- [20] X. Wu, H. Chen, J. Zhou, and T. Guo, "Study of the mainlobe misorientation of the first-order steerable differential array in the presence of microphone gain and phase errors," *IEEE Signal Processing Lett.*, vol. 21, no. 6, pp. 667–671, Jun. 2014.
- [21] S. Doclo and M. Moonen, "Design of broadband beamformers robust against gain and phase errors in the microphone array characteristics," *IEEE Trans. Signal Process.*, vol. 51, pp. 2511–2526, Oct. 2003.