Joint Channel Estimation for Three-Hop MIMO Relaying Systems

Italo Vitor Cavalcante, André L. F. de Almeida, Senior Member, IEEE, and Martin Haardt, Senior Member, IEEE

Abstract—We propose a novel joint channel estimator for a relaying MIMO communication system. Considering a three-hop relaying protocol, our combined alternating least squares (Comb-ALS) algorithm obtains cooperative diversity by fully exploiting the tensor algebraic structures of the available cooperative MIMO links. This is achieved by coupling the tensor data for the different relay-assisted links to iteratively estimate the channel matrices. Simulation results corroborate the effectiveness of the proposed tensor-based joint channel estimator in comparison with a sequential tensor-based method and a sequential LS estimator.

Index Terms—Channel estimation, cooperative communications, relaying, tensor decomposition.

I. INTRODUCTION

C OOPERATIVE communications are well-known solutions to increase diversity and/or signal power at the receiver, leading to increased capacity and coverage in wireless communication systems [1], [2]. In this context, amplify-and-forward (AF) relaying techniques are attractive solutions, which avoid decoding at the relays, being preferable when complexity and/or latency issues are of importance [3], [4].

In cooperative relaying systems, the reliability of signal detection at the destination strongly depends on the accuracy of the channel state information (CSI) for all the links involved in the communication process. Moreover, the use of precoding techniques at the source and/or destination [5], [6] often requires the instantaneous CSI knowledge of all links to carry out transmit optimization. In [7], [8] a singular value decomposition (SVD) based algorithm is proposed to provide the destination with the knowledge of the channel matrices involved in a two-hop cooperative communication scenario. A few additional works resort to tensor-based approaches to solve the channel estimation problem. In [9], a supervised tensor-based channel estimation scheme was proposed for two-way relaying cooperative systems with multiple antennas at the relay nodes. In [10], multiuser receivers based on a trilinear model have been proposed for uplink cooperative diversity systems employing an antenna

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array at the destination node (base station). A more recent work [11] proposes a blind receiver for an uplink multiuser cooperative diversity system employing CDMA-based relaying. All these works consider two-hop relaying, and the channel estimation problem is concerned with the joint estimation of the channel matrices of both hops. To further extend coverage and combat channel impairments such as path-loss and shadowing, it may be advantageous to introduce additional hops while exploiting multiple relaying links, whenever they are available [1], [12]. To this end, [13] proposes a method to jointly estimate the channel matrices in a relaying MIMO communication system considering a three-hop scenario with a four-phase communication protocol. The main drawback is the high computational complexity of the channel estimation task.

In this paper, we tackle the three-hop relaying scenario and derive a simple and effective algorithm to jointly estimate the partial channels involved in the communication. The proposed algorithm fully exploits the tensor algebraic structures of the available MIMO links, by combining PARAFAC [14] and *Tucker3* [15] decompositions to iteratively estimate the channel matrices. Moreover, a simple design of the AF relaying matrices based on rank-one approximations is proposed to ensure identifiability of all the channels involved in the communication. Our simulation results corroborate the effectiveness of the proposed channel estimator, which exhibits a similar performance as the one of [13] under a reduced computational complexity and an improved spectral efficiency. Compared to conventional matrix-based channel estimators, the proposed joint channel estimator can operate under more flexible antenna configurations while offering an improved channel estimation accuracy. Such gains come from the exploitation of the multilinear (tensor) structure of the signals received at the destination, which allows a simultaneous estimation of all the channels without error accumulation as in sequential LS estimation schemes.

Notation: Column vectors, matrices, and tensors are denoted by boldface lower-case, boldface capital, and calligraphic letters, respectively. The operators $(.)^T$, $(.)^H$, $(.)^{\dagger}$ stand for the transpose, Hermitian transpose and Moore-Penrose pseudo-inverse, respectively, and $\|.\|_F$ denotes the Frobenius norm. The operators $Tr\{\cdot\}$ and $E\{\cdot\}$ denote the trace and expected value, respectively. The Diag(a) operator forms a diagonal matrix by putting the vector **a** on its main diagonal, and $A_{(k,:)}$ denotes the kth row of A. The Kronecker product and the Khatri-Rao (columnwise Kronecker) product between two matrices A and **B** are symbolized by $A \otimes B$ and $A \diamond B$, respectively. We denote $\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ as third-order tensor of size I_j along dimension (mode) j. A n-mode fiber of $\boldsymbol{\mathcal{A}}$ is a vector that collects the elements of this tensor by varying the *n*-th index and keeping all the other indexes fixed. The *n*-mode unfolding (matricization) of $\boldsymbol{\mathcal{A}}$ is obtained by collecting all the *n*-mode fibers into a matrix symbolized by $[\mathcal{A}]_{(n)}$ [16]. The *n*-mode product between

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Fig. 1. Three-hop MIMO relaying system with two relay groups.

a tensor $\mathcal{A} \in \mathbb{C}^{I \times J \times K}$ and a matrix $\mathcal{B} \in \mathbb{C}^{J \times R}$, symbolized by $\mathcal{A} \times_n \mathcal{B}$, consists in multiplying the *n*-mode unfolding of \mathcal{A} by the matrix \mathcal{B} , i.e. $[\mathcal{X}]_{(n)} = \mathcal{B}[\mathcal{A}]_{(n)}$, where $[\mathcal{X}]_{(n)}$ is the *n*-mode unfolding of the tensor $\mathcal{X} \in \mathbb{C}^{I \times R \times K}$ that results from this operation. We use the shorthand notation $[\mathcal{C} \sqcup_n \mathcal{D}]$ to denote the concatenation of two matrices along the *n*-th mode. We can construct a third-order tensor by concatenating matrices along its the first, second, and third modes.

II. SYSTEM MODEL

We consider a one-way three-hop MIMO relaying system where a source node transmits information to a destination node with the aid of two relaying groups. Fig. 1 illustrates the considered scenario. The source and destination nodes are equipped with N and M antennas, respectively. There are two groups of single-antenna amplify-and-forward (AF) relays with M_1 and M_2 relays in the first and second groups, respectively. In this paper we assume that the relays operate in a half-duplex mode (i.e., each relay does not receive and transmit signals simultaneously) and due propagation path-loss, the signals transmitted by the source node cannot be received by the destination node.

The communication process is divided into three phases. In the first transmission phase, the source node transmits a signal vector $\boldsymbol{x} \in \mathbb{C}^{N \times 1}$ and the signal is received by the first and second relaying groups. In the discrete-time baseband notation, the signal received by the first relaying group is given by:

$$\boldsymbol{y}_{r_1} = \boldsymbol{H}_{sr_1}\boldsymbol{x} + \boldsymbol{v}_{r_1} \in \mathbb{C}^{M_1 \times 1}$$
(1)

where $\boldsymbol{H}_{\mathrm{sr}_1} \in \mathbb{C}^{M_1} \times N$ is the channel matrix linking the source to the relays of the first group, while $\boldsymbol{v}_{\mathrm{r}_1}$ is the corresponding additive Gaussian noise vector.

In the second transmission phase, the relays of the first group amplify the received signal vector (1) by means of a diagonal $M_1 \times M_1$ matrix F and forward the result to the second relaying group and to the destination. The signals received at the second relaying group and destination node are given, respectively, by:

$$\boldsymbol{y}_{r_2} = \boldsymbol{H}_{r_1 r_2} \boldsymbol{F} \boldsymbol{H}_{sr_1} \boldsymbol{x} + \boldsymbol{H}_{r_1 r_2} \boldsymbol{F} \boldsymbol{v}_{r_1} + \boldsymbol{v}_{r_2} \in \mathbb{C}^{M_2 \times 1} \quad (2)$$

$$\boldsymbol{y}_{\mathrm{d},1} = \boldsymbol{H}_{\mathrm{r}_{1}\mathrm{d}}\boldsymbol{F}\boldsymbol{H}_{\mathrm{sr}_{1}}\boldsymbol{x} + \boldsymbol{H}_{\mathrm{r}_{1}\mathrm{d}}\boldsymbol{F}\boldsymbol{v}_{\mathrm{r}_{1}} + \boldsymbol{v}_{\mathrm{d},1} \in \mathbb{C}^{M \times 1} \quad (3)$$

where $\boldsymbol{H}_{r_1d} \in \mathbb{C}^{M \times M_1}$ and $\boldsymbol{H}_{r_1r_2} \in \mathbb{C}^{M_2 \times M_1}$ are the channel matrices that model, respectively, the links from the first relaying group to the destination and from the first relaying group to the second relaying group, respectively.

Finally, in the third transmission phase, the relays of the second group amplify the received signal vector (2) using a diagonal $M_2 \times M_2$ matrix **G** and forward the result to the destination node. In this phase, the signals received at the destination node are then given by:

$$\boldsymbol{y}_{\mathrm{d},2} = \boldsymbol{H}_{\mathrm{r}_{2}\mathrm{d}}\boldsymbol{G}\boldsymbol{H}_{\mathrm{r}_{1}\mathrm{r}_{2}}\boldsymbol{F}\boldsymbol{H}_{\mathrm{sr}_{1}}\boldsymbol{x} + \boldsymbol{H}_{\mathrm{r}_{2}\mathrm{d}}\boldsymbol{G}\boldsymbol{H}_{\mathrm{r}_{1}\mathrm{r}_{2}}\boldsymbol{F}\boldsymbol{v}_{\mathrm{r}_{1}} \\ + \boldsymbol{H}_{\mathrm{r}_{2}\mathrm{d}}\boldsymbol{G}\boldsymbol{v}_{\mathrm{r}_{2}} + \boldsymbol{v}_{\mathrm{d},2} \in \mathbb{C}^{M \times 1}$$
(4)

III. PROPOSED APPROACH

As in [9] and [17], we divide the training phase into K timeblocks and, for all K time-blocks, the same training sequence matrix $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_L] \in \mathbb{C}^{N \times L}$ is transmitted by the source node and the signals are conveyed from the source to the destination using the protocol described in Section II, where Lis the number of symbols transmitted by the source node at the *k*-th time-block. We assume that the training sequence matrix is orthogonal $(\mathbf{X}\mathbf{X}^H = \mathbf{I}_N)$ and that all the channels are quasi-static, i.e. the channel coefficients are constant during the K time blocks. Let $\mathbf{W} \in \mathbb{C}^{K \times M_1}$ and $\mathbf{T} \in \mathbb{C}^{K \times M_2}$ be matrices whose rows contain the amplification factors used by the relays of the first and second groups, during the K time blocks. Thus, we can recast the data model in the destination node (equations (3) and (4)) by collecting the L received data vectors into two matrices as follows:

$$\begin{aligned} \boldsymbol{Y}_{d,1}^{(k)} &= \boldsymbol{H}_{r_{1}d} D_{k}(\boldsymbol{W}) \boldsymbol{H}_{sr_{1}} \boldsymbol{X} + \boldsymbol{H}_{r_{1}d} D_{k}(\boldsymbol{W}) \boldsymbol{V}_{r_{1}} \\ &+ \boldsymbol{V}_{d} \in \mathbb{C}^{M \times L}, k = 1, \dots, K \end{aligned} \tag{5} \\ \boldsymbol{Y}_{d,2}^{(k)} &= \boldsymbol{H}_{r_{2}d} D_{k}(\boldsymbol{T}) \boldsymbol{H}_{r_{1}r_{2}} D_{k}(\boldsymbol{W}) \boldsymbol{H}_{sr_{1}} \boldsymbol{X} \\ &+ \boldsymbol{H}_{r_{2}d} D_{k}(\boldsymbol{T}) \boldsymbol{H}_{r_{1}r_{2}} D_{k}(\boldsymbol{W}) \boldsymbol{V}_{r_{1}} + \boldsymbol{H}_{r_{2}d} D_{k}(\boldsymbol{T}) \boldsymbol{V}_{r_{2}} \\ &+ \boldsymbol{V}_{d} \in \mathbb{C}^{M \times L}, k = 1, \dots, K \end{aligned}$$

where $D_k(.)$ is the operator that constructs a diagonal matrix by selecting the *k*th row and putting it on the main diagonal. The matrices **W** and **T** should be of rank M_1 and M_2 , respectively, which means the amplification factors are assumed to change between time blocks.

A. Tensor-Based Formulation

At the destination node, multiplying both sides of (5) by \mathbf{X}^{H} from the right-hand side, we obtain $\tilde{\mathbf{P}}^{(k)} = \mathbf{P}^{(k)} + \mathbf{V}_{1}^{(k)} \in \mathbb{C}^{M \times N}, k = 1, \dots, K$, where $\mathbf{P}^{(k)} = \mathbf{H}_{r_{1}d}D_{k}(\mathbf{W})\mathbf{H}_{sr_{1}}$ is the signal component for our data model and $\mathbf{V}_{1}^{(k)} = \mathbf{H}_{r_{1}d}D_{k}(\mathbf{W})\mathbf{V}_{r_{1}}\mathbf{X}^{H} + \mathbf{V}_{d}\mathbf{X}^{H}$ is the noise component. Our model can be expressed in compact form using the tensor notation. To this end, let us introduce [18]

$$\boldsymbol{\mathcal{P}} = [\boldsymbol{P}^{(1)} \sqcup_3 \boldsymbol{P}^{(2)} \dots \sqcup_3 \boldsymbol{P}^{(K)}] \in \mathbb{C}^{M \times N \times K}, \qquad (7)$$

which can be expressed as

$$\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{I}}_{3,M_1} \times_1 \boldsymbol{H}_{r_1 d} \times_2 \boldsymbol{H}_{sr_1}^T \times_3 \boldsymbol{W}, \qquad (8)$$

where \mathcal{I}_{3,M_1} is the identity tensor of size $M_1 \times M_1 \times M_1$. This is the parallel factor (PARAFAC) decomposition of \mathcal{P} [16]. The 1-mode and 2-mode unfoldings of the tensor (8) are given, respectively, by:

$$[\boldsymbol{\mathcal{P}}]_{(1)} = \boldsymbol{H}_{r_1 d} (\boldsymbol{W} \diamond \boldsymbol{H}_{sr_1}^T)^T \in \mathbb{C}^{M \times NK}, \qquad (9)$$

$$[\boldsymbol{\mathcal{P}}]_{(2)} = \boldsymbol{H}_{\mathrm{sr}_1}^T (\boldsymbol{W} \diamond \boldsymbol{H}_{\mathrm{r}_1 \mathrm{d}})^T \in \mathbb{C}^{N \times MK}.$$
 (10)

It is worth mentioning that the estimates of the matrix pair $\{H_{r_1d}, H_{sr_1}\}$ could be obtained from the PARAFAC decomposition of the tensor in (8), by means of a standard ALS-PARAFAC fitting algorithm, as in [17]. However, by exploiting cooperative diversity, our aim is to combine each one of these tensors with that obtained via the complete source-relay-destination link, as explained in the sequel. Such a combined approach generally translates into more relaxed identifiability conditions and an improved performance, as will be corroborated later.

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Multiplying both sides of the equation (6) by \boldsymbol{X}^{H} from the right-hand side yields $\tilde{\boldsymbol{R}}^{(k)} = \boldsymbol{R}^{(k)} + \boldsymbol{V}_{2}^{(k)} \in \mathbb{C}^{M \times N}, k = 1, \ldots, K$, where

$$\boldsymbol{R}^{(k)} = \boldsymbol{H}_{r_2 d} D_k(\boldsymbol{T}) \boldsymbol{H}_{r_1 r_2} D_k(\boldsymbol{W}) \boldsymbol{H}_{s r_1} \in \mathbb{C}^{M \times N}$$
(11)

is the signal component of our data model and $V_2^{(k)} = H_{r_2d}D_k(T)H_{r_1r_2}D_k(W)V_{r_1}X^H + H_{r_2d}D_k(T)V_{r_2}X^H + V_dX^H$ is the noise component. Note that (11) is a frontal slice of a PARATUCK2 tensor [16].

Let us introduce $\mathbf{r}_k \triangleq \operatorname{vec}(\mathbf{R}^{(k)}) \in \mathbb{C}^{MN \times 1}$. Applying the property $\operatorname{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\operatorname{vec}(\mathbf{B})$, and using the equivalence $\operatorname{vec}(D_k(\mathbf{T})\mathbf{H}_{r_1r_2}D_k(\mathbf{W})) = \operatorname{Diag}(\operatorname{vec}(\mathbf{H}_{r_1r_2}))(\mathbf{W}_{(k,:)}^T \otimes \mathbf{T}_{(k,:)}^T)$, we get

$$oldsymbol{r}_k = (oldsymbol{H}_{ ext{sr}_1}^T \otimes oldsymbol{H}_{ ext{r}_2 ext{d}}) ext{Diag}(ext{vec}(oldsymbol{H}_{ ext{r}_1 ext{r}_2}))(oldsymbol{W}_{(k,:)}^T \otimes oldsymbol{T}_{(k,:)}^T)$$

Defining $\boldsymbol{R} \triangleq [\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_K]^T \in \mathbb{C}^{K \times MN}$, we obtain

$$\boldsymbol{R} = (\boldsymbol{W}^T \diamond \boldsymbol{T}^T)^T \text{Diag}(\text{vec}(\boldsymbol{H}_{r_1 r_2})) (\boldsymbol{H}_{\text{sr}_1}^T \otimes \boldsymbol{H}_{r_2 d})^T \\ \in \mathbb{C}^{K \times MN}.$$
(12)

Comparing (12) with (9) and (10), R can be viewed as the 3-mode unfolding of a tensor \mathcal{R} , which admits the following structured *Tucker3* decomposition [16]:

$$\boldsymbol{\mathcal{R}} = \boldsymbol{\mathcal{H}} \times_1 \boldsymbol{H}_{r_2 d} \times_2 \boldsymbol{H}_{sr_1}^T \times_3 (\boldsymbol{W}^T \diamond \boldsymbol{T}^T)^T \in \mathbb{C}^{M \times N \times K},$$
(13)

where $\mathcal{H} \in \mathbb{C}^{M_2 \times M_1 \times M_1 M_2}$ is the corresponding core tensor, the 3-mode unfolding of which is given by $[\mathcal{H}]_{(3)} = \text{Diag}(\text{vec}(\mathcal{H}_{r_1r_2}))$. We are interested in the 1-mode and 2-mode unfoldings of this tensor that are given by

$$[\mathcal{R}]_{(1)} = \boldsymbol{H}_{r_{2}d}[\mathcal{H}]_{(1)}((\boldsymbol{W}^{T} \diamond \boldsymbol{T}^{T})^{T} \otimes \boldsymbol{H}_{sr_{1}}^{T})^{T} \in \mathbb{C}^{M \times NK}$$

$$(14)$$

$$[\mathcal{R}]_{(2)} = \boldsymbol{H}_{sr_{1}}^{T}[\mathcal{H}]_{(2)}((\boldsymbol{W}^{T} \diamond \boldsymbol{T}^{T})^{T} \otimes \boldsymbol{H}_{r_{2}d})^{T} \in \mathbb{C}^{N \times MK}$$

$$(15)$$

B. Channel Estimation Algorithm

Let $\boldsymbol{r} \triangleq \operatorname{vec}(\boldsymbol{R}^T) \in \mathbb{C}^{MNK \times 1}$. From (12) we have:

$$\boldsymbol{r} = ((\boldsymbol{W}^T \diamond \boldsymbol{T}^T)^T \diamond (\boldsymbol{H}_{\mathrm{sr}_1}^T \otimes \boldsymbol{H}_{\mathrm{r}_2\mathrm{d}})) \mathrm{vec}(\boldsymbol{H}_{\mathrm{r}_1\mathrm{r}_2}), \qquad (16)$$

where we have applied again the property $\operatorname{vec}(\boldsymbol{ABC}) = (\boldsymbol{C}^T \otimes \boldsymbol{a})\operatorname{vec}(\boldsymbol{B})$. Defining $\boldsymbol{h}_{r_1r_2} \triangleq \operatorname{vec}(\boldsymbol{H}_{r_1r_2}) \in \mathbb{C}^{M_1M_2 \times 1}$, (16) leads to the following LS estimate of $\boldsymbol{H}_{r_1r_2}$:

$$\hat{\boldsymbol{h}}_{r_1r_2} = \left[\left(\boldsymbol{W}^T \diamond \boldsymbol{T}^T \right)^T \diamond \left(\hat{\boldsymbol{H}}_{sr_1}^T \otimes \hat{\boldsymbol{H}}_{r_2d} \right) \right]^{\dagger} \tilde{\boldsymbol{r}}, \qquad (17)$$

where $\tilde{\mathbf{r}}$ is the noisy version of (12) after adding the additive noise component and replacing the true channel matrices by the estimated ones. Combining (10) and (15) the following LS estimate of \mathbf{H}_{sr_1} can be obtained:

$$\hat{\boldsymbol{H}}_{\mathrm{sr}_{1}} = \begin{bmatrix} (\boldsymbol{W} \diamond \hat{\boldsymbol{H}}_{\mathrm{r}_{1}\mathrm{d}}) \\ ((\boldsymbol{W}^{T} \diamond \boldsymbol{T}^{T})^{T} \otimes \hat{\boldsymbol{H}}_{\mathrm{r}_{2}\mathrm{d}}) [\hat{\boldsymbol{\mathcal{H}}}]_{(2)}^{T} \end{bmatrix}^{\dagger} \begin{bmatrix} [\tilde{\boldsymbol{\mathcal{P}}}]_{(2)}^{T} \\ [\tilde{\boldsymbol{\mathcal{R}}}]_{(2)}^{T} \end{bmatrix} \quad (18)$$

where $\hat{\mathcal{P}}$ and $\hat{\mathcal{R}}$ are noisy versions of \mathcal{P} and \mathcal{R} , respectively, after adding the additive noise components and replacing the true channel matrices by the estimated ones. Moreover, from (14) we find a LS estimate of \boldsymbol{H}_{r_2d}

$$\hat{\boldsymbol{H}}_{r_{2}d} = [\tilde{\boldsymbol{\mathcal{R}}}]_{(1)} \Big[[\boldsymbol{\mathcal{H}}]_{(1)} ((\boldsymbol{W}^{T} \diamond \boldsymbol{T}^{T})^{T} \otimes \boldsymbol{H}_{sr_{1}}^{T})^{T} \Big]^{\dagger}.$$
(19)

TABLE I Comb-ALS Algorithm

<i>Initialization</i> : Set $i = 0$; Initialize randomly the matrices $H_{sr_1}^{(i=0)}$,
$\hat{H}_{\mathrm{r_1d}}^{(i=0)}$ and $\hat{H}_{\mathrm{r_2d}}^{(i=0)};$
1) Set $i = i + 1$.
2) Estimate $\hat{H}_{r_1r_2}^{(i)}$ using $\hat{H}_{sr_1}^{(i-1)}$ and $\hat{H}_{r_2d}^{(i-1)}$ from equation (17);
3) Estimate $\hat{H}_{sr_1}^{(i)}$ using $\hat{H}_{r_1d}^{(i-1)}$, $\hat{H}_{r_2d}^{(i-1)}$ and $\hat{H}_{r_1r_2}^{(i)}$ from
equation (18);
4) Estimate $\hat{H}_{r_2d}^{(i)}$ using $\hat{H}_{sr_1}^{(i)}$ and $\hat{H}_{r_1r_2}^{(i)}$ from equation (19).
Then, estimate $\hat{H}_{r_1d}^{(i)}$ using $\hat{H}_{sr_1}^{(i)}$ from equation (20);
5) Repeat steps 1 to 4 until convergence.

Similarly, from (9) we obtain a LS estimate of H_{r_1d} :

$$\hat{\boldsymbol{H}}_{r_{1}d} = \tilde{[\boldsymbol{\mathcal{P}}]}_{(1)} \left[\left(\boldsymbol{W} \diamond \hat{\boldsymbol{H}}_{sr_{1}}^{T} \right)^{T} \right]^{\dagger}.$$
(20)

Table I briefly summarizes the proposed algorithm, referred to as combined ALS (Comb-ALS). The per iteration complexity of the Comb-ALS estimator can be calculated by summing the individual complexities associated with the computation of the left pseudoinverses in (17)–(20) which involve, respectively, the inverses of matrices of dimensions $M_1M_2 \times M_1M_2$, $M_1 \times M_1$, $M_2 \times M_2$, and $M_1 \times M_1$.

C. Design of the AF Relaying Matrices and Identifiability

We choose $\boldsymbol{U} = (\boldsymbol{W}^T \diamond \boldsymbol{T}^T)^T \in \mathbb{C}^{K \times M_1 M_2}$ as a Vandermonde matrix whose columns are the first K rows of a $M_1 M_2 \times$ M_1M_2 DFT matrix. With U fixed, the individual AF matrices $W \in \mathbb{C}^{K \times M_1}$ and $T \in \mathbb{C}^{K \times M_2}$ are determined by means of a LS Khatri-Rao factorization procedure which consists of K successive rank-1 factorizations steps (see, in this case, [9]). The AF matrices are normalized such that $Tr\{D_k(\boldsymbol{W})^H D_k(\boldsymbol{W})\} =$ γ_1 and $\operatorname{Tr}\{D_k(\boldsymbol{T})^H D_k(\boldsymbol{T})\} = \gamma_2$, where $\gamma_i = E\{1/\|\boldsymbol{y}_{r_i}\|^2\}$, i = 1, 2, are long-term average amplification factors to ensure that the average transmit power constraint at the relays of the first and second group are not violated [2]. From the chosen design, the identifiability issue can be addressed. First, note that H_{sr_1} and H_{r_1d} is identifiable in the LS sense from (18) and (20), respectively, if we choose $K = \min(M_1, M_2)$. This means that at least one AF relaying matrix (i.e., W or T) must have full column rank. Without loss of generality, we assume that W has full column rank. On the other hand, the identifiability of $h_{r_1r_2}$ requires the Khatri-Rao product in (17) to be left-invertible. Since the channel matrices have full rank (e.g., i.i.d. channels), satisfying $KMN \ge M_1M_2$ ensures the identifiability of $h_{r_1r_2}$. Along the same lines, the identifiability of H_{r_2d} from (19) implies satisfying the condition $KN \ge M_2$. In this work we do not intend to optimize the AF matrices, since our interest in on channel estimation. Optimized designs are examined in [21].

IV. SIMULATION RESULTS AND DISCUSSION

We evaluate the performance of the Comb-ALS channel estimator through computer simulations. To this end, we assume a system where the source and destination have N = 2 and M = 2 antennas, respectively. The communication process is assisted by 2 relaying groups having 2 antennas each (i.e., $M_1 = M_2 = 2$). The Comb-ALS estimator is compared with a sequential estimation method that consists in first applying a PARAFAC-ALS stage to estimate H_{r_1d} , H_{sr_1} from the unfoldings (9) and (10), respectively, followed by a PARATUCK2-ALS stage to estimate $H_{r_1r_2}$ and H_{r_2d} from (17) and (19), respectively, conditioned on the previous estimation of H_{sr_1} . This estimator, herein referred

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Fig. 2. NMSE vs. training SNR for $K = 2, N = 2, M = M_1 = M_2 = 2$.

to as PARAFAC-PARATUCK2, has some similarity with the sequential PARAFAC/PARATUCK2 receiver proposed in [20] for a two-hop MIMO relaying system. Note that the PARAFAC-PARATUCK2 estimator does not combine the links from the two relaying groups towards the destination to estimate H_{sr_1} , in contrast to the Comb-ALS estimator. Note also that the Comb-ALS estimator has a similar complexity per iteration compared with the PARAFAC-PARATUCK2 estimator, the difference between these two algorithms is that $H_{\rm sr_1}$ is estimated from (18) in the Comb-ALS estimator and from (10) in PARAFAC/PARATUCK2 estimator, although in both cases, the pseudoinverse needs the computation of a $M_1 \times M_1$ inverse, thus the complexity of this step differs only in the number of multiplications (in the Comb-ALS algorithm the number of multiplications is twice that of the PARAFAC-PARATUCK2 estimator). This small increase in the complexity of the Comb-ALS algorithm is compensated by the improved performance as shown in our simulation results. We also compare our estimator with a conventional LS estimation method similar to that of [12], which sequentially estimate the matrices H_{r_2d} , $H_{r_1r_2}$ and H_{sr_1} , and H_{r_1d} . Since the conventional LS estimator needs 6 transmission phases to estimate all the channel matrices, we set K = 2 for the Comb-ALS and PARAFAC-PARATUCK2 estimators in order to have the same spectral efficiency for all the solutions. The average signal-to-noise ratio (SNR) at all the relays and the destination are assumed equal. In each Monte Carlo run, the channel and symbol matrices are normalized to have unit norm, while the variance of the noise term is set to ensure the desired SNR value. The AF matrices for the PARAFAC-PARATUCK2 and Comb-ALS are the same, while for the conventional LS method we assume identity matrices. All the channel matrices are normalized to ensure the transmit power constraint at the relays, for all methods. The accuracy of the channel estimation is evaluated in terms of the normalized mean square error (NMSE) given by NMSE $(\hat{H}_{eff}) = \frac{\|\hat{H}_{eff} - H_{eff}\|_F^2}{\|\hat{f}_{f}\|_F}$, where the $\|\hat{\pmb{H}}_{ ext{eff}}\|_{F}^{2}$ effective channel H_{eff} is defined in equation (21) on this page. The results are averaged over 5000 runs, each corresponding to a realization of all channel matrices and the noise tensor.

Fig. 2 depicts the NMSE of the effective channel as a function of the training SNR. Note that the proposed method yields more accurate channel estimates compared to the PARAFAC-PARATUCK2 and conventional LS methods. For high SNRs, the channel estimation error of the PARAFAC-PARATUCK2 receiver saturates with the increase of the SNR, while the Comb-ALS has a linear decrease in the NMSE, corroborating its superiority in terms of channel esti-



Fig. 3. BER vs. data SNR with a ZF receiver. Training SNR = 25 dB.

mation. The conventional LS estimator has a similar behavior in terms of NMSE compared to the proposed Comb-ALS estimator only for lower SNR values, while it exhibits an error floor as the SNR increases. Such an error floor is due to the accumulation of the residual channel estimation errors in the conventional LS estimator. This problem is not present in the Comb-ALS estimator since all the channel matrices are jointly estimated. Fig. 3 depicts the symbol error rate (SER) performance as a function of the SNR of the transmitted symbols, assuming a 4-QAM modulation and a zero forcing (ZF) receiver designed from the estimated channel matrices. To provide a reference, this figure also shows the results obtained with a ZF receiver operating under perfect knowledge of all the channels, and designed by combining the links from the two relaying groups the destination. The ZF solution can be written as

$$\hat{\boldsymbol{x}}_{ZF}(t) = \underbrace{\begin{bmatrix} \hat{\boldsymbol{H}}_{r_{1}d} \boldsymbol{F} \hat{\boldsymbol{H}}_{sr_{1}} \\ \hat{\boldsymbol{H}}_{r_{2}d} \boldsymbol{G} \hat{\boldsymbol{H}}_{r_{1}r_{2}} \boldsymbol{F} \hat{\boldsymbol{H}}_{sr_{1}} \end{bmatrix}}_{\boldsymbol{H}_{eff}}^{\mathsf{T}} \begin{bmatrix} \boldsymbol{y}_{d}(t+1) \\ \boldsymbol{y}_{d}(t+2) \end{bmatrix}, \quad (21)$$

where the estimated channel matrices may be obtained either with Comb-ALS, PARAFAC-PARATUCK2 or with the conventional LS estimators. We can see that Comb-ALS outperforms PARAFAC-PARATUCK2, being closer to the perfect CSI case for low and medium SNR values. Note also that the proposed estimator exhibits a SER performance similar to the sequential LS method. Assuming that orthogonal training sequences are used in all training stages, the LS method requires $M \geq M_2$ and $M \geq M_1$ to estimate the first three channel matrices and $N \ge M$ to estimate \boldsymbol{H}_{r_1d} , from which we conclude that $N \ge M \ge \max(M_1, M_2)$ is necessary. On the other hand, the proposed tensor-based method does not impose such a restriction on the antenna configuration, resulting in more flexible operation conditions. The performance gain requires a slightly higher computational complexity associated with matrix inversions in the channel estimation algorithm.

V. SUMMARY

The proposed joint channel estimator for three-hop MIMO relaying systems is based on alternating least squares estimation by coupling PARAFAC and *Tucker3* tensor models for the received signals. Our results corroborate the improved performance of the proposed estimator over the PARAFAC-PARATUCK2 and conventional LS competitors, while operating in less restrictive antenna configurations. A perspective of this work is the use of orthogonal space-time block codes at the source and/or the relays [22], [23].

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